Surface Gravity Anomalies Due to Buried Bodies

Simple analytical solution may be derived for bodies with uniform density contrast simple shape, such as:

- Sphere
- Horizontal/vertical cylinders
- Infinite or semi-infinite plates
- And more ...
Last week we obtained a general solution:

\[ \Delta g_z = \gamma \Delta \rho \int \frac{1}{r^2} \cos \alpha dV \]

where: \( \gamma \) is the gravitational constant, \( \Delta \rho \) is the density anomaly, \( r \) is the distance to the observation point, \( \alpha \) is the angle between the vertical and \( r \), and \( V \) is the volume.

**Question:** Express the gravity anomaly in Cartesian coordinates.
In Cartesian coordinates:

\[ \Delta g_z = \gamma \Delta \rho \frac{z}{(x^2 + z^2)^{3/2}} \int \int \int dx dy dz \]

A sphere:

\[ \Delta g_z = \gamma \Delta \rho \frac{z}{(x^2 + z^2)^{3/2}} \frac{4\pi a^3}{3} \]
A 2D vertical column:

\[ \Delta g_z = 2\gamma \Delta \rho b \ln \left( \frac{r^2}{r^1} \right) \]

A series of 2D vertical columns:

\[ \Delta g_z = 2\gamma \sum_i \Delta \rho_i b_i \ln \left( \frac{r^{2_i}}{r^{1_i}} \right) \]
In the preceding pages we looked at the results of forward modeling.

In practice, however, inverse modeling is of greater importance.

**Question:** Can the data be inverted to obtain the density, size and shape of the buried anomaly?
Inspection of the solution for a buried sphere reveals a non-uniqueness of that problem. The term $\Delta \rho a^3$ introduces an ambiguity to the problem. This is because there are infinite combinations of $\Delta \rho$ and $a^3$ that can give the same $\Delta \rho a^3$.

This highlights the importance of adding geological and geophysical constraints!
Question: What should be the size of the surveyed region?

Going back to the sphere anomaly, we see that the signal due to a sphere buried at depth $Z$ can only be fully resolved at distance out to 2-3 $Z$. 

![Diagram](image_url)
Thus, to resolve details of density structure of the lower crust (at say 20-40 km), gravity measurements must be made over an extensive area, as the width of the these anomalies is much larger than the width of upper crust anomalies.
Gravitational acceleration on a rotating oblate spheroid

Question The acceleration of gravity is greater on the poles or on the equator?
The Earth is an oblate spheroid
- fatter at the equator and thinner at the poles
- excess mass under the equator

The Earth is rotating. Centrifugal acceleration reduces gravitational attraction. The further you are from the rotation axis, the greater the centrifugal acceleration is.

\[ g = 9.83 \text{ m/s}^2 \]

\[ g = 9.78 \text{ m/s}^2 \]
Geoid Anomaly

Reference Geoid is a mathematical figure whose surface is an equipotential of the theoretical gravity field of a symmetric spheroidal earth model with realistic radial variations in density, plus the centrifugal potential. The international gravity formula that gives the value of $g$ on such a spheroid is:

$$g(\lambda) = g_e(1 + \alpha \sin^2 \lambda + \beta \sin^4 \lambda)$$

where:

$g_e$ is the gravitational acceleration at the equator

$\lambda$ is the latitude

$\alpha = 5.278895^{-3}$

$\beta = 2.3462^{-5}$
Geoid is the measured reference equipotential surface that defines the sea level.
Geoid Anomaly is the difference in elevation between the measured geoid and the reference geoid.

Question what are the units of geoid anomaly?
Geoid height anomaly

This shows that deviations between the Geoid (observed) and the reference Geoid (mathematically calculated) are as large as 80 meters.
SEASAT map (showing water topography)

Largest features are associated with the trenches. This is because 10 km deep and filled with water rather than rock.
Corrections

Geoid anomaly contains information regarding the 3-D mass distribution. But first, a few corrections should be applied.

- Free-air correction
- Bouguer
- Terrain
Free-Air correction, $\delta g_{FA}$:

This correction accounts for the fact that the point of measurement is at elevation $h$ rather than at the sea level on the reference spheroid.
Since:
\[ g(h) = g(\lambda) \left( \frac{R(\lambda)}{R(\lambda) + h} \right)^2 \approx g(\lambda) \left( 1 - \frac{2h}{R(\lambda)} \right), \]
where \( g(\lambda) \) is the gravity at sea level of the reference spheroid, and \( R(\lambda) \) is the radius of the reference spheroid at \( \lambda \).

The free-air correction \( \delta g_{FA} \), which is to be added to the measured value in order to correct it to the sea-level value is:
\[ \delta g_{FA} = g(\lambda) - g(h) = g(\lambda) \frac{2h}{R(\lambda)} \]

This correction amounts to \( 3.1 \times 10^{-6} \) m s\(^{-2} \) per meter of elevation.
The free-air anomaly $g_{FA}$ is the measured anomaly, $g_{obs}$, with the two corrections applied:

$$g_{FA} = g_{obs} - g(\lambda) + \delta g_{FA}$$
Bouguer correction, $\delta g_B$:

This correction accounts for the gravitational attraction of the rocks between the point of measurement and the sea level:

$$\delta g_B = 2\pi \gamma \rho h,$$

where $\gamma$ is the constant of gravitation, $\rho$ is the density of material point and the sea-level, and $h$ is the height of measurement point above sea-level.
For crustal density of $2.7 \times 10^3$ kg m$^{-3}$, we obtain a Bouguer correction of $1.1 \times 10^{-6}$ m s$^{-2}$ per meter of elevation.

- **Terrain correction, $\delta g_T$:**

This correction accounts for the deviation of the surface from an infinite horizontal plane. It is calculated analytically by use of templates and topographic maps.

The terrain correction is small, and except for area of mountainous terrain, can often be ignored.
The Bouguer anomaly $g_B$ is the measured anomaly, $g_{obs}$, with the four corrections applied:

$$g_B = g_{obs} - g(\lambda) + \delta g_{FA} - \delta g_B + \delta g_T$$

Bouguer anomaly for offshore gravity:

- Replace the water with rock
- Apply terrain correction for seabed topography

After subtracting these effects, the ”corrected” signal contains information regarding the 3-D distribution of mass in the earth interior. But the ambiguity is still there...