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1 Introduction

The field of cold atoms in physics is only about 20 years old. First, laser cooling of atoms was developed in the 80’s of the last century by Steven Chu (1; 2), Claude Cohen-Tannoudji (3) and William D. Phillips (4) and they are the 1997 Nobel prize laureates. Then, atoms were cooled to nano-Kelvin temperatures using the method of evaporative cooling. This enabled the observation of the phase transition which is called Bose-Einstein condensation (BEC). This phase transition was observed in a series of experiments with vapors of rubidium ($^{87}\text{Rb}$) (5) and sodium ($^{23}\text{Na}$) (6). The Nobel prize of 2001 was given to Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman for this accomplishment, which confirmed a 1925 prediction by Bose (7) and Einstein (8; 9).

A BEC can be held in vacuum by external magnetic and light potentials. The light potentials may be created with a micron resolution by focusing the beams or creating lattices with standing waves. These kinds of potentials led to different directions such as using the cold atoms to simulate solid state transitions, such as the Mott insulator transition (10) and Anderson localization (11; 12). Magnetic potentials, on the other hand, are created from outside the vacuum, and hence complicated and fine potentials cannot be created. A solution is enabled by bringing the magnetic field sources into the vacuum, very close to the atoms. This is done with atomchips which are the topic of this research proposal.

Atomchips (13; 14; 15; 16) enable the engineering of complicated potentials for manipulating atomic quantum states, both external and internal degrees of freedom, including beamsplitters, interferometers, lattices etc. (17; 18; 19). Bringing the atoms close to the atomchip surface, near the sources of these potentials, enables tight traps with low power consumption, and may enable a new tool for fundamental studies as well as numerous applications such as clocks, sensors and quantum information processing. Indeed, interferometry with both external and internal degrees of freedom, has already been demonstrated. However, as the atoms approach the dielectric or metallic surface, they are perturbed by atom-surface interactions and by temporal and spatial magnetic field fluctuations. On one hand, this enables surface microscopy studies using ultra-cold atoms (20; 21; 22) and studies of dispersion forces, including the Casimir-Polder interaction (23; 24; 25; 26; 27), but on the other hand this destroys atomic coherence and introduces heating, trap loss and potential corrugation (28; 29; 30).
Part I

Present research summary

In this part, I present the experiment we have planned and built in the past 2 years. I start by describing the experimental apparatus - the vacuum and lasers setups, the computer control of the experiment and our imaging system. Following that I explain and give experimental examples of the different stages needed to achieve the phase transition to Bose-Einstein condensation (BEC). I conclude this part by presenting experimental data from our new system showing clear evidence that we have reached the phase transition.

2 Experimental setup

Bose-Einstein condensation in dilute gases happens when the atomic sample, in the form of a dilute gaseous cloud, is cooled until all of the atoms occupy the ground state of the trap. This extreme situation sets the constraints when designing the new experimental apparatus. For example, the cooling is done by laser cooling and RF evaporative cooling (as discussed in sections 3 and 4.3), which requires optical access for the laser beams while requiring 3 pairs of coils to create the necessary magnetic fields. In addition, as room temperature background atoms and molecules may heat the sample upon collision, the cooled atomic sample can exist only in a ultra-high vacuum (UHV) environment, thus the experiment must be done under such conditions (Sec. 2.1). In our system we reach $10^{-12}$ mbar.

The initial magnetic trapping of the atoms can be done in a static trap, but manipulating the cloud requires changing many parameters at specific times. We do that by controlling the electronic equipment (which controls the lasers, magnetic fields, etc.) with analog and digital signals and those signals are controlled with a complex computer control that has both special software and dedicated hardware that works with a precision of $1\mu$s (Sec. 2.4). After trapping and manipulating the atoms, we extract the data by imaging the atoms and analyzing the resolved image (Sec. 2.5).

2.1 Vacuum

Our vacuum system (Fig. 1) is made of a 6-way cross with all the other vacuum parts connected to this cross. The frame is designed to hold rigidly the vacuum system and the magnetic coils with as less vibrations as possible together with being far enough so there is easy optical access from all directions. To the bottom of the 6-way cross we connect the science chamber and from the top we insert the mount with the atomchip (Sec. 2.3). On two of the sides we have the molecular turbo pump and the ion pump. The last two sides have a titanium sublimation pump (TSP) and a blank. In Fig. 1(b) the scientific chamber is on the bottom and the ion pump (the TSP) is on the right (left).

2.2 Lasers

A $^{87}$Rb BEC experiment needs laser beams with four different frequencies (cooler, repumper, optical pumping and imaging). Because an Acousto-Optic modulator (AOM) can change the laser light frequency up to a few hundred MHz, only two lasers are needed. The main laser, a TOPTICA DLX110, can give up to 1W, but even the 700mW we operate at are more...
2 EXPERIMENTAL SETUP

Figure 1: Design and a picture of the vacuum setup from January, 2007. (a) The design shows the vacuum parts (gray) held by a frame (dark blue) and 3 pairs of coils (cyan). The vacuum parts that are shown are the 6-way cross, the science chamber and a short nipple from the side of the chamber for the $^{87}\text{Rb}$ dispensers. (b) Picture of the setup from the same angle. In addition to the design, a long nipple for the ion pump and another one for the titanium sublimation pump (TSP) are connected to the 6-way cross. Magnetic coils and optics are not assembled yet.

than enough for the main cooling beam and for the optical pumping and imaging beams. The second laser, used for the repumper, is home made and its power is $\sim 50\text{mW}$. Fig. 2(a) presents the laser box and the beams’ path (each laser has polarization spectroscopy for locking).

Outside of the lasers box, the main beam is divided into three different beams. Each one of the beams passes through an AOM (the cooler and imaging beams pass twice through their AOMs). Every AOM shifts the light frequency, and then the light is coupled to an optical fiber. The repumper beam also passes through an AOM, and is then coupled into the cooler fiber. This is done because for the laser cooling stage (Sec. 3) both of the beams are required, and while the repumper can be delivered independently, a joint optical path to the chamber is simpler.

Each beam is focused into the AOM and has also a mechanical light shutter\(^1\) to close the light completely. The focusing is important for two reasons: (i) When the light is focused into the AOM, the closing and opening of the light is faster, but the total efficiency of the first order\(^2\) is lower. (ii) The more important reason for focusing is that after the second lens, which re-collimates the beam, all the orders are parallel, regardless of their frequency shift. This fact, especially in the cooler beam which reflects into the AOM for a second pass, provides the ability to change the frequency of the laser light with the AOM while not moving it in space. This is important because after the AOM the beam is coupled into an optical fiber, and if the beam will move even slightly in space, the coupling efficiency will drop dramatically. Even though the AOM can close the light very fast, it cannot close it completely. This is because the applied RF signal to the AOM can only be reduced and not

---

\(^1\)The mechanical shutters are made from small speakers\(^{[31]}\).

\(^2\)When the laser beam passes through the AOM, it interacts with the phonons inside the crystal, and is split into different orders. The zero order is the same as the input beam, both in direction and frequency. The first order (the one used for the experiment), has some angle relative to the main beam and its frequency is shifted from the main beam frequency by the same amount as the RF frequency fed into the AOM crystal. Its amplitude is controlled by the amplitude of the RF signal.
2 EXPERIMENTAL SETUP

Figure 2: Lasers box, (a), and AOMs setup, (b). (a) A TOPTICA DLX110 is used for the main laser beam (cooler, optical pumping and imaging) and for the repumper beam a home made laser is utilized. The beam paths (including both of the spectroscopies) are shown with red (DLX110) and blue (home made) arrows. (b) The four lines of focusing lenses, mechanical shutters and AOMs are shown. From top to bottom - repumper, cooler, imaging and optical pumping.

Figure 3: AOMs setup. Outside of the lasers box the beams (here only the cooler and repumper are shown) are manipulated in both the frequencies and amplitudes with AOMs and then injected into fibers.

turned off. The small leakage in the RF signal from the AOM driver causes some leakage of laser light to the first order. Some light is also simply reflected to the first order from dirt and imperfections in the crystal. We therefore use the mechanical shutters to block completely this small leakage.

Fig. 3 is a scheme of the cooler and repumper beams paths and Fig. 2(b) shows a picture of the whole optical system, including AOM lines and optical fibers. In Fig. 4 the four laser beams are plotted with respect to the $^{87}$Rb D2 transition hyperfine structure. It illustrates how the AOMs shift differently the cooler, imaging and optical pumping beams from the locking point of the laser.

2.3 The atomchip mount

The atomchip is the most important part in the experiment. It contains all the current carrying wires (gold) creating the magnetic fields for the experiments with the BEC, and is also used as a mirror for the Magneto-Optical trap (MOT) (Sec. 3.2). For these two reasons,
2 EXPERIMENTAL SETUP

Figure 4: The four laser beam frequencies are superposed on a scheme of $^{87}$Rb D2 transition hyperfine structure. The inset shows an absorption spectroscopy signal (top) compared to a polarization spectroscopy signal (bottom).

Figure 5: The atomchip mount. (a) A picture of the copper structure before the atomchip was glued on the white Macor. (b) A picture of the atomchip mount. The copper rods carry the high current for the copper structure while the ribbon is made of thin wires for the atomchip. (c) A zoom on the top of the mount. The chip wires are bonded to pins which are connected to the ribbon wires. The copper rods are connected to the copper structure that is hidden beneath the atomchip.

it needs to be held, rigidly, in the middle of the science chamber. In our experiment, the chip is positioned on the top of a mount, which is inserted into the vacuum from the top part of the 6-way cross (see Sec. 2.1 and Fig. [1]).

The mount is shown in Fig. 5. It was designed in our group and was produced in the workshop of the university. It uses a CF160 blank flange, where 8 holes for high current feed-troughs and 1 for 35-pin connector - were drilled into it. After drilling the holes, the feed-through and the 35-pin connector were welded, by UHV welding, to the blank. The high current feed-troughs are used for the copper wires shown in Fig. 5(a). These copper wires create the robust potentials needed for the MOT and initial magnetic trap stages (Sec. [32]
2 EXPERIMENTAL SETUP

Figure 6: Experimental control interface. The window is divided into two main parts. The left one controls the analog channels and the right one controls the digital channels.

and Sec. 4.2, respectively). The small wires, shown as a ribbon in Fig. 5(b), and which can carry up to 2A of current, connect the chip wires (Fig. 5(c)) to the 35-pin connector.

2.4 Experimental control

The different experimental stages require accurate control, both in time and value, of many electronic devices. These devices usually can be controlled by analog or digital signals. We use a dedicated computer, National Instruments PXI, that runs in real time mode and sends the signals that control the experimental sequence. In addition, we use a LabView program, whose interface is shown in Fig. 6 to load the experimental sequence to the PXI. This interface was written by an electronic engineer who works in our group, and it can control analog and digital channels. Digital channels just need to be set to an ON/OFF state, with any arbitrary time sequence. Analog channels are more complicated. We can control them by linear ramps only, where each ramp needs a start time, change time and end value. After inserting the wanted data, the program will collect it, organized it in the correct format and transmit it to the PXI.

2.5 Imaging

After trapping and manipulating the atomic cloud, its properties are measured. This is done by imaging the cloud with laser light. When light propagates through the atomic cloud, it is absorbed and phase shifted (33):

\[ \vec{E} = t\vec{E}_0 e^{i\phi}. \]

The transmission, \( t \), and the phase shift, \( \phi \), are:

\[ t = e^{-\tilde{D}/2} = \exp\left(-\tilde{n}\sigma_0 \frac{1}{2} \frac{1}{1 + \delta^2}\right), \quad \text{and} \]

\[ \phi = -\delta \frac{\tilde{D}}{2} = -\frac{\tilde{n}\sigma_0 \delta}{2} \frac{1}{1 + \delta^2}. \]
2 EXPERIMENTAL SETUP

Figure 7: Imaging scheme. Red lines represent the imaging laser beam and the blue lines represent the "shadow". The main magnification imaging axis uses the \( f_1 \) and \( f_2 \) lenses. Side imaging, with lower magnification, can be done by putting a mirror and diverting the beam (red and blue dotted lines) to a side camera with shorter focal length lens, \( f_3 \). Putting the mirror exactly in the focus is not a necessity.

\[ \tilde{D} = \frac{\tilde{n}\sigma_0}{1 + \delta^2} \] is the off-resonance optical density, \( \tilde{n} = \int n \cdot dz \) is the column density, \( \sigma_0 \) is the resonance cross-section\(^3\) and \( \delta = \frac{\omega - \omega_0}{\Gamma/2} \) is the detuning of the laser light.

These two effects lead to different imaging techniques. In one type of method, off-resonance laser light illuminates the cloud, so that the atoms hardly absorb the light (so they’re not heated and can be imaged again), while they still give rise to a measurable phase shift. This kind of imaging is called non-destructive because several images of the same atomic cloud can be taken. Phase-contrast and dark-ground imaging are such methods \(^3\).

Another option is to shine an on-resonance laser light on the atoms. This time, the atoms will absorb a non-negligible amount of light and re-emit it in a random direction, through spontaneous emission. Florescence imaging is when the re-emitted light is being collected into a CCD camera or a photo-diode. Alternatively, it is possible to collect the “missing” light, the shadow. This method is called absorption imaging and this is the main method we are using in our lab.

In absorption imaging, the laser light that passes through the atomic cloud, is magnified (or demagnified, depending on the object size) and projected onto a CCD camera. Since a CCD camera measures the intensity of the light, eq. \([4]\) becomes Beer’s law:

\[ I(x, y) = I_0(x, y) e^{-\tilde{D}(x, y)} \]  

\( x \) and \( y \) stress the fact that this operation is done for each pixel on the CCD (\( z \) is the imaging axis and is being integrated upon).

Fig. [7] shows the scheme of our imaging system which is capable of imaging in two different magnifications (not simultaneously) by adding or removing a mirror\(^4\).

2.5.1 Analysis of the image

In eq. \([4]\), \( \tilde{D} \) is the parameter that holds the information on the atomic cloud. In order to extract \( \tilde{D} \), two images are needed - one for \( I \), and one for \( I_0 \). \( \tilde{D} \) is the logarithm of the ratio of these two images. This procedure is done for each pixel, and the outcome is the absorption

\(^3\)for two-level atoms \( \sigma_0 = \frac{6\pi\lambda^2}{\hbar} \), and if more levels exist, \( e.g. \), Zeeman sublevels, several resonances can contribute (see \([32]\)).

\(^4\)This mirror is held by a magnetic stage so that it may be returned every time to the same position, without any need of realignment.
2 EXPERIMENTAL SETUP

Figure 8: Absorption imaging. In all, the \( y = 0 \) is the position of the atomchip. (a) An image of the laser beam after propagating through the atomic cloud. The shadow of the cloud is clearly visible. (b) The corresponding background image (no atoms). (c) The resulting absorption image. Even though both of the images (with atoms and background) contain many fringes, these are canceled when taking the ratio and the absorption image is smooth and clean.

image. \( I \) is recorded after the laser beam propagates through the cloud, and it contains the shadow of the cloud. \( I_0 \) is the laser intensity before propagating through the cloud. This background image is taken by shining the laser, exactly in the same experimental procedure as was done for \( I \), but a second after releasing the atoms, so there are no atoms in the image. We assume that the laser beam didn’t change its properties (e.g., spatial distribution, intensity, etc.) during that second.

An example of such a set of three images is shown in Fig. 8. It is clear from these images that the cloud doesn’t have to be in the center of the beam, nor does the beam have to have a perfect Gaussian shape. These images were taken with a large diameter imaging beam, and when it passed through the windows of the chamber (in and out), unavoidable fringes are imprinted on the beam. In addition, the bonding of the chip wires are also visible in the imaging beam (on the top of the images). Both of these patterns on the imaging beam don’t appear in the absorption image (Fig. 8(c)) because they are the same in both of the images (Fig. 8(a-b)) and they are canceled out when taking the ratio. In the later stages of the experiment, when the cloud is smaller, a smaller imaging beam is used and the strong fringe pattern reduces.

Forming the absorption image is only the first step in the analysis of the image. The image is then fitted and more information can be extracted. For example, a thermal cloud is fitted to a Gaussian, and (i) the position of the center of the cloud, (ii) the size of the cloud, and (iii) the peak optical density are extracted from the fit.

After constructing the absorption image, calculating the total atom number is done by multiplying the column density \( \tilde{n} = \bar{D}/\sigma_0 \) (assuming \( \delta = 0 \)) with the pixel size (in the object plane), \( A \), and summing over the image.

\[
N = \frac{A}{\sigma_0} \sum_{\text{pixels}} \text{absorption}. \tag{5}
\]

The analysis software, whose interface is shown in Fig. 9 was written by me in MATLAB, and requires two saved image files (with atoms and background) in a known place on the hard drive and in a defined format. The software then reads these files, creates the absorption
image, fits the data to the wanted fit and plots the results on screen. Many of its parameters, e.g., fit type, plot type (absorption, with atoms, background), etc., can be adjusted after analyzing the images, and it can save and read the analysis results for later usage.

On the left side of the interface screen there is the image with two 1-dimensional cuts along the center of the cloud. The right side is dedicated to different panels that control the behavior of the software. The main two are on the top and they are called "Plotting and Fitting" and "Analyze Option". The first controls which type of fit the software will use (1-dimensional fits along the center, 2-dimensional fit on the entire image, Gaussian fit for thermal atoms, inverted parabola for BEC) and which image to plot (absorption, with atoms, background, magnification of the absorption). The second controls more advanced analysis that requires analyzing a series of images (e.g., temperature, life time of the cloud, position and size of the cloud for different parameters, etc.).

2.5.2 Temperature measurement

One of the most fundamental measurements when cooling atoms is the temperature measurement. It is done by doing a time of flight (TOF) measurement, whereby the atomic cloud is released and left free to expand while falling under the influence of gravity. By utilizing the increasing size of the cloud we can calculate its temperature. TOF measurement is a series of images of the cloud, prepared each time in exactly the same way and then released from the trap and imaged. The only difference between the images in the series is the different times elapsing between the release and when the image is taken. In addition to the free fall, the cloud also expands because of the atoms’ kinetic energy, and this expansion depends on the
An intuitive explanation uses the relation
\[ \frac{1}{2} m \sigma_v^2 = \frac{1}{2} k_B T. \]  
(6)

From the fit of the cloud to a Gaussian, \( \sigma_x(t) = \sigma_v t \) is obtained. Inserting this relation into eq. (6) gives
\[ \sigma_x^2 = \frac{k_B T}{m} t^2. \]  
(7)

This relation is qualitatively correct, but it doesn’t consider the initial size of the cloud. For that, the partition function, \( \rho(x, v) = \exp(-\beta H(x, v)) \) (one particle in 1-dimension), needs to be calculated, where \( H = 1/2m \omega_x^2 x^2 + 1/2mv_y^2 \) is the Hamiltonian. Using again eq. (6), the partition function at \( t = 0 \) becomes (up to normalization to number of atoms) \( (34) \),
\[ \rho(x, v) \propto \exp \left( \frac{x^2}{2\sigma_x^2} \right) \exp \left( \frac{v^2}{2\sigma_v^2} \right). \]  
(8)

Because when the cloud is imaged, only its spatial distribution is measured (for different times), we integrate over the velocity, so that
\[ \rho(x, t) \propto \int \exp \left( \frac{(x - vt)^2}{2\sigma_x^2} \right) \exp \left( \frac{v^2}{2\sigma_v^2} \right) dv. \]  
(9)

Solving this integral,
\[ \rho(x, t) \propto \exp \left( -\frac{x^2}{2\sigma_x^2(t)} \right) \quad \text{with} \]  
(10)
\[ \sigma_x^2(t) = \sigma_x^2(t = 0) + \sigma_v^2 t^2, \]  
(11)
results in the correct function which we use (eq. (6) and eq. (11)).

An example of a temperature measurement is shown in Fig. 10. The two fits represent the two axes of the cloud in the image. The vertical difference between the fits reflects the difference in the initial size of the cloud in the radial and longitudinal directions, which caused by the asymmetry of the trap. The difference in the temperatures is because the cloud is not in thermal equilibrium.
3 Laser cooling

The phase transition to Bose-Einstein condensation occurs when the phase space density (defined later, Sec. 4) becomes larger than one. The RF evaporation technique is able to bring the atomic cloud to the needed parameters. However, this technique requires a trap with a controllable depth, namely, that the height of the trap walls may be varied so hot atoms may be expelled. In our experiment, we use for this purpose a magnetic trap. These traps are typically not deep enough to trap atoms at room temperature and pre-cooling, to \( \sim 100 \mu K \), is necessary. In our system, we use a magneto-optical trap to cool and trap an atomic sample (from the background gas), as a preparatory step for the magnetic trap.

3.1 Theory of light and matter interaction

i Light force on two level atoms

When an atom moves in a laser light field, it makes a transition to an exited state while absorbing a photon (if the laser frequency is close to an atomic transition). Assuming low light intensity, hence no stimulated emission, the atom will spontaneously decay back to the ground state by emitting a photon in a random direction. During the absorption and emission cycle, the atom gets two momentum kicks. The first one is from the laser light in the direction of the beam. The second is when spontaneously emitting a photon in a random direction. Because this cycle is very fast, 23ns for the cooling cycle in \(^{87}\text{Rb}\), finding the force on the atoms requires summing on many such cycles. Summing over \( N \) cycles results with \( \vec{p} = N \hbar \vec{k} \) from the absorption since it will absorb a photon from the same direction each time. On the other hand, since the emitting is in a random direction, the momentum from the spontaneous emission is \( \vec{p} = \sum < \vec{p} > = 0 \). From the last two equalities, the total momentum the atom will get is in the direction of the beam, hence it is being pushed by the beam.

The total force on the atom is \([35]\):

\[
\vec{F} = \frac{d\vec{p}}{dt} = \hbar \vec{k} \gamma_{\text{abs}}, \quad \text{where} \quad \gamma_{\text{abs}} = \frac{s_0 \Gamma/2}{1 + s_0 + \left( \frac{\delta + \omega_D}{\Gamma/2} \right)^2}.
\]

\( \gamma_{\text{abs}} \) is the absorption rate of photons by the atom, and \( \Gamma \) is the linewidth of the atomic transition. \( \delta \equiv \omega_l - \omega_a \) is the detuning of the laser light from the atomic transition. \( s_0 = I/I_{\text{sat}} = 1/(\pi \hbar \Gamma c/3 \lambda^3) \) is the saturation parameter. The last parameter, \( \omega_D = -\vec{k} \cdot \vec{v} \), is the Doppler shift.

Notice that the force has a Lorentzian shape, and it’s maximized when the Doppler shift is canceled by the detuning of the laser.

ii Optical molasses

So far I explained how a close to resonance laser beam can push an atom. Consider a 1-dimensional case, an atom and two counter-propagating laser beams, which create a standing wave. The idea of cooling requires that the atoms should be pushed always against the direction of its velocity. For one beam, and an atom that moves toward the beam, this will dictate a red detune laser light, as shown by eq. \( [12] \). In the case of two beams, if both of them are red detuned, the atom will always feel a force stopping it. This result is due to the
Doppler shift that will blue shift the front beam toward resonance while it will red shift the rear beam even more off-resonance (1, 35).

Summing the forces from the two beams we find (35, 36):

$$\vec{F}_{OM} = \vec{F}_+ + \vec{F}_- \cong \frac{4\hbar k^2 s_0 \delta/\Gamma}{(1 + s_0 + |\delta/\Gamma|^2)^2} \vec{v} \equiv -\beta \vec{v},$$ \hspace{1cm} (13)

where terms of $(kv/\Gamma)^4$ and higher have been neglected. This force is proportional to the velocity (for small velocities) and behaves like a friction or damping force.

Notice that this force will damp the velocity only for a red detuned laser light. For blue detuning, the light will increase the atom’s velocity.

In order to have 3-dimensional cooling, 3 pairs of red detuned counter-propagating laser beams are needed.

### iii Doppler limit

From eq. (13) it may look as if the atoms are quickly decelerated to $v = 0$, meaning $T = 0$. In reality this does not occur, implying there are other processes at work which heats the atoms and stop the cooling. This heating is a result of the discrete size of the momentum kicks the atoms get, when absorbing and emitting photons.

One way to quantify this is to compare the cooling rate, $\vec{F}_{OM} \cdot \vec{v}$, to the heating rate, $4\hbar \omega_r \gamma_{abs}$. $2\hbar \omega_r$ is for the kinetic energy the atom gets after one cycle of absorption and emission, and $2\gamma_{abs}$ is the absorption rate from the two laser beams (in 1-dimension). The steady-state kinetic energy is $\frac{\hbar}{4} \left( \frac{2|\delta|}{\Gamma} + \frac{\Gamma}{2|\delta|} \right)$, the minimum is at $\delta = -\Gamma/2$ (the lasers are red detuned), and the temperature limit is

$$T_D = \frac{1}{k_B} \frac{\hbar \Gamma}{2}.$$ \hspace{1cm} (14)

Another way is to consider only the random emission. The atom performs a random walk in velocity space, and even though $\langle \vec{v} \rangle = 0$, $\langle v^2 \rangle \neq 0$ and will increase with the number of scattered photons (1, 37). Increasing $\langle v^2 \rangle$ corresponds to heating. Equating the cooling and heating terms, results in the same steady-state temperature as in eq. (14).

For $^{87}\text{Rb}$ atoms, $\Gamma = 6 MHz$ and the Doppler limit is $T_D \approx 150 \mu K$.

### iv Sub-Doppler cooling

So far, the theory considered only two-level atoms. It was sufficient for many experiments, but when temperatures below the Doppler limit were measured (1), a new theory that includes the multilevel structure of the atoms was established (3). This theory considered also the sublevels of the atom, e.g., hyper-fine structure and Zeeman levels, and the optical pumping an atom will undergo by the laser light.

The theory is described in (3), and it contains two models. One for two linearly polarized beams, called Lin⊥Lin configuration, and one for a $\sigma^+ - \sigma^-$ configuration. The first was used in (4), where they’ve reached sub-Doppler temperatures. The second is commonly used in MOT experiments.

The basic concept of this cooling is that because of the specific polarization of the light and the complex structure of the atoms, atoms at different sublevels interact differently with the light. In the Lin⊥Lin configuration, the two counter-propagating beams create a high
polarization gradient in space (from linear to circular, and vice versa). Different sub-levels of the atom feel different light shifts, hence they have different probabilities to absorb a photon. The result is that after the atom "climbs" the potential created by the light shift, it absorbs a photon with circular polarization and is pumped into a different state. In the new state, the atom is in the bottom of the potential and it needs again to "climb" the potential, resulting in an endless climbing of the potential. Consequently, this cooling is named \textit{Sysiphus} cooling.

In the $\sigma^+ - \sigma^-$ configuration, there is no polarization gradient. The polarization is linear and it rotates in space. In this configuration the cooling is due to different probabilities to absorb a $\sigma^+$ or a $\sigma^-$ photon due to Clebsch-Gordan coefficients.

These two new cooling mechanisms can cool below the Doppler limit because they are based on different absorption probabilities due to different populations in the ground state and are not based on on-resonance and off-resonance beams due to Doppler shifts.

\section{Recoil limit}

After going below the Doppler limit, a new limit appears, the recoils limit. This is the temperature associated with the kinetic energy an atom at rest will get when recoiling while absorbing or emitting one photon. The temperature at this limit is:

\[ T_r = \frac{1}{k_B} \frac{p^2}{2m} = \frac{1}{k_B} \frac{\hbar^2 k_L^2}{2m} = \frac{1}{k_B} \frac{\hbar^2}{2m \lambda_L^2}, \]

(15)

where $m$ is the $^{87}\text{Rb}$ mass and $k_L$, $\lambda_L$ are the laser wavenumber and wavelength.

Cooling with laser light below the recoil limit is not possible\textsuperscript{6}, and for further cooling of the atoms, they need to be exposed to evaporative cooling, which in our case is realized in a dark magnetic trap (Sec. 4).

For $^{87}\text{Rb}$ atoms, the recoil temperature is $T_r \approx 350\text{nK}$.

\subsection{Magneto-optical trap (MOT)}

\subsection{Theory}

In the last section, the theory of laser cooling was explained, but with light alone one can only cool the atoms, and not trap them. The interaction between the light to the atoms is velocity dependent alone. If an atom will diffuse from the center there is no force to bring it back, the force is the same everywhere. Even when doing 3-dimensional optical molasses, namely, beams are impinging on the atom from all 6 directions, there is no force that brings the wondering atoms back to the center.

In a MOT \textsuperscript{2,35}, the position dependent force is introduced by adding a magnetic field, a quadrupole, and using opposite circular polarization for the cooling beams (see Fig. 11). Note that here we define the polarization of the photons and the atoms relative to the lab frame. An equally valid explanation may be given when the photon polarization is taken relative to their direction of motion and the atoms’ polarization is relative to the local magnetic field direction. The basic concept is that an atom at the ground state ($m_g = 0$) has three excited states ($m_e = 0, \pm 1$), and the probability to be excited to either one of them is not equal. For example, as shown in Fig. 11 when the atom is at $z'$, its $m_e = -1$ state is closer to resonance, hence, it will absorb more photons from the $\sigma^-$ beam.\footnote{Cooling below the recoil limit with laser light was achieved by pumping the atoms to a dark state. Other laser cooling techniques for neutral atoms, such as side-band cooling, also exist.}
3 LASER COOLING

Figure 11: Light and magnetic fields in a MOT in 1-dimension. The horizontal dashed line is the laser light frequency, \( \omega_l \), and it’s detuned from the atomic transition by \( \delta_l \). The energies of the excited states, \( m_e = 0, \pm 1 \) are marked by solid lines and they follow the quadrupole field direction. All indices are relative to the lab frame. An atom at position \( z' \), will feel the \( m_e = -1 \) state closer to resonance, hence, the probability to absorb a \( \sigma^- \) photon is higher. (from (35)).

Figure 12: 6-beam MOT and mirror MOT. In all plots, 2 beams along the x axis are not shown. Here, each beam is marked as right (left) hand circular, RHC (LHC), and not as \( \sigma^+ \) and \( \sigma^- \), as in Fig. 11, but this is completely equivalent. (a,b) Standard configuration of the 6-beam MOT. The quadrupole and 4 beams are shown. (c,d) The atomchip is used as a mirror, and the reflection of 2 beams replaces the 2 that are blocked by the atomchip. (from (38)). The mirror-MOT works because the circular polarization is flipped by the mirror upon reflection. Note that different from the previous figure, in this figure we use the alternative frame of reference (see text) to define the photon polarization.

Because the cooling transition is \( F = 2 \rightarrow F' = 3 \), for any Zeeman sublevel in the ground state, there are three available sublevels in the excited state and the example in Fig. 11 of \( m_g = 0 \) is applicable to any other sublevels.

Introducing the magnetic force into eq. (12), will change \( \delta + \omega_D \) to \( \delta + \omega_D + \mu' Bh \), where \( \mu' = \mu_B(g_e m_e - g_g m_g) \) and \( B = B'z \). Solving \( \mathbf{F}_+ + \mathbf{F}_- \) for Doppler and Zeeman shifts smaller than the detuning, results in a similar result to eq. (13)

\[
\mathbf{F} = -\beta \mathbf{v} - \kappa \mathbf{r},
\]  

(16)

where \( \beta \) is equivalent to eq. (13) and \( \kappa = \frac{\mu' B'}{\hbar k} \). \( \beta \) and \( \kappa \) are the damping coefficients in momentum and position space, respectively.
3.2.2 Mirror-MOT and U-MOT

In Fig. 12(a, b), a scheme of a 6-beam MOT is shown. Note that Fig. 11 is in the lab frame and Fig. 12 is in the local frame, where the photon polarization is relative to the photons momentum direction, and the atoms polarization is relative to the direction of the local magnetic field. Fig. 12(c,d) shows how the atomchip blocks two of the beams. Using the reflections of the other two beams, the atoms are exposed to 6 beams in the correct polarization and direction. This is called a mirror-MOT.\(^7\)

In Fig. 12 in the 6-beam MOT, and also while using the atomchip, the quadrupole magnetic field was created by external coils, in an anti-Helmholtz configuration. Fig. 13 shows how a quadrupole field can be created in a different way. If one takes a straight current \(I_w\) carrying wire, circular magnetic field lines will be generated around it. Adding an external bias field \(B_b\) will cause the cancelation of the total field at a certain point, and linear increase close by. This is the definition of a quadrupole field, and as long as the atomic cloud doesn’t extend beyond the linear regime, it feels a quadrupole field. The field produced by a straight wire is a quadrupole in two dimensions. The confinement in the third dimension is done by bending the wire into a "U"-shaped wire. The magnetic fields from the two "legs" of the U will cancel each other at the center and the field will increase linearly to both directions. Fig. 13(a) shows the magnetic lines of a straight wire and a U-shaped wire, and (b) shows the absolute magnetic field. Changing the value of \(B_b\) and/or \(I_w\) will change the position of the minimum, \(d_0\), and the gradients of the quadrupole, \(B'\). The basic equations which describe the above are:

\[
\begin{align*}
d_0 & \propto \frac{I_w}{B_b}, \quad \text{and} \quad \frac{B'}{B_b} \propto \frac{B_b^2}{I_w}. \\
\end{align*}
\]

As in a standard quadrupole, adding an additional bias field perpendicular to \(B_b\) will move the minimum.

Fig. 13(c) shows where the central part of the U-shaped wire is located relative to the atoms and that the magnetic lines that the atoms feel are the same as if external quadrupole coils were used (see Fig. 12(d)). The needed bias field, \(B_b\) in (a) and \(B_y\) in (c), is created by a pair of coils in an Helmholtz configuration. In our system, there are 3 pairs of coils, all in

---

\(^7\)The idea to use the reflection as another beam started in a different configuration that uses only one beam in what is called a pyramid-MOT.\(^{39}\)
the Helmholtz configuration, to provide the required bias fields in any needed direction (see Fig. 1(a)).

### 3.2.3 Experiment

In our setup, the cooler and repumper light is coupled into one fiber (see Figs. 2(b), 3). The output coupler of the fiber near the science chamber expands the beam to 20 mm in diameter. Then, the beam is split into 4 beams with 3 polarizing beam splitters (PBS). Before each PBS there is a λ/2 waveplate to control the power in each beam. These waveplates are mounted in a mount with an actuator for very sensitive control over the beam balance. In addition, each beam, just before entering the chamber, passes through a λ/4 waveplate that changes its linear polarization into circular polarization. Each one of the beams has ~ 30 mW of cooler laser and ~ 3 mW of repumper. One pair of beams is directed at 45° to the atomchip, as in Fig. 1(b). The other pair is parallel to the chip, along the x direction in Fig. 1(b), and their center is ~ 5 mm below the surface of the atomchip.

The cooler light is ~ 3Γ red detuned from the $F = 2 \rightarrow F' = 3$ transition. This is a closed transition, as from the $F' = 3$ the atoms can only decay back to the $F = 2$ state. Since the laser has a finite linewidth and the distance between the $F' = 2$ and the $F' = 3$ is only 266 MHz, some of the atoms will be excited to the $F' = 2$ state. From there, some will decay to the $F = 1$ state. This event happens in 1 out of 1000 cycles, but since the cycle is very fast (23 ns), in < 1 ms all the atoms will fall to the $F = 1$ state. For this reason we use a second laser, the repumper, which is locked to the $F = 1 \rightarrow F' = 2$ transition and it brings the atoms occupying the $F = 1$ state back to the cooling cycle.

The copper U wire is shown in Fig. 5(a). Its legs are 2 × 2 mm, its main part is 10 × 1 mm and its center is ~ 2 mm above the atomchip. The magnetic bias fields are created by 3 pairs of square coils, very close to a Helmholtz configuration (see Fig. 1(a)). The dimensions of the coils are: (i) The x coils, (356, 269) mm in width and height, and 210 mm distance between them. (ii) The y coils are (356, 333) mm in size and 210 mm apart. (iii) The z coils are (291, 291) mm and 163 mm. In the area in which the atoms are trapped and cooled, these coils give a homogeneous magnetic field. According to simulations the magnetic field is constant, up to 1 mG, in an area of ± 5 mm from the center. Measurement of the magnetic fields showed a change of up to 10 $-20$ mG from the expected value at center, but 10 mG is the precision of our Gauss meter and it doesn’t show a stable measurement at this accuracy. Measurement of the G/A ratio of the magnetic coils give (0.835, 0.834, 1.007) G/A for the (x, y, z) coils.

In order to have atoms in the chamber we run current in two $^{87}$Rb dispensers connected in parallel. We run 16 A for 6.5 s and 11 A for another 4.5 s. The loading of the MOT takes 20 s. The 9 s delay from the closing of the dispensers to the end of the loading stage, like the two ramps of the dispenser, is found after optimization. The optimization aims at having as many atoms in the MOT as possible, while having a long lifetime in the magnetic trap, dependent on the vacuum level. The delay enables the dispensers to cool down and allows the ion pump time to pump the background gas and lower the background pressure for the next stages of the experiment (Sec. 4).

To trap the $^{87}$Rb atoms from the background gas, we run 40 A in the U wire, ~ 5.4 G in the y bias coils and ~ 1.5 G in the z bias coils. Due to the asymmetry of the U, the quadrupole center is not directly above the U wire, and by applying a small z bias it moves closer to the center of the atomchip. For the same reason the wide part of the U wire isn’t centered compared to the white Macor and the atomchip. These parameters create a MOT
Figure 14: Cooler AOM frequency and imaging detuning. (a) At each frequency two data points were taken. The frequency is the cooler AOM RF frequency, thus, it’s only half of the laser shift. The maximum is at 97.7MHz, which is 17.6MHz ($\sim 3\Gamma$) from the $F = 2 \rightarrow F' = 3$ transition. See Fig. 4. (b) At each detuning at least two data points were taken. The detuning is relative to the transition $F = 2 \rightarrow F' = 3$. The fit is to a Lorentzian. The transition width returned from the fit, $7.4 MHz$, is a convolution of the transition natural linewidth with the laser linewidth. The center is shifted a little from the transition (see text). See Fig. 4.

of $\sim (75 \pm 5) \cdot 10^6$ atoms, $\sim 6.5 mm$ from the atomchip.

Optimization of the MOT involves numerous parameters, and the goal is obviously to maximize the atom number in the MOT. Two of these parameters are the cooler frequency and the magnetic gradients. Changing the minimum position will change also the gradients (eqs. 17 and 18), hence, the cooler frequency needs to be adjusted according to eq. 16 and Fig. 11. On the other hand, after the minimum position is fixed, only the gradients can be changed to match the cooler detuning (usually $\sim 3\Gamma$).

In addition, the balance between each pair of cooling beams and between the horizontal and the 45° beams is a crucial parameter. If the beams are imbalanced, the MOT can move away from the quadrupole center and the efficiency of the cooling drops. This balance is also important in the molasses stage, Sec. 3.3.

Another parameter involves the beams polarization, but usually the $\lambda/4$ waveplates are aligned independently to give circular polarization, and are not changed from that alignment.

Fig. 14(a) shows an example of an optimization procedure. In that figure, the best cooler frequency was searched for (to maximized the atom number). Using the experimental control software, Sec. 2.4, the system runs in a "loop" mode, where only one parameter is changed between different runs. This parameter is the RF frequency of the cooler laser AOM. The graph is fitted to a parabola and the maximum atom number is found at $97.7 MHz$, which is $\sim 3\Gamma$ from the $F = 2 \rightarrow F' = 3$ transition (after double pass in the AOM).

Fig. 14(b) presents the maximizing procedure for the imaging signal, namely, the optical density (see Sec. 2.5). Since the optical density has a strong dependence on the detuning of the laser, finding the on-resonance frequency can improve the signal significantly. Changing the imaging laser frequency and fitting the data to a Lorentzian enables to find the linewidth and the center. The linewidth is a convolution of the transition natural linewidth with the laser linewidth. The center, in the figure, is shifted from the expected transition frequency. This may be due to magnetic fields during the imaging or in the laser locks. It can also be due to a slightly inaccurate calibration of the experimental control software signals relative to
the expected input of the AOM driver. This will result in an offset in the RF frequency the AOM gets. Some of these shifts can be measured independently and then compensated, but this approach will not always give the best result. Sometimes it’s easier and better simply to measure the total shift, as was done in the figure, and then to compensate for all of the different shifts together.

3.3 Compression and molasses

The MOT is trapped \( \sim 6.5\,\text{mm} \) from the atomchip. It is where we found the atom number collected by the MOT to be maximized, and not surprisingly it is also approximately the center of the region in which all beams impinge on the atoms. The next stages, mainly the magnetic trap, require the atoms to be closer to the atomchip, \( \sim 2.5\,\text{mm} \) from it. For this reason, the atomic cloud is brought closer to the chip. This is done by increasing the currents in the U wire and the y bias to 60\( \text{A} \) and \( \sim 19.5\,\text{G} \) respectively. While bringing the cloud closer to the atomchip, it is also being compressed to match its size to the magnetic trap size. This is called mode matching and is done very thoroughly. Having a good mode matching between the compressed MOT and the magnetic trap means a high efficiency in the transition and low heating.

The compression is done in 100\( \text{ms} \), and at its end there is a short, 3\( \text{ms} \), ”gray” MOT. In the so-called gray MOT, one takes the cooler light to be more off-resonance, from \( \sim 3\Gamma \) to \( 6\Gamma \) (can be up to 10\( \Gamma \)). The intensity of the cooler is also reduced. The purpose of the gray MOT is to again cool the atomic cloud, after it heats during the compression, and to further cool it beyond the original MOT temperatures, by decreasing the heating due to light-atom scattering.

To establish the magnetic trap by ramping down the MOT quadrupole field, and by ramping up the magnetic fields of the magnetic trap, takes \( \sim 6 - 7\,\text{ms} \), during which time the atomic cloud should be held in place, or it will fall under gravity. Falling will reduce the loading efficiency of the magnetic trap. The optical molasses (Sec. 3.1(iii)), which is now introduced to further cool the atoms, also helps keep the atoms in place. With the velocity dependent light pressure, the cloud will fall much slower than under gravity alone.

The main goal of the optical molasses stage is the sub-Doppler temperature (Sec. 3.1(iv)). Reaching a low temperature before loading the magnetic trap will improve the loading efficiency and, also very important, the temperature of the cloud after loading to the magnetic trap is lower. Having a lower temperature in the magnetic trap will put the atomic cloud in a better starting point for RF evaporation cooling, the last stage before the phase transition to Bose-Einstein condensation (Sec. 4.3).

The molasses is done in 5\( \text{ms} \), and in this 5\( \text{ms} \) the cooler light is taken even more off-resonance than in the gray MOT. At the end of the molasses the laser light is 12.5\( \Gamma \) from the transition. The shifting of the frequency is accompanied by reduction of the light intensity. The reduction of light intensity is a side effect of changing the RF frequency of the AOM, but it’s a desired effect, as the light intensity should be reduced for good molasses to take place. Unlike in the gray MOT, where the frequency shift is linear, in the molasses the sequence is more complicated. When closing the magnetic fields, the laser light, cooler and repumper, are closed. This is done because the cloud is heated and deformed if the laser light is on while closing the magnetic fields. The laser light is reopened after 1.7\( \text{ms} \), at 6\( \Gamma \) detuning. In 2.5\( \text{ms} \)

\*The current in the x bias is ramped from 0G to \( \sim 1\,\text{G} \) in order to match also the position in the x direction.
the cooler frequency is ramped to $8.5 \Gamma$, and after an additional $0.6\text{ms}$ to $12.5 \Gamma$. After that the magnetic fields are opened and the optical pumping pulse is executed. These stages will be explained next, in Sec. 4. For an image of the gray MOT and the molasses pulses as they appear on a scope, see Fig. 15.

The temperature of the MOT is $\approx 300 - 350 \mu K$, and after a gray MOT it can be as low as $50 \mu K$, but the compression stage heats it up. After the compression the gray MOT and the molasses, the temperature of the cloud is $\approx 100 \mu K$. The molasses can lower the temperature even further, but in our experience, it is accompanied by atom loss. For us, the atom number is more important than the temperature, especially when the efficiency of the loading to the magnetic trap is high ($\sim 70\%$).

## 4 Magnetic trap

Reaching the phase-space density (PSD) needed for a Bose-Einstein condensation means cooling and compressing the atoms. PSD incorporates the density in position and momentum space and it is defined as

$$\Phi = n\lambda_{dB}^3,$$  \hspace{1cm} (19)

$$\lambda_{dB} = \frac{h}{\sqrt{2\pi mk_B T}},$$  \hspace{1cm} (20)

$n$ is the density of the atomic cloud, $\lambda_{dB}$ is the thermal wavelength, $m$ is the mass, $h$ and $k_B$ are the Planck and Boltzmann constants, and $T$ is the temperature.

Compression can be done in a MOT or in a dark magnetic trap whereas cooling with light is limited to the recoil temperature. The cooling method able to reach a PSD $>1$ is evaporative cooling (Sec. 4.3). In our setup this is achieved with a magnetic trap (Sec. 4.2) and with an "RF knife" that is able to expel atoms above any desired energy from the trap. The interaction between the atoms and the magnetic field is through the atomic magnetic moment, and it is

$$U = -\vec{\mu} \cdot \vec{B} \approx \mu_B g_F m_F B,$$  \hspace{1cm} (21)

where, $\mu_B$ is Bohr magneton, $g_F$ is Landé factor and for $^{87}\text{Rb}$ atoms at the $F = 2$ state $g_F = 1/2$. $m_F$ denotes the Zeeman sublevel and $B$ is the absolute magnetic field. The use of the modulus of the magnetic field is called the adiabatic approximation. It is valid only when the magnetic moment of the atoms can follow the changes in the direction of the magnetic field. In this case, the rest frame of the moving atom $\vec{\mu}$ will always be in the direction of the magnetic field and $\vec{\mu} \cdot \vec{B} \rightarrow \mu |B|$. The adiabatic approximation can be applied when $\omega \ll \omega_L$, where $\omega$ is the trap frequency and $\omega_L$ is Larmor frequency, which is defined as

$$\omega_L = \frac{\mu_B B}{\hbar},$$  \hspace{1cm} (22)

where $B$ is the absolute magnetic field at the minimum of the trap.

From eq. (21) it is clear that in the $F = 2$ ground level, only $m_F = 1$ and $m_F = 2$ are states that will be trapped at the minimum of the magnetic field as they are low field seeking states, namely, the minimum energy they are attracted to occurs at the minimum of the field. It is better to put all the atoms in the $m_F = 2$ state to prevent spin-flip collisions. It is not possible to trap atoms in the high field seeking states ($m_F = -1$ and $m_F = -2$) because there...
cannot be a maximum of the magnetic field in free space (without sources) according to the Earnshaw theorem \[^{41}\]. The optical pumping stage, Sec. 4.1, pumps all the atoms to the \( m_F = 2 \) state.

From eq. (22) it is clear that quadrupole is not a good magnetic trap. At the quadrupole center there is a zero, hence, \( \omega_L \to 0 \), and the adiabatic approximation fails. In a quadrupole, when an atom in \( m_F = 2 \) passes through the quadrupole center, it feels no magnetic field and all its Zeeman sublevels are degenerate. In this situation, the atom can change its \( m_F \) state easily to an untrapped state (Majorana spin-flips). This change of spin states causes atoms to be lost from the trap, and is more pronounced when the atomic cloud is colder and stays longer at the bottom of the trap \[^{42}\]. One solution is to use a time-average orbiting potential. In this type of trap, the atoms feel the time-averaged potential, with no zero at the minimum. Another solution is to create a DC magnetic trap with a non-zero field at the center or elsewhere. These traps are usually referred to as Ioffe-Pritchard (IP) traps. It is this type of trap which we use in our experiment and it will be explained in Sec. 4.2.

4.1 Optical pumping

As was explained earlier, in order to trap the atoms they need to be in the \( m_F = 2 \) state\[^{9}\]. The pumping of the atoms to the \( m_F = 2 \) state is done with a short pulse of \( \sigma^+ \) laser light\[^{10}\]. When an atom in an \( m_i (i = -2, \ldots, 2) \) absorbs a \( \sigma^+ \) photon, it is excited to the \( F', m_{i+2} \) state. From this state it decays spontaneously, with equal probabilities, to \( m_i, m_{i+1} \) or \( m_{i+2} \). On average it will decay to the \( m_{i+1} \), and this introduces the pumping. After a few cycles all the atoms will be at the \( m_F = 2 \) state.

Regarding the experiment there are two comments. The first is that when the atoms are

---

\[^{9}\]It is possible to trap atoms directly from the molasses, but then 3/5 of the atoms will be lost immediately, and the rest will have short life-time in the trap due to spin-flipping collisions among themselves.

\[^{10}\]In order to remove the degeneracy of the Zeeman sublevels, and to define \( \sigma^+ \), a small bias field (in the direction of the laser light) is applied.
in the $m_F = 2$ state, the probability to absorb $\sigma^+$ light is very high (the highest from all the states). From the $m_F = 2$ state the atoms will be excited to the $F' = 3$, $m_F = 3$ state, and they can decay only to the $m_F = 2$ state again. This is a closed cycle so the pumping is still efficient, but this cycle will heat the atoms. The optical pumping stage is after the molasses and there is no more cooling of the atoms before loading them to the magnetic trap, hence, any heating will reduce the efficiency of the loading. For this reason the optical pumping light is not tuned to the $F = 2 \rightarrow F' = 3$ transition, like the cooler and the imaging light. It is tuned to the $F = 2 \rightarrow F' = 2$ transition (see Fig. 4). Now, an atom in the $m_F = 2$ cannot absorb a $\sigma^+$ photon, as there is no $m_F = 3$ state at the exited state. The atoms in the $m_F = 2$ state are in a so-called ”dark state”. They don’t ”see” the laser light anymore and they are not heated.

The second comment concerns the repumper laser. Since the optical pumping transition is $F = 2 \rightarrow F' = 2$, half of the atoms decay to the $F = 1$ state. For this reason, the optical pumping, even if it is a very weak pulse (less than 0.5$mW$), needs a relatively strong repumper light.

Fig. 15 shows an image taken from an oscilloscope and which summarizes the molasses and optical pumping stages. The first signal is that of a photo-diode (PD) that measures the intensity of the cooler and repumper laser light, together. The second is another PD which measures the intensity of the optical pumping and the imaging lasers (there is no imaging at all, so it can be ignored). The third and fourth are two different fast current sensors that measure the current in the y bias coils and the z wire (creating the magnetic trap). Since the signals are from PDs and current sensors, the y axis values are arbitrary, unless they are calibrated (something that is not necessary for understanding the sequence). The time 0 represents the trigger of the oscilloscope at the start of the molasses stage. Following the cooler/repumper intensity shows the reduced power at the gray MOT stage (3$ms$ before the start of the molasses), and the two different ramps of the molasses itself. Both the cooler and the repumper are closed just before closing the magnetic fields and reopened after 1.7$ms$. The optical pumping pulse length is 400$\mu$s and it’s executed only after the y bias field is starting to open but while the Z-wire is closed so there is a homogeneous magnetic field, not a trap.
The repumper pulse is also visible together with the optical pumping pulse. Fig. 16 shows a Stern-Gerlach experiment that we do in order to determine the relative population of the different sublevels. The experiment is done after the optical pumping pulse. We close the y bias and then run current in the Z copper wire. This is done to prevent creating a trap which can remove any untrapped states from the cloud. In the image, the $m_F = 0$ falls under gravity. $m_F = 1$ and $m_F = 2$ fall with different accelerations ($m_F = 2$ feels a force twice as strong). $m_F = -1$ and $m_F = -2$ also feel a force, but directed upwards, toward the atomchip. Due to this force, these states will accelerate upwards, hit the atomchip and disappear from the image. Fig. 16(a) shows how the distribution looks like without an optical pumping pulse, (b) with un-optimized optical pumping, where residual atoms at the $m_F = 1$ are clearly visible, and (c) after optimization, where all the atoms are in the $m_F = 2$ state.

### 4.2 The magnetic trap

As was explained previously, a quadrupole field cannot be used for magnetic trapping due to the Majorana spin-flips the atoms at the trap center will undergo. If one will add a constant magnetic field to a quadrupole, the zero in the field will only change it’s position, but there will still be a zero. The way to remove the zero is to remove the quadrupole in one of the axeses and in that direction to add a constant field. If one will add a magnetic field with a minimum at the center and increasing to both sides, e.g. a parabola shape, there will be confinement in 3 dimensions. These traps are called Ioffe-Pritchard (IP) traps. Bending a wire to a "Z"-shape, and not to a U-shape, will create an IP trap \cite{14, 17, 19, 43, 44}.

Fig. 17(a,b) shows the difference in the magnetic fields between the U-wire and the Z-wire traps. Unlike the U trap, in the Z trap the "legs" of the Z create a bias field in the center, and the magnetic field increases in both directions. There is no cancelation of the magnetic field, hence, there is no zero and no quadrupole. Fig. 17(c,d) shows contour plots of numerical calculations of the potential created by a Z-wire (the present copper wire we have in our atomchip mount, see Fig. 5(a)) and a y bias field. The trap minimum is 100µm from the
surface of the atomchip. The value of $B_0$ at the center of the trap is set to 1G. This minimum field value is reached by adding an external bias field (in x direction) that cancel most of the field created by the two legs of the Z-Wire.

The value of $B_0$ sets two parameters. The first is the Majorana spin-flip rate. When an atom goes through a regime of no magnetic field, $B_0 = 0$, it can change its spin easily, and where there is a non-negligible magnetic field this probability reduces. This rate decays exponentially with magnetic field and it is (14, 15):

$$\Gamma_M \approx \frac{\pi \omega}{2} \exp\left(-\frac{2\mu B_0 + \hbar \omega}{2\hbar \omega}\right),$$  \hspace{1cm} (23)

where $\omega$ is the trap frequency. Usually 1G is enough to reduce Majorana spin-flip rate to negligible rates.

The other parameter $B_0$ sets is the trap depth. For a straight wire and a bias field, $B_b$, close to the wire the magnetic field $\rightarrow \infty$. Far from the wire, though, the magnetic field from the wire $\rightarrow 0$, and the total magnetic field equals the bias field. For this reason the trap depth is the same as the bias field (up to $\mu$ to convert magnetic field to energy) (14). This is correct when $B_0 = 0$. If $B_0 > 0$, far from the wire the field is $\sqrt{B_0^2 + B_b^2}$, and the trap depth is $\sqrt{B_0^2 + B_b^2} - B_0$. If $B_0$ is too big, the trap depth reduces to zero (the same logic shows that lowering $B_0$ will create a tighter trap). For this reason, keeping the trap as deep as possible, $B_0$ should be as small as possible while keeping the Majorana spin-flip rate low.

When transferring the atoms from the MOT (after compression, molasses and optical pumping) to the magnetic trap, the trap position and size should be mode matched to the magnetic trap being formed. This is the reason for the compression stage. After transferring the atoms to the magnetic trap, the RF evaporation may start, as explained in Sec. 4.3. In our setup, the RF evaporation is a long procedure of 15.5s, and some atoms are lost from the trap, even without applying the RF. Most of the atoms are lost due to collisions with room temperature atoms from the background gas. For this reason the current in the dispensers is optimized in a way that will give high number of atoms in the magnetic trap after 15 – 20s. It is very tempting to increase the current in the dispensers, so as to have a larger MOT and more atoms in the magnetic trap. In our experience, this usually leads to an even smaller number of atoms in the magnetic trap after 20s.

The magnetic trap is often called a dark magnetic trap, unlike the magneto-optical trap. This is to emphasize that there is no laser light, hence, sub-recoil temperatures can be reached. If there is a leakage of light it will cause heating and losses, even before starting to cool the atoms. Hence, the science chamber is separated from the optics part with a black screen, to prevent any reflection of laser light onto the atoms.

Fig. 18 shows a life-time measurement of the atomic cloud. The data is fitted to two exponents because usually there is a fast decrease in the number of atoms in the first few seconds due to imperfect optical pumping and loading. The second exponent is a measure of the life-time due to long processes, e.g., background pressure and laser light leakage. The life-time measurement was done twice, one in the morning and one in the evening of the same day, to check that there is no build up of the pressure during the day. The build up is due to too high current in the dispensers and it will effect the life-time during the day. In the figure, the second life-time measurement has about the same life-time (up to measurement error) but the number of atoms in the trap is higher. This is due to a ”warming up” of the system. The pressure is getting lower during the night and it reaches back to the usual working pressure
Figure 18: Life-time and temperature of the cloud at different dark times. The two curves of the life-time were done in the morning and the evening of the same day, to check for buildup of pressure during the day. Both were fitted to two exponents to separate the losses due to imperfect loading and due to background pressure. The temperatures in the radial and axial are different immediately after loading, and they equilibrate later on and continue cooling during the day. In addition, wires and coils return to their working temperatures, meaning some small deformations. This is normal and it happens because the optimization of the system is usually done after the system has already ”warmed up”.

Fig. 18 also shows how the temperature of the cloud behaves in the first 15s after the loading. The temperature of the cloud in the two axes is different since the temperature represents kinetic energy in the two axes of the cloud, and the cloud is not in equilibrium. Immediately after loading, the two axes have very different temperatures since the trap is tighter in the radial axis ($T_r$ in the figure). 2s after the loading the cloud equilibrates, but then the temperature in the two axes flips. The radial axis becomes colder, and both of them continue to cool. This perhaps indicates the existence of a cooling mechanism in the radial direction, e.g. the atoms evaporate by hitting the surface or overcoming the bias field barrier against gravity.

After loading the atomic cloud to the magnetic trap, its temperature in the radial direction is $250 - 280 \mu K$ and the trap depth is $1.2 m K$ (according to simulations). The trap depth is only 4-5 times higher than the cloud temperature. Since the cloud is thermal, there are atoms in the tail of the Maxwell-Boltzmann distribution that have higher temperature than the trap depth. These atoms will be evaporated from the trap, reducing the total temperature of the cloud (see Sec. 4.3 for more details). For comparison, in the RF evaporation the tail of the Maxwell-Boltzmann distribution is cut at energies 6-7 times the temperature of the cloud. The evaporation from the magnetic trap is not efficient but it reduces the radial temperature. As the cloud is trying to reach equilibrium also the temperature in the axial axis follows the radial temperature.

4.3 RF evaporation

Evaporation is a widespread phenomenon in the ordinary life. It typically describes the transfer of atoms or molecules from a liquid state to a gaseous state. Evaporation happens
when an ensemble of atoms, in a Maxwell-Boltzmann distribution, is held in a finite potential. There will always be atoms at the tail of the distribution with higher energy than the potential barrier. These atoms will evaporate. Since the most energetic atoms disappear from the trap, and they carry their energy with them, the average energy of the rest of the ensemble decreases, and it is cooled. This is the basic phenomenon behind the cooling of a cup of tea, and is also what we use to cool the atoms in the magnetic trap.

i Theory

In our system, forced RF evaporation is being applied to the atomic cloud. The evaporation is forced, and not natural, because it happens due to an RF signal inserted into the system. It is still considered evaporation, even though there is no transition from liquid to gaseous, since the tail of the Maxwell-Boltzmann distribution is being cut and the rest of the ensemble thermalize to a lower temperature.

Evaporation reduces the atomic cloud temperature at the expense of atom number. Since the number of atoms in the cloud is limited, the evaporation should be done wisely, or all the atoms will be lost before the phase-space density (PSD) needed for the BEC will be reached. In the evaporation process there are two important parameters, $\alpha$ and $\eta$ (the theory is explained in detail in [46] and we will repeat it here only briefly). $\alpha$ is the temperature decrease per atom loss, such that:

$$\alpha = \frac{d(\ln T)}{d(\ln N)} = \frac{\dot{T}}{\dot{N}/N}. \tag{24}$$

$\eta$ is the truncation parameter, which means that atoms with energy $> \eta k_B T$ are evaporated from the trap.

The average energy of the trapped atoms is $3k_B T$ ($3/2$ from the kinetic energy and $3/2$ from the trapping harmonic potential, in 3-dimensions). The average energy of a lost atom is approximated by $(\eta + 1)k_B T$. There is a simple relation between $\alpha$ and the average energy of the lost atom, namely,

$$\alpha = \frac{\eta + 1}{3} - 1. \tag{25}$$

From that relation, it is clear that a higher $\eta$ is better. For a higher $\eta$, the system will lose more energy for each lost atom. Taking this to an extreme, one can wait until one particle will hold all the energy of the system, and then evaporate this atom. Clearly, waiting for this event will take some time. Considering an actual experimental life-time means that all of the atoms will be lost before this event will happen. This extreme example reveals the required balance between the evaporation time and the life-time of the system. This balance is expressed as $R$, the good to bad collision ratio. Bad collisions are inelastic collisions, meaning a collision of a trapped atom with a background gas atom at room temperature. These collisions cause atoms loss. Good collision are elastic collisions. This are the collisions that cause the thermalization of the cloud after the tail of the Maxwell-Boltzmann distribution is cut. This ratio determines how fast, or slow, the evaporation should be.

The bad collision rate is the inverse of the life-time. The good collision rate is

$$\Gamma_{el} = n_0 \sigma \sqrt{2\bar{v}}, \tag{26}$$

where, $n_0$ is the peak density of the cloud, $\sigma$ is the elastic cross section and $\sqrt{2\bar{v}}$ is the average velocity between two atoms ($\bar{v} = \sqrt{\frac{8 k_B T}{\pi m}}$). This rate depends on the density and
the temperature, and sometimes it is better to compress the cloud even if it increases the temperature.

ii Experiment

The atomic cloud, after the optical pumping pulse, is in the $|F, m_F\rangle = |2, 2\rangle$ state, and is trapped in a magnetic trap. When an atom oscillates in the trap, it feels different magnetic fields, according to its position in the trap, and the energy difference between its Zeeman sublevels changes. Of course, hotter atoms have more energy, hence they can go farther in the trap, and consequently feel higher magnetic fields and the energy difference between their Zeeman sublevels will be higher.

If an RF signal, with a frequency matching exactly that of the Zeeman splitting, is applied, atoms can absorb the RF photon and change their Zeeman sublevel to an untrapped state. Applying an RF signal with energy that matches the Zeeman splitting of the hottest atoms, farthest from the trap center, will couple only to these atoms and will change their state to an untrapped state. This way only the hottest atoms will be evaporated from the trap and the rest will thermalize.

Fig. 19 shows how the potential looks like for the sublevels $m_F = 0, \pm 1$ (the atoms we trap have also $m_F = \pm 2$ sublevels but this just adds two more curves to the image and does not change the explanation qualitatively). In the left part of the figure, $m_F = 1$ is a trapped state, and the hottest atoms travel the farthest and feel the highest splitting. The applied RF signal couples the sublevel of these atoms to an untrapped state. The right side of the figure shows the thermalization of the cloud after the hottest atoms are lost.

In our experiment, we start with $(50 \pm 5) \cdot 10^6$ atoms in the magnetic trap, $\sim 2.5 mm$ from the atomchip. The RF is transmitted from the U-wire, which is near the atoms and idle. Its frequency starts from $50 MHz$, and reduces until it reaches $0.695 MHz$ (only $25 kHz$ above the trap bottom) and the phase transition to BEC occurs. During the RF ramp we also change the RF amplitude. The RF signal doesn’t open the trap, it rather opens a hole in the trap at a distance from the center (the distance depends on the frequency).

If the RF signal is too strong, power broadening will occur and the evaporation will be less efficient. This effect is noticed mainly close to the transition to BEC. In this regime, $25 - 100 kHz$ above the trap bottom, if the signal is broadened to $\sim 10 - 20 kHz$, it is impossible to have the resolution of $10 kHz$ and even $5 kHz$ needed for the end frequency of the RF signal.

$^{11}$The magnetic fields in the trap are a few ten of Gauss. At this field only the linear regime of the Zeeman splitting is considered, and $\Delta E = \mu_B g_F \Delta m_F B$. 

Figure 19: RF evaporation scheme. On the left, the hottest atoms are coupled to an untrapped state. On the right, the rest of the cloud thermalize.
Fig. 20 shows the different ramps of the RF frequency and amplitude. The amplitude is that of the signal to the function generator, hence its units are arbitrary. The inset is a zoom-in on the last second. We close the RF signal with a GPIB command and consequently there is an arbitrary delay. In order to finish the RF ramps deterministically, we raise the RF frequency so it doesn’t evaporate the atoms anymore, and then the signal is closed (the signal in the graph is before the GPIB delay).

In addition to changing the frequency and amplitude, we also compress the cloud. After loading the atomic cloud to the magnetic trap, its temperature is high and its full width $(2\sigma_r)$ is $\sim 2.5\text{mm}$. Bringing it closer than $2.5\text{mm}$ from the surface will cause atoms to be lost. On the other hand, compressing the cloud will increase the good collision rate and increase the efficiency of the evaporation. For this reason, we compress the cloud in a few stages, according to its temperature and size. In the compressions we keep the $B_0$ at the trap bottom to be $\sim 1\text{G}$. The last stage of the compression brings the cloud to $300\mu\text{m}$ from the atomchip surface, with a radial trapping frequency of $\omega_r = 2\pi \times 565\text{Hz}$. The radial trapping frequency is measured by “pushing” the cloud away from the atomchip, letting it oscillate in the trap and imaging it after different times (see Fig. 23).

Measuring the trap bottom with the RF signal is possible. The sequence is to evaporate the cloud to $\sim 1\mu K$, then change very fast (50$\mu$s) the frequency to a low value, below the trap bottom. If this value is below the trap bottom nothing will happen. Increasing this value to the trap bottom value will cause the cloud to disappear, all the atoms will feel the RF signal when passing through the center of the trap. If the RF amplitude is not too high, the change from a cloud to no atoms in the trap can happen in a $1k\text{Hz}$ difference. This sensitivity tells us that the RF amplitude is the correct one. With this technique we measure at the end of the evaporation the trap bottom to be $0.67 \text{MHz}$, which is $\sim 1G$ ($\mu_B \times 1G = h \times 0.7 \text{MHz}$).

5 Bose-Einstein condensation (BEC)

Bose-Einstein condensation is a phase transition which occurs when a macroscopic amount of atoms occupies the ground state of the system. In our case, the phase transition takes place when taking energy (cooling) from an atomic cloud (of bosons) with a Maxwell-Boltzmann

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Footnote: GPIB is a communication protocol, like network or USB. Similar to any other protocols, they are not working in real time, they have an arbitrary delay of 100 – 200$\mu$s.
distribution. A detailed explanation of the transition to BEC of non-interacting atoms in free space can be found in [47]. In our experiment the atoms are trapped in an harmonic trap, which adds an external potential to the Hamiltonian of the system. In addition, $^{87}$Rb are interacting atoms. The analysis of the transition of interacting atoms in an harmonic trap (and much more) can be found in [48; 49].

In this section I will explain briefly the theory of the transition, for non-interacting and interacting bosons, in an harmonic trap, and give experimental results from our experiment showing the phase transition to BEC.

5.1 Theory of Bose-Einstein condensation

i Ideal Bose gas in an harmonic trap

Bose-Einstein condensation of non-interacting atoms in an harmonic potential is a textbook problem. The external potential, $V_{\text{ext}}(\vec{r})$, and two associated frequencies, $\omega_{ho}$ and $\bar{\omega}$, are

$$
V_{\text{ext}}(\vec{r}) = \frac{m}{2}(\omega_x x^2 + \omega_y y^2 + \omega_z z^2)
$$

$$
\omega_{ho} = \left(\omega_x \omega_y \omega_z\right)^{1/3}
$$

$$
\bar{\omega} = \frac{\omega_x + \omega_y + \omega_z}{3}
$$

(27)

where $m$ is the mass and $\omega_{x,y,z}$ are the frequencies in the x,y,z direction. Since the atoms are non-interacting, the many-body Hamiltonian is a sum of many single-particle Hamiltonians with energies

$$
e_{n_x,n_y,n_z} = (n_x + 1/2)\hbar \omega_x + (n_y + 1/2)\hbar \omega_y + (n_z + 1/2)\hbar \omega_z.
$$

(28)

The a ground-state wave function is

$$
\phi(\vec{r}_1, ..., \vec{r}_N) = \prod_i \varphi_0(\vec{r}_i), \quad \text{with}
$$

$$
\varphi_0(\vec{r}) = \left(\frac{m \omega_{ho}}{\pi \hbar}\right)^{3/4} \exp\left(-\frac{m}{2\hbar}(\omega_x x^2 + \omega_y y^2 + \omega_z z^2)\right).
$$

(29)

Finding the transition parameters requires starting with the grand-canonical formalism of an atomic cloud at temperature $T$. The total number of atoms, $N$, is given by the sum

$$
N = \sum_{n_x,n_y,n_z} \frac{1}{\exp(\beta(e_{n_x,n_y,n_z} - \mu)) - 1}
$$

(30)

where $\mu$ is the chemical potential and $\beta = (k_B T)^{-1}$. Separating the lowest state, $e_{000}$, from eq. (30), one finds

$$
N_0 = \frac{1}{\exp(\beta(\frac{3}{2}\hbar \bar{\omega} - \mu)) - 1}.
$$

(31)

This number, the total number of atoms in the ground state, can become macroscopic at a critical value of the chemical potential,

$$
\mu_c = \frac{3}{2} \hbar \bar{\omega}.
$$

(32)
Inserting eqs. (31) and (32) into eq. (30), and replacing the sum with an integral, results in

\[ N - N_0 = \int_0^\infty \frac{dn_x dn_y dn_z}{\exp (\beta \hbar (\omega_x n_x + \omega_y n_y + \omega_z n_z)) - 1}. \]  

The solution of this integral, after changing variables \( \beta \hbar \omega \rightarrow \tilde{n}_x \), is

\[ N - N_0 = \zeta(3) \left( \frac{k_B T}{\hbar \omega_{ho}} \right)^3. \]  

\( \zeta(n) \) is a Riemann \( \zeta \) function and \( n = 3 \) because the energy of the trapped cloud is \( 3k_B T \) (a free cloud will have \( \zeta(3/2) \)). The transition temperature is found by imposing \( N_0 = 0 \)

\[ T_c = \frac{\hbar \omega_{ho}}{k_B} \left( \frac{N}{\zeta(3)} \right)^{1/3} = 0.94 \frac{\hbar \omega_{ho}}{k_B} \left( N \right)^{1/3}. \]  

ii Interacting atoms: ground state

When the atoms are interacting, the Hamiltonian cannot be separated into a sum of single-particle expressions. The new Hamiltonian of \( N \) interacting bosons in an external potential, \( V_{ext}(\vec{r}) \), is

\[
\hat{H} = \int d\vec{r} \hat{\Psi}^\dagger(\vec{r}) \left( -\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(\vec{r}) \right) \hat{\Psi}(\vec{r}) + \frac{1}{2} \int d\vec{r} d\vec{r}' \hat{\Psi}^\dagger(\vec{r}) \hat{\Psi}^\dagger(\vec{r}') V_{int}(\vec{r} - \vec{r}') \hat{\Psi}(\vec{r}) \hat{\Psi}(\vec{r}'),
\]

where \( V_{int}(\vec{r} - \vec{r}') \) is the two-body interaction potential and \( \hat{\Psi}^\dagger(\vec{r}) \) and \( \hat{\Psi}(\vec{r}) \) are the creation and annihilation boson field operators.

The next step is to write \( \hat{\Psi}(\vec{r}, t) \) (evolution in time of the field operator)

\[
i\hbar \frac{\partial}{\partial t} \hat{\Psi}(\vec{r}, t) = \left[ \hat{P}, \hat{H} \right] = \left[ \hat{P}, \hat{\Psi}, \hat{\Psi}^\dagger \right] = \left[ \hat{\Psi}, \hat{H} \right] = \left[ \hat{\Psi}, \left( \frac{\hbar^2}{2m} \nabla^2 + V_{ext}(\vec{r}) + \int d\vec{r}' \hat{\Psi}^\dagger(\vec{r}', t) V_{int}(\vec{r} - \vec{r}') \hat{\Psi}(\vec{r}', t) \right) \hat{\Psi}(\vec{r}, t) \right] \hat{\Psi}(\vec{r}, t),
\]

and then to replace it with \( \Phi(\vec{r}, t) \). The function \( \Phi \) is a classical field and has a meaning as the "wave function of the condensate". It is defined as

\[ \Phi(\vec{r}, t) \equiv \langle \hat{\Psi}(\vec{r}, t) \rangle, \quad \text{and} \quad n_0(\vec{r}, t) = |\Phi(\vec{r}, t)|^2. \]

In general, replacing \( \Psi \) with \( \Phi \) in the integral containing the atom-atom interaction, in eq. (37), is a poor approximation. However, the BEC cloud is a dilute and cold gas and only low energy, binary collisions are relevant. These collisions are characterized by the (constant) s-wave scattering length, \( \sigma_s \). Considering the last insight, the next approximation can be applied

\[ V_{int}(\vec{r} - \vec{r}) = g \delta(\vec{r} - \vec{r}), \quad \text{with} \quad g = \frac{4\pi \hbar^2 \sigma_s}{m}. \]
Inserting the effective potential, eq. (40), into the integral, eq. (37), together with replacing \( \Psi \) with \( \Phi \), yields the Gross-Pitaevskii equation (50; 51; 52),
\[
i\hbar \frac{\partial}{\partial t} \Phi(\vec{r}, t) = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(\vec{r}) + g|\Phi(\vec{r}, t)|^2 \right] \Phi(\vec{r}, t).
\] (41)

Finding the ground state is done by writing \( \Phi(\vec{r}, t) = \phi(\vec{r}) \exp\left(-i\mu t/\hbar\right) \) and solving the Gross-Pitaevskii equation (eq. (41))
\[
\left[ -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(\vec{r}) + g\phi^2(\vec{r}) \right] \phi(\vec{r}) = \mu \phi(\vec{r}).
\] (42)

\( \mu \) is the chemical potential and \( \phi \) is a real function normalized to the atom number, \( n(\vec{r}) = \phi^2(\vec{r}) \).

iii Thomas-Fermi approximation

In the non-interacting case, the wave function of the BEC was the standard wave function of a particle in a harmonic oscillator, a Gaussian. Introducing the interactions into the system, produces a non-linear Schrödinger equation. The non-linear term depends on the interaction between the atoms, and this depends on the s-wave scattering cross section, \( \sigma_s \). \( \sigma_s \) can be large or small, it can be positive or negative. Each one will cause the wave function to behave differently. Positive scattering length correspond to repulsive interactions and will cause the atomic cloud to "blow up", namely, to increase its size and reduce its density. Negative scattering length corresponds to attractive interactions and it will cause the cloud to collapse.

An interesting limit is when the repulsive interaction energy is much larger than the kinetic energy. This limit is interesting because it is usually within the experimental parameters. The limit is defined as \( N\sigma_s/\sigma_{ho} \gg 1 \), where \( \sigma_{ho} = \sqrt{\hbar/m\omega_{ho}} \) is the size of the harmonic oscillator ground state (the external potential ground state). If the kinetic term is neglected from eq. (42), the density profile becomes
\[
n(\vec{r}) = \phi^2(\vec{r}) = \frac{\mu - V_{\text{ext}}(\vec{r})}{g}, \quad \mu < V_{\text{ext}}(\vec{r})
n(\vec{r}) = 0, \quad \text{elsewhere.}
\] (43)

This is known as the Thomas-Fermi approximation. The normalization condition of \( n(\vec{r}) \) relates the chemical potential to the number of atoms
\[
\mu = \frac{\hbar \omega_{ho}}{2} \left( \frac{15N\sigma_s}{\sigma_{ho}} \right)^{2/5}.
\] (44)

The density profile, eq. (43), has an inverted shape of the potential, and since the trap is harmonic, the density of the cloud has a shape of an inverted parabola. This situation is somewhat analogous to water "filling up" the potential up to the "height" of the chemical potential. Comparing this approximation to numerical calculations and experimental data shows very good agreement. As long as the number of atoms is high the approximation is valid. This approximation breaks down at very small clouds, where the interaction is not strong enough to neglect the kinetic energy. It is also not valid near the edges of the cloud. At the edge, the approximation result, an inverted parabola, is replaced with a smoother transition, a tail.
Since the density is defined in the regime of $\mu < V_{\text{ext}}(\vec{r}) = 1/2m\omega_0^2r^2$, it leads to a definition of the Thomas-Fermi size of the cloud, $\sigma_{TF}$, as

$$\sigma_{TF} = \frac{1}{\omega_0} \sqrt{\frac{2\mu}{m}}.$$  \hspace{1cm} (45)

### 5.2 Signatures of Bose-Einstein condensation

After cooling the atomic cloud through the phase transition, one should observe the differences between a Bose gas and a gas in a Maxwell-Boltzmann distribution. As long as the thermal gas does not reach the phase transition, or is very close to it, it is still considered an ideal gas that obeys the Maxwell-Boltzmann distribution. The velocity distribution has a Gaussian shape, and after release it expands isotropically. See Sec. 2.5 for an example image and explanation.

When the atomic gas condenses, both the shape and the expansion after release change. In a non-interacting BEC, the shape of the condensed atoms is still a Gaussian, eq. (29), but much sharper than the thermal distribution. If the atoms are interacting, the condensate gets the trap shape - an inverted parabola, eq. (43). This parabola is much wider than the ground state Gaussian, but it is still much narrower than the broad thermal distribution.

In addition, when releasing a BEC from the trap, it’s expansion depends significantly on the trap shape, setting the initial conditions. The cloud expands faster in the axis which was narrower in the trap, i.e. the tighter axis. When the atoms are non-interacting, this effect can be explained as a result of the uncertainty principle, $\Delta x \Delta p > \hbar$. In the tighter axis, $\Delta x$ is known more precisely, hence $\Delta p$ has a larger uncertainty. If the atoms are interacting this anisotropic expansion is mostly due to the interaction energy converted into kinetic energy.

In the following I review these signatures as observed in my experiment.

#### 5.2.1 Bimodal distribution

The first signature of the BEC is the change of the atom density function of the cloud. When the atoms are thermal, their distribution is a Maxwell-Boltzmann distribution and the density function is a Gaussian. At the critical temperature, a macroscopic number of atoms are in the ground state, and their density function is their wave function squared, eq. (43). This transition between the wave functions, is more noticeable after 15 ms of time of flight (TOF), where the thermal atoms expand faster.

Fig. 21 shows three images of a cloud before the phase transition. All the images were fitted to a bimodal function, a Gaussian for the thermal cloud and a modified inverted parabola for the condensate. Eq. (43) describes the condensate wave function as an inverted parabola, in 3 dimensions. When the cloud is imaged, it’s density is integrated in the imaging axis. This integration modifies the imaged density function. For the Gaussian part, this integration doesn’t change the shape. Regarding the parabola, there is a change. The total function used for the bimodal fitting is

$$n(x, y) = n_c(x, y) + n_{th}(x, y), \quad \text{where}$$

$$n_c(x, y) = A_T F \cdot \max \left( 1 - \frac{(x-x_0)^2}{w_x^2} - \frac{(y-y_0)^2}{w_y^2}, 0 \right)^{3/2}, \quad \text{and}$$

$$n_{th}(x, y) = A_G \cdot \exp \left( -\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(y-y_0)^2}{2\sigma_y^2} \right).$$  \hspace{1cm} (46)
Figure 21: BEC transition. One is able to observe from top to bottom how the inverted parabola peak increase and becomes more dominant relative to the thermal Gaussian background.
$A_{TF}$ and $A_G$ are the general optical density amplitudes, for the condensate and the thermal parts, respectively. $x_0, y_0$ are the center coordinates and $w_x$, $w_y$, $\sigma_x$, $\sigma_y$ are the widths of the parabola and the Gaussian.

The last functions are 2-dimensional fits to the data. This can be computational intensive and less accurate, especially when there are many fitting parameters. A simple solution is to find the center and then to take a column and a row of the data. These two vectors are fitted, independently, to 1-dimension functions. 1-dimension fitting is faster and more robust. The images in Fig. 21 were fitted with this 1-dimension function

$$n(x) = A_{TF} \cdot \max \left( 1 - \frac{(x-x_0)^2}{w_x^2}, 0 \right)^{3/2} + A_G \cdot \exp \left( -\frac{(x-x_0)^2}{2\sigma_x^2} \right) + C_x. \quad (47)$$

The row and the column are fitted with the same function, changing $x$ with $y$. The addition of $C_x$ in eq. (47), compared to eq. (10), is due to small intensity fluctuations between the two images taken for the analysis, see Sec. 2.5. When the two images don’t have exactly the same intensity, in the region without atoms, this will result in a small background level. The constant should fix this problem.

The images in Fig. 21 were taken after RF evaporation, where each image depicts a slightly different end frequency. The end frequencies are 0.705, 0.7 and 0.695 MHz for (a), (b) and (c). The images were fitted to eq. (47). The results are plotted in the right side of the imaging analysis interface. The left part shows the absorption image and two cuts, through the center of the cloud, on the top and right. The cuts show the data and the fit. In the fit, the black curve is the Gaussian part and the red curve is the parabola part (in addition to the Gaussian).

In (a), most of the cloud is still thermal, and in (b) and (c) the condensate part is increasing as the RF end frequency is lowered. Lowering the end frequency of the RF results in lower temperatures. For example, $\hbar \times 5kHZ = k_B \times 240nK$. This relation gives an indication of the temperature change between the images. The fraction of the condensate can be estimated from the images directly or from the fit, from the ratio of $A_{TF}$ to $A_G$. In addition, the total number of atoms becomes smaller as the RF end frequency is made lower.

Another important signature can be noticed in this set of images. When looking at the thermal part, its size is equal in the $x$ and $y$ directions. Opposite to that, the condensation part is asymmetric. Its radial size, $w_y$ in the interface, is more than 3 times larger than its axial size. This difference is enhancing as the cloud is cooling and the BEC part increasing. This asymmetry is explained in the next section.

### 5.2.2 Anisotropic expansion

#### i Theory

In the previous section, the BEC wave function was described, but only for the ground state. Imaging the cloud in-situ, in the trap, is possible, but the data that can be extracted from such an image is limited. The size of the condensate, $\sigma_{TF}$ (eq. (15)), can be extracted from the images. This size, according to calculations is $\sim 2.2\mu m$. For comparison, in our imaging system, the magnification is 1.5 and the pixel size is $6\mu m$, which leads to an effective pixel size of $4\mu m$. With this effective pixel size nothing below $4\mu m$ can be measured and in order to achieve an accurate measurement, the object needs to be larger than $\sim 10\mu m$. Therefore, in our system we need to let the cloud fall and expand.
The evolution of the wave function after release from the trap follows a simple re-scaling of its parabolic shape. The derivation is described in (33, 34). This solution assumes a cigar-shaped cloud, namely an elongated cloud, for which the size in two axes is much smaller than in the third. This is equivalent to the differences in the trap frequencies. In these traps, it is convenient to change the coordinate system to cylindrical coordinates. In this coordinate system the trap has radial, $\omega_r$, and axial, $\omega_z$, frequencies with a ratio of $\omega_r/\omega_z = \epsilon^{-1}$. The evolution in time of the half-length of the condensate in the radial and axial directions is (ignoring terms with $\epsilon^4$ and $\tau = \omega_r t$)

$$\rho(t) = \rho_0 \sqrt{1 + \tau^2}, \quad \text{and}$$

$$z(t) = \epsilon^{-1} \rho_0 \left(1 + \epsilon^2 (\tau \arctan \tau - \ln \sqrt{1 + \tau^2})\right). \quad (48)$$

This solution describes 3 stages of the expansion.

- $\tau < 1$. After the release, $t \ll \omega_r^{-1}$ and the lengths evolve as a square-root function. This indicates a radial acceleration, where the interaction energy is converted to kinetic energy.

- $1 < \tau < \epsilon^{-2}$. The radial direction expands linearly while the axial direction doesn’t expand noticeably.

- $\epsilon^{-2} < \tau$. Both radial and axial axes expand linearly with an asymptotic aspect ratio of $z(t)/\rho(t) = \pi \epsilon^2 / 2$.

We note that for times $t \gg \omega_r^{-1}$ the velocity of the radial expansion satisfies

$$\frac{1}{2} m v_r^2 = \mu. \quad (49)$$

**ii Measurements**

Fig. 22 shows a series of images taken after different (increasing from left to right) time-of-flight (TOF) periods following the release the trap. The figure is made of different stripes taken from different images and put together to create one image. Each image is normalized to the peak density of its own cloud. This leads to a few effects: The cloud looks clear in all the images, relative to the background, and it falsely seems as if the density of the cloud doesn’t reduce with longer TOF. This normalization causes the background noise level to increase with longer TOF. The reason for this normalization is that the optical density of the BEC is 0.7 after 5 ms and only 0.17 after 22 ms. If all the images were normalized to the first image peak optical density, the images would look more real but the cloud after a long TOF will be barely observable.

The main phenomenon observed in this series of images is that the cloud expands mostly in the radial direction.

**iii Analysis**

In order to analyze the data, the trap frequencies need to be known. The radial frequency measurement is shown in Fig. 23. This measurement was done with a $1 \mu K$ cloud. The cloud was "pushed" in the trap, started to oscillate, and was imaged after different times. The "pushing" is done by lowering the current in the $y$ bias coil and returning it to the previous
Figure 22: BEC time of flight. A series of images of increasing TOF. Each image is normalized to the peak density of the cloud in that image, hence there is no visible reduction of the optical density with the expansion. For the same reason, the level of the background noise is increasing with the TOF. The anisotropic expansion is apparent. In the trap (not shown) the cloud is elongated with the longitudinal (horizontal) axis being longer. As it falls the radial axis expand faster and after $5\,\text{ms}$ the cloud looks round. After $11\,−\,13\,\text{ms}$ the cloud starts to look elongated in the radial (vertical) direction, a tendency which becomes clearer with longer TOF.

value. When the $y$ bias field value is lower, the trap is farther from the atomchip, and the cloud starts to move toward the new minimum. After restoring the previous value of the $y$ bias, the trap position returns to the original position and the cloud follows the new minimum again. If these two changes happen slowly, the cloud will manage to follow the minimum of the trap as it changes and no oscillations will appear. Doing this change faster than the speed that the cloud can follow, the change in the trap minimum will cause the atoms suddenly to feel a different trap. In this new trap the cloud is not in the center, but on the “side” of the trap. From that position the cloud will move to the center, changing its potential energy to kinetic energy, and since there is no friction it will oscillate. The rate of change of the magnetic field the atoms can follow is the inverse of the trap frequency, $w_r^{-1}$.

Fig. 23 shows a measurement of the radial trap frequency. It shows the distance from the atomchip for different oscillation times. This distance is normalized to the initial position, so a simple fit to a sine function is feasible. The images were taken from $2\,\text{ms}$ after the ”pushing”, up to $9\,\text{ms}$.

From the fit of the data to a $A\sin(\omega t + \phi)$ function, the frequency is

$$\omega_r = \omega \cdot 10^3 = 2\pi \times 565\,\text{Hz}.$$  \hspace{1cm} (50)

The longitudinal frequency cannot be measured in our system. To measure this frequency the cloud needs to be pushed to the side. If the currents in the $y$ bias or the Z-wire will be changed, the trap will only move up-down. Changing the $x$ bias will only cause the value of the magnetic field at the minimum to change, without any spatial movement. Also using a z bias will not cause any motion in the longitudinal direction. This frequency was taken from our simulation to be

$$\omega_z = 2\pi \times 45\,\text{Hz}.$$  \hspace{1cm} (51)

After knowing the frequencies, and using eqs. (48), the times of the different stages in the
Figure 23: Trap frequency measurement. In this measurement the cloud was "pushed" away from the atomchip (kicked from the center of the trap) and its distance from the atomchip was monitored. In the figure, the distance is normalized to the initial position and fitted to a sine function.

![Graph showing oscillation time vs. oscillation amplitude](image)

Figure 24: Gravity and chemical potential measurement of a BEC. (a) After being released from the trap the cloud falls under gravity. (b) Measuring the temperature of a BEC has no meaning. There is no thermal distribution, hence there is no temperature. From this fit only the slope of the radial axis (y in the figure) should be considered and used as $v_r^2$ in eq. (49).

expansion can be found. The condition for the end of the first stage is

$$\tau = \omega_r t = 1 \rightarrow t \approx 0.28 \text{ms}. \quad (52)$$

This shows that the TOF measurement presented previously, which started after 5ms, is in the radial linear expansion regime. The third stage will start after

$$\tau = \omega_r t = \epsilon^{-2} \rightarrow t \approx 45 \text{ms}. \quad (53)$$

The third stage takes place only after the TOF measurement ends, hence in this measurement, the longitudinal axis is not expected to expand at all. Indeed, the size of the condensate in the longitudinal axis hardly changed. Being in the second stage, the system follows eq. (49).

Fig. 24 shows two different fits of the same TOF measurement of Fig. 22. In (a) the distance from the atomchip was fitted to a parabola. This is done to find several parameters. The first is to see that the cloud is falling under gravity. If it doesn’t, there might be a
problem with the calibration of the imaging system magnification. All the calculations are done with an effective pixel size which is the real pixel size divided by the magnification. If the magnification is wrong, it will look like the cloud is falling faster or slower than what is expected under gravity. Second, the fit finds also the initial position, which is the same as imaging the cloud in-situ. Third, the fit has a free parameter for $v_0$, initial velocity. This parameter describes if the cloud was ”pushed” during the release and it is expressed in the figure in \( \text{cm/s} \) and also as temperature units. The temperature is found as $T = \frac{mv^2}{k_B}$ (a convenient unit to measure energy).

Fig. 24(b) shows a temperature measurement of a BEC\(^{13}\). In general, measuring the BEC temperature is wrong, since the BEC has no temperature. It’s not a thermal gas with a Maxwell-Boltzmann distribution anymore, and hence its temperature is not well defined. This measurement and fit are done to measure the chemical potential. Eq. (49) relates the chemical potential to the speed the cloud expands. A temperature measurement relates the same speed to the temperature, and hence the mechanism is the same, only the interpretation is different. From eq. (49), and knowing that $v_r = r/t$, it is clear that the chemical potential is the slope of the fit times $m/2$, namely,

$$\mu = \frac{1}{2}m \times 2.64 \cdot 10^{-5} \approx h \times 2.9k\ Hz. \tag{54}$$

The last conversion is to conveniently compare $\mu$ to the trap frequencies and the trap ground state energy. It is worth noting that the same chemical potential is extracted from the temperature found from the fit (after converting units)

$$\mu = \frac{k_B T}{2} = \frac{k_B}{2} \times 275.5nK \approx h \times 2.9k\ Hz, \tag{55}$$

where the Boltzmann constant is divided by 2 because $\mu = 1/2mv^2 = 1/2k_B T$.

6 Summary and conclusions

As described in part II of this document, when I started my master studies, we had an empty laboratory, as can be seen in Fig. 1. With the help of a post doctoral fellow, Dr. Plamen Petrov, I built the whole apparatus (see different images in Sec. 2), optimized the different stages (sections 3, 4) and achieved the first signal of a Bose-Einstein condensation in September, 2008.

The current status of the system is that we have a stable Bose-Einstein condensation of $\sim 2 \cdot 10^4$ atoms, and the system is ready to start doing the proposed experiments, as they are described in part I of this document.

\(^{13}\)The size of the cloud was taken from a 2-dimension Thomas-Fermi fit. A similar fit to eq. (49) without the thermal part.
References


REFERENCES


