Bose-Einstein Condensate on the Atom Chip

Thesis submitted in partial fulfillment of the requirements for the degree of "DOCTOR OF PHILOSOPHY"

By

Ran Salem

Submitted to the Senate of Ben-Gurion University of the Negev

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Approved by the advisor Prof. Ron Folman
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January 2010

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Faculty of Natural Science.
Abstract

The atom chip is a micro-fabricated, integrated device in which electric, magnetic and optical fields can confine, control and manipulate ultracold atoms and Bose-Einstein condensates (BECs). The atom chips are fabricated using the well-established technologies taken from the semiconductors industry. They allow the design and generation of a variety of potentials for the trapped atoms. The atom chip is especially advantageous at very small atom-surface separations, where higher trap gradients are achieved and allow a better control in the designed potentials. However, interactions with the nearby surface become important in such small atom-surface separations, and they can limit the minimal separation from the surface. For example, spatial and temporal magnetic field fluctuations, due to electron scattering and Johnson noise, respectively, limit the minimum atom-surface distance, as they cause potential corrugations, spin flips, heating and decoherence.

The main aim of the current work was to design and build the atom chip experiment in Ben-Gurion University (BGU) (the first in Israel), and to further enhance our understanding of the effects limiting atom-surface separations in atom chips. The atom chip experimental apparatus in BGU was built from scratch. We have achieved a BEC of $30 \times 10^3$ $^{87}\text{Rb}$ atoms, and successfully loaded it to the atom chip traps, attaining atom-surface separations as low as $5\mu\text{m}$. This experimental apparatus is a crucial milestone that will set the base for the future experiments that are discussed in this thesis.

Furthermore, we have made a comprehensive experimental and theoretical study of the spatial corrugations in the static magnetic field generated by the atom chip trapping wire. This is measured by the density variation of the trapped atomic sample. Based on the performed atomic magnetometry, we could present a general theory of current deviations in straight current-carrying wires with random imperfections, as well as to explain the measured long range organized patterns oriented at $\pm45^\circ$ relative to the mean current flow. The surprising length of these patterns and their orientation is explained as a direct consequence of universal scattering properties at defects. The observed amplitude of the current direction fluctuations scales inversely to that expected from the relative thickness variations, the grain size, and the defect concentration, all determined independently by standard methods.

We also present an analysis of magnetic traps for ultracold atoms based on current-carrying wires with sub-micron dimensions. The physical limitations of these conducting wires are analyzed. We study the effects of surface thermal noise, electron scattering within the wire, and the Casimir-Polder force. We show that wires with cross sections as small as a few tens of nanometers should enable robust operating conditions for coherent atom optics (e.g. tunneling barriers for interferometry). In particular, trap sizes of the order of the deBroglie wavelength become accessible, based solely on static magnetic fields, and thereby bringing the atom chip a step closer to fulfilling its promise of a compact device for complex and
accurate quantum optics with ultracold atoms.

Finally, we present an overview of the current status of our atom chip experiment that implements some of the above results. Here we use a single wire on the atom chip to generate a periodic magnetic potential. Localization of the atomic ground state in this periodic potential can then be controlled and we discuss the interference pattern expected after releasing the atom cloud. The design of this experiment required special considerations of the trapped BEC time evolution, once released from the trapping potential. We have learned that the trap release rate plays an important role in the formation of the atomic density interference patterns. This experiment is already in progress.

Our work resulted in the following publications:


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To my wife Shimrit,
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Chapter 1

Introduction

In 1925 Einstein (extending Bose’s ideas for photons) predicted that at low temperatures particles in a gas would all reside in the same quantum state [5, 6]. This peculiar gaseous state, a Bose-Einstein condensate (BEC), was produced in the laboratory for the first time in 1995 [7, 8] using powerful laser cooling [9, 10, 11] and evaporative cooling [12] methods developed in recent years.

Condensates exhibit quantum phenomena on a macroscopic scale. The study of BECs encompasses a variety of different subfields in physics, including atomic, condensed matter, nuclear, and quantum physics. In this introductory chapter I will briefly review the basic concepts of the BEC. Deeper introductions to the theory and practice of BECs can be found in several textbook and review papers [13, 14, 15, 16].

In 1999 a new tool for the manipulation, control, and detection of ultracold atoms was introduced. Called the atom chip, it is a micro-fabricated, integrated device in which electric, magnetic and optical fields can confine, control and manipulate cold atoms, thereby miniaturizing BEC and ultracold atom experiments. The atom chip is our main experimental tool, and I will also review its main properties in this short introduction. Here again, deeper reviews can be found in [17, 18, 19].

1.1 Bose Einstein condensation

Quantum mechanics associates a deBroglie wavelength with particles; for a thermal distribution at temperature $T$ this wavelength is $\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$, where $m$ is the particle mass. When the deBroglie wavelength is on the order of the inter-particle distance $d$, or when

$$n\lambda_{dB}^3 > 1$$  \hspace{1cm} (1.1)

where $n$ is the density, there is, in some sense, an overlap between the particle wave functions. In the case of indistinguishable particles governed by Bose statistics, this leads to a condensation of a macroscopic fraction of the particles in the ground state.
Einstein was not sure whether this effect could be observed in nature. In the 1940s BEC was considered as the mechanism for superfluidity in liquid helium, but this issue was under great dispute. In any case the liquid helium “condensate”, if it existed, was far from that of an ideal gas or even a weakly interacting one. At the same time, a theory of weakly interacting BEC was developed in the context of superfluidity, pioneered by Bogoliubov [20]. Finally in 1995 experimental groups at MIT [7] and JILA [8] succeeded in condensing a gas of weakly interacting particles, in both cases the vapor of alkali metals. There has since been an exponential growth in both theoretical work and experiments studying and using such systems.

1.1.1 The quest for ultralow temperatures

Cold atoms have many advantages for studying quantum phenomena. A major one is that in the BEC, essentially all atoms occupy the same quantum state, and the condensate can be described very well in terms of a mean-field Hartree-Fock theory for the entire ensemble of atoms. The physics which emerge from this effective Hamiltonian includes inter-atomic interactions, and is therefore both more accurate and exhibits richer effects than the ideal gas. A BEC behaves in many ways like a macroscopic coherent matter wave, similar to a coherent radiation field (a sample of atoms released from a BEC is therefore also referred to as an atom laser [21]), allowing sensitive measurements of the BEC quantum phase through interference experiments.

It is important to remember that the equilibrium phase of the alkali metals, like the Rb we use for BEC, is not gaseous but is, rather, solid. The vapor is metastable and when possible, the gas will solidify. Luckily, energy and momentum conservation forbid molecule formation as a result of two-body collisions, so three-body interactions are required for nucleation. Therefore the gas phase can be stable for BEC formation if the density is low enough. Conversely however, the BEC condition Eq. (1.1) requires \( n > 1/\lambda_{dB}^3 \). Since the density must be small, the only way to achieve BEC is to make \( \lambda_{dB} \) large by working at very low temperatures. For our experiments this temperature turns out to be \( \sim 300 \text{ nK} \).

Although the atomic density in BECs is low, two-body interactions do play a significant role. They change the ground-state wave function, and introduce many-body correlations as described below.

1.1.2 The many-body Hamiltonian

The single-particle Hamiltonian for atoms in a potential is:

\[
H_{sp} = -\frac{\hbar^2}{2m} \nabla^2 + V_{tr}(\mathbf{r}, t). \tag{1.2}
\]

Typically experimental traps can be approximated by a harmonic trapping potential \( V_{tr}(\mathbf{r}) = \frac{1}{2} \sum_{j=x,y,z} m \omega_j^2 r_j^2 \), but this discussion is valid for different trapping potentials as well as for time-dependent potentials.
1.1. BOSE EINSTEIN CONDENSATION

A realistic discussion of the many-body BEC must account for atom-atom interactions. For macroscopic samples, such interaction potentials are in general very hard to calculate. Two major approximations may then be made: neglecting three-body interactions, and assuming only $s$-wave scattering. Although three-body interactions do play a role, this process is suppressed by the diluteness of the atomic cloud, since the probability of a three-body interaction for a given atom is proportional to $n^2$. Furthermore, BEC temperatures are typically well below those required for $d$-wave scattering.

For dilute gases at low temperatures the atomic pairwise interaction $V(|r_1 - r_2|)$ can be modeled as a pseudo-potential $V_{\text{int}} = g\delta(r_1 - r_2)$ [14, 15], where the interaction prefactor $g$ is related to the $s$-wave scattering length $a$ by $g = 4\pi\hbar^2a/m$ and $\delta$ is the Dirac delta function ($a = 5.052$ nm for $^{87}$Rb [22]). The effective many-body Hamiltonian in a static trap may be written as:

$$H = \sum_{i=1}^{N} \left[ -\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{tr}}(r_i) \right] + g \sum_{i<j} \delta(r_i - r_j). \quad (1.3)$$

For the typical parameters of our BEC, we can restrict the discussion to zero temperature. The many-body wave function of the condensate in a static trap, $\Psi$, can be described using a mean-field Hartree-Fock ansatz [13, 14, 15]:

$$\Psi(r_1...r_N) = \prod_{i=1...N} \phi(r_i), \quad (1.4)$$

where $\phi(r_i)$ are the single-particle wave functions.

1.1.2.1 Gross-Pitaevskii equation

The mean-field approximation is useful for explaining many physical effects. By multiplying both sides of the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi = H \Psi \quad (1.5)$$

for the many-body Hamiltonian [Eq. (1.3)] by $\prod_{i=2...N} \phi(r_i)$, and integrating over coordinates $i = 2...N$, we get the Gross-Pitaevskii equation for $\phi$:

$$i\hbar \frac{\partial}{\partial t} \phi(r, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{tr}}(r) + gN |\phi(r, t)|^2 \right] \phi(r, t), \quad (1.6)$$

with $N - 1 \simeq N$. The wave function $\phi$ is often referred to as the condensate wave function. The Gross-Pitaevskii equation has a form similar to that of the Schrödinger equation, in which the external potential acting on the particles $V_{\text{tr}}(r)$ is supplemented by a non-linear term $gN |\phi(r)|^2$ that includes the effective mean-field potential produced by the other bosons. The ground state of $\phi$ is then found by the time-independent Gross-Pitaevskii equation:

$$\mu\phi(r) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{tr}}(r) + gN |\phi(r)|^2 \right] \phi(r) \quad (1.7)$$
where the chemical potential $\mu$ is the ground state energy.

### 1.1.2.2 Thomas-Fermi approximation

Under typical experimental conditions the atomic interactions are dominant $[N g |\phi(0)|^2 \simeq \mu \gg \hbar \omega_{j}$ for $j = x, y, z]$ and we can neglect the kinetic energy term in comparison to the interaction energy term. In this regime there are no longer any operators on the right-hand side of Eq. (1.7) and we obtain the Thomas-Fermi approximation:

$$
\phi_{\text{TF}}(r) = \begin{cases} 
\left(\frac{\mu - V_{\text{tr}}(r)}{Ng}\right)^{1/2}, & \text{for } \mu \geq V_{\text{tr}}(r) \\
0, & \text{otherwise.}
\end{cases}
$$

The chemical potential $\mu$ is determined by the normalization of $|\phi\rangle$:

$$
\mu = \frac{1}{2} \hbar \bar{\omega} \left(15Na \sqrt{\frac{m\bar{\omega}}{\hbar}}\right)^{2/5},
$$

where $\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$. The Thomas-Fermi approximation becomes invalid when a BEC is confined by a very elongated 1D trap, in which the transverse frequency $\omega_{r}$, perpendicular to the cylindrical axis, becomes much larger and $N g |\phi(0)|^2 \simeq \mu \leq \hbar \omega_{r}$. This regime is often reached in atom chip traps, for which the Gross-Pitaevskii equation must instead be solved numerically.

### 1.2 The atom chip

Ultracold atoms are extremely sensitive to their environment, yet their trapping and isolation allows them to approach room-temperature objects, thus opening the possibility of a device operating at ambient temperature that can be used to control and manipulate the atoms. The atom chip combines two modern techniques to achieve this goal: on the one hand controlling ultracold atoms and BECs, and on the other hand, fabricating a solid-state chip using the mature technology of the semiconductor industry to enable miniaturization. The principle of the atom chip is simple: to create the electric [23], magnetic [17, 18, 19, 24, 25], light [26, 27, 28, 29, 30, 31, 32, 33, 34, 35], RF [36, 37] and MW [38] fields needed in order to trap, manipulate and measure atoms in the quantum realm where they behave as matter waves. The atoms can be trapped and manipulated just a few microns above surfaces that are designed and fabricated specifically for these purposes.

In 2001 BEC was first achieved using atom chips [39, 40]. The potentials generated by the atom chip have led to outstanding achievements at atom-surface distances of $1 - 100 \mu m$, e.g., spatial interference [36, 37, 38] and hyperfine state interferometry [41]. These techniques have also allowed access to coherence measurements in reduced-dimensional BECs [42].
1.2. Neutral atom trapping

Once cooled to ultralow temperatures, neutral atoms can be manipulated by means of their interaction with magnetic, electric, optical, and RF fields. In this work we will only consider magnetic potentials, which are the atom chip’s most common trapping tool. The magnetic fields used for trapping and guiding the cold atoms on an atom chip are produced by microscopic current carrying wires fabricated on the chip’s surface. The power of these devices lies in the flexibility of the magnetic fields that can be created: almost any planar arrangement of conductors can conveniently be fabricated with lithographic techniques pioneered in the electronics industry.

Trapping atoms in magnetic traps is possible due to the coupling (Zeeman) energy between atoms and the external magnetic field:

\[ U = -\mu \cdot B \simeq g_F m_F \mu_B |B|, \]  

where \( m_F \) is the projection of the atomic spin onto the external field direction, \( g_F \) is the Land \( \acute{\text{e}} \) factor of the atomic (hyperfine) state, \( \mu_B = h \times 1.4 \text{ MHz/G} \) is the Bohr magneton, and \( B \) is the magnetic field. Different internal atomic states will interact differently with the field, and in the context of trapping may be categorized by the direction of the magnetic moment relative to the magnetic field: strong-field seekers are drawn towards regions of high fields, while weak-field seekers are attracted towards field minima. Since a three dimensional magnetic maximum cannot be produced, only weak-field seeking states can be magnetically trapped. I will now describe the simplest case of generating a magnetic field minimum using current-carrying wires on the atom chip.

1.2.2 Magnetic potentials

The simplest form of a magnetic field suitable for trapping atoms generated by a current-carrying wire is a “wire guide”. In this configuration, the magnetic field generated from a straight current-carrying wire is combined with a homogeneous external bias field \( B_y \) perpendicular to the wire. Because the field of a (thin) wire decays as \( 1/r \) with the distance \( r \) from the wire, \( B_y \) will always compensate the wire field at some distance \( r = h \). This leads to a two-dimensional quadrupole field, confining the atoms along a line parallel to the wire at the distance \( h \). For very cold atoms, this field must be modified, since near the zero of the magnetic field spin flips between trapped states (weak-field seekers) and untrapped states (strong-field seekers) may occur. These so-called Majorana spin flips occur with greater probability as the two states approach degeneracy at low field strengths. A non-vanishing magnetic minimum is accomplished by adding a bias field \( B_x \) parallel to the wire (Ioffe-Pritchard field) [17].

The wire guide described above generates a cylindrical potential (a guide). In order to generate a three dimensional trap, the wire can be bent (or fabricated) in a Z shape. The leads of the Z-shaped wire generate a confining field at both ends of the wire guide, thus serving as “end caps” for the three-dimensional potential trap.
1.2.3 Limitations of the atom chip

Traps with large gradients (frequencies) are advantageous; for example, they give rise to smaller vibrational heating rates since the energy levels become more widely separated. They also enable more tightly confined ground state wave functions and thus higher resolution in tailoring potentials. Further decreasing the atom-surface distance should increase trap gradients sufficiently to construct tunneling barriers with widths on the order of the atomic deBroglie wavelength enabling, e.g., atom chip interferometry based solely on static magnetic fields. Furthermore, sub-micron distances are also important for technological advantages such as low power consumption and high-density arrays of traps. However, it remains important to understand what ultimately limits the atom-surface distance.

At small atom-surface distances, interactions with the nearby surface become important. For example, spatial and temporal magnetic field fluctuations, due to electron scattering and Johnson noise respectively, limit the minimum atom-surface distance, as they cause potential corrugations, spin flips, heating and decoherence. There have been several experiments utilizing cold atoms to study these interactions [43, 44, 45, 46, 47, 1, 48], and many suggestions on how to overcome their damaging effects [49, 50, 51, 2, 3, 52].

Also becoming prominent for small atom-surface distances is the Casimir-Polder (CP) force [53]; normally attractive, it reduces the magnetic barrier and allows atoms to tunnel to the surface, as already observed experimentally [54, 55].

Finally, let us note that the limitations could also be turned around to become advantages. At very small distances the atoms may also serve as a sensitive probe for surface phenomena. One example is electron scattering which we observed and is described in this thesis. Another example are plasmons which are expected to affect the atomic external and internal degrees of freedom and may also become observable [56].
1.3 Outline of this thesis

The initial work of my Ph.D. was to build, from scratch, the atom chip experimental setup at BGU. Together with another Ph.D. student, Tal David, we built this setup and achieved a BEC. I then continued to successfully load the BEC to the atom chip wire traps, and to trap and probe the atoms at minimal atom-surface separations (as low as $5 \mu m$), as well as to conduct both experimental and theoretical studies of atoms so close to surfaces.

The thesis is organized as follows:

The experimental setup we have built, and the experimental procedures optimized to achieve a BEC and to load it to the atom chip traps, are presented in Ch. 2.

Chapter 3 presents a study of static potential corrugations in atom chips. We present a full model for potential corrugations arising from current directional deviations in the trapping wire. We also present the surprising observation of long-range correlations forming organized patterns oriented at $\pm 45^\circ$ relative to the mean current flow. This chapter is based on the following publications:


In Ch. 4 we present an analysis of magnetic traps for ultracold atoms based on current-carrying wires with sub-micron dimensions. We analyze the physical limitations of these conducting wires, as well as how such miniaturized magnetic traps are affected by the nearby surface due to tunneling to the surface, surface thermal noise, electron scattering within the wire, and the Casimir-Polder force. We show that wires with cross sections as small as a few tens of nanometers should enable robust operating conditions for coherent atom optics (e.g., tunneling barriers for interferometry). This chapter is based on


In Ch. 5 we present an overview of the current status of our atom chip experiment that implements some of the above results. Here we use a single wire on the atom chip to generate a periodic magnetic potential. Localization of the atomic ground state in this periodic potential can then be controlled and we discuss the interference pattern expected after releasing the atom cloud.

A summary and an outlook are given in Ch. 6.
In addition to the work presented in this thesis, I played an active role in studying static magnetic corrugations and surface-induced noise in anisotropic conductors. This topic will not be discussed in this thesis but is presented in Ref. [57] and in:

Chapter 2

The BGU atom chip experiment

In this chapter an integrated setup for loading a BEC to atom chip traps is presented (Sec. 2.1). A wire-based magneto-optical trap (MOT) allows simplified trapping and cooling of a large number of atoms near a material surface. With a U-shaped current-carrying copper structure generating a magnetic quadrupole field, more than $10^8 ^{87}\text{Rb}$ atoms are collected in our mirror MOT. These atoms are subsequently loaded to a Z-wire magnetic trap where they are cooled to a BEC by means of evaporation cooling (Sec. 2.2). Properties of the BEC are discussed in Sec. 2.3, and loading to the wire-based trap of the atom chip is discussed in Sec. 2.4.

This chapter presents the major part of my studies which was devoted to designing the apparatus, building it from scratch, and then operating it. Additional experimental and theoretical results, as well as further experimental proposals, are discussed in the following chapters. Since general introductions to the experimental methods we use are available in several textbooks [16, 58, 59], and as most aspects of our setup have already been described in the theses of my partner to this project and an MSc student working with us [57, 60], I will, in the following, only briefly describe the experimental apparatus itself.

2.1 Experimental setup

Generating a BEC is routinely achieved in many laboratories all over the world. Standard techniques of laser cooling of neutral atoms are used to collect a large number of atoms in a MOT. These pre-cooled atoms are transferred to a magnetic or optical trap where Bose-Einstein condensation is achieved by evaporative cooling. These experiments are performed inside an ultra-high vacuum (UHV) chamber. The following presents each major apparatus component in the same sequence as its actual use during the experimental cycle.
CHAPTER 2. THE BGU ATOM CHIP EXPERIMENT

2.1.1 Vacuum system

In order to maintain isolation of the cold atomic sample it is essential to maintain ultra high vacuum (UHV). In our system the measured pressure is routinely below $10^{-11}$ torr. All the vacuum components are made of 316LN stainless steel, due to its low magnetic permeability. The experimental chamber has an octagonal cross section with a height of 71 mm and a diameter of 200 mm. This main science chamber is connected to the vacuum pumps and the vacuum gauge by a 6-way cross (Fig. 2.1). The atomic sample is optically accessed through high-quality windows: a large window is mounted on the bottom (clear diameter 11 cm) and seven smaller windows (clear diameter 28 mm) are mounted around the circumference. These windows have high optical quality in terms of flatness and are anti-reflection coated for the Rb D2-line excitation wavelength (780 nm).

The experimental procedure requires introducing a specific amount of Rb into the chamber. The Rb source consists of four copper feedthroughs, whereby each pair is connected to two Rb dispensers in parallel. These dispensers are heated up by a direct current and the Rb, which is present as a crystalline salt in the dispenser core, is chemically reduced to the metallic element and evaporates. It is important to maintain good vacuum in the experimental chamber, especially during the time the atoms are trapped in the magnetic trap. Therefore, the dispensers are operated in a pulsed mode: for the first 10 s of the experiment, while the MOT is loaded, the dispensers are heated by running 15 A through them, and the current is switched off for the rest of the experimental cycle. With this mode of operation a lifetime of $\sim 50$ s of the magnetic trap is achieved (Sec. 2.2.2.1).

Several vacuum pumps are needed to achieve UHV; a dry scroll pump (Varian TriScroll 300) and a turbo pump (Varian turbo-71, 70 l/s) are used to pump from atmospheric pressure to high vacuum (down to $\approx 10^{-8}$ torr) (see [57, 60] for more details). Once high vacuum is achieved, we start pumping with an ion pump. Further lowering the pressure to the desired UHV level ($< 10^{-11}$ torr) requires a week of ‘baking’, a process in which the vacuum system is heated to $150^\circ$ C to remove condensed gases such as water vapor. We use a large (300 l/s) ion pump to maintain UHV and to allow fast vacuum recovery throughout the experimental cycle. Because the ion pump needs strong permanent magnets which could interfere with the experiment, it is attached to the 6-way cross via a long connector tube (‘nipple’) to take it farther away from the experimental chamber. To maintain the UHV we also use a titanium sublimation pump (TSP), typically at weekly intervals.

2.1.2 Laser system

The laser setup used in the experimental apparatus can be divided into three sections: the first section consists of two diode lasers, each of which has a spectroscopy setup (required for the laser frequency stabilization). In the second stage

---

1 Measured with Varian UHV-24p made Bayard-Alpert type ionization gauge
2 Varian VaIon Plus 300 pump, noble diode
2.1. EXPERIMENTAL SETUP

![Diagram of vacuum chamber](image)

Figure 2.1: Schematic drawings of the vacuum chamber. The main chamber consists of a stainless steel octagon which allows good optical access through seven side windows and a large window at the bottom. All the vacuum parts are made of 316LN stainless steel. In this setup we reach a vacuum of approximately $10^{-12}$ torr and achieve a trap lifetime of approximately 50 sec.

of the laser setup, the two light beams are split and shifted in frequency by four acousto-optical modulators (AOMs). After modulation, the light is sent into the third stage by means of optical fibers, where it is guided into the vacuum chamber in which the experiments are being performed. Each stage of the laser setup is briefly discussed in the following.

2.1.2.1 Laser lock

To achieve laser-cooling of neutral atoms, intense light having a wavelength close to a strong atomic transition is needed. In $^{87}$Rb the D2-line ($\lambda \sim 780$ nm) provides a strong cooling transition. The corresponding near-infrared light can easily be generated by semiconductor laser diodes which allow small and robust laser setups. These laser diodes have to be frequency-stabilized and narrowed below the natural linewidth of the corresponding Rb transition, which is $\Delta \Gamma \sim 6$ MHz. By monitoring the absorption of the laser light in a Rb vapor cell, an error signal can be generated that is proportional to the deviation of the laser frequency from the atomic resonance. This electronic signal is then used in a feedback loop to lock the laser frequency. We use two laser systems to generate the four different light frequencies required for the experiment; a commercial tapered amplifier laser system (Toptica TA-100), that can generate an output power of 800 mW, is used for the cooler beam, optical pumping beam, and the imaging beam (Fig. 2.2). A second laser, (Toptica DL-100), capable of generating an output power of 70 mW, is used for the re-pumper beam. The error signals discussed above are generated by a weak 20 MHz modulation of the diode laser input current. In our setup, this technique is the most effective in terms of stability when compared to other methods, such as polarization lock and light frequency modulation by an electro-optic modulator (EOM).

Both lasers are locked to a cross-over peak near the atomic hyperfine-structure
transitions. The cooling laser is stabilized to the $F = 2$ to $F' = 1, 3$ cross-over peak and the repumper laser is stabilized to the $F = 1$ to $F' = 1, 2$ cross-over. The laser frequencies are shifted close to the desired Rb transition using AOMs, as discussed in the next paragraph.

The laser frequencies are shifted close to the desired Rb transition using AOMs, as discussed in the next paragraph.

![Diagram of hyperfine structure of the D-line of $^{87}$Rb. In our experiment we use the D2-line. The different light beams required for the experiment are shown on the right.](image)

**Figure 2.2:** Hyperfine structure of the D-line of $^{87}$Rb. In our experiment we use the D2-line. The different light beams required for the experiment are shown on the right.

### 2.1.2.2 Light manipulation

The wavelength of the laser light, which is locked to the cross-over peaks, can be shifted by an AOM to the precise atomic transition frequency, or to a desired detuning with respect to it. A radio-frequency acoustic wave in the non-linear AOM crystal causes density modulation, thus diffracting the incident laser light. As a result, the frequency shift (and the deflection angle) depends on the diffraction order. Typically, the first diffraction order is used and the light is shifted by $50 - 150$ MHz with an efficiency of 70%-80%. In the setup shown in Fig. 2.3 the AOMs are used either in a single-pass or in a double-pass configuration. The double-pass configuration is advantageous if the frequency shift of the laser light has to be changed during the experiment, because the deflection angle is automatically compensated as the beam is retro-reflected (cooler beam and imaging beam). Single-pass AOMs are usually operated at a fixed frequency (repumper beam and optical pumping beam). In our setup, the light provided by the TA-100 is divided into three paths: one is shifted by a double-pass AOM operated at 98 MHz to be 16 MHz red-detuned with respect to the cooling transition. The second beam is

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3These additional peaks arise from the pump and the probe beam being resonant with different transitions of a velocity class ($v \neq 0$), whereas the standard Lamb dips appear when pump and probe beams are both resonant with the same transition of a velocity class $v = 0$. Thus the cross-over peaks appear at frequencies $(\omega_1 + \omega_2)/2$ where $\omega_i$ is the frequency of the atomic hyperfine transition. This provides additional reference frequencies which can be used to stabilize the lasers.
shifted in a double-pass AOM by 106 MHz to be directly on the atomic resonance for imaging purposes. The last beam is shifted in a single-pass AOM by 50 MHz to be resonant with the optical pumping transition. Finally the light provided by the repumper laser is shifted in a single-pass AOM by 80 MHz to be resonant with the repumper transition. In addition, these AOMs provide very fast intensity modulation and even on/off switching of the laser light in less than 1 µs with an extinction ratio of $10^{-3}$ for the single-pass configuration and $10^{-6}$ for the double-pass configuration. These fast switches are accompanied by relatively slow mechanical shutters (500 µs switching time) to block the small amount of residual light. All the light beams are coupled into single-mode fibers and fed to the experimental chamber. The imaging light is divided into two fibers for the different imaging axes (see Sec. 2.1.3).

2.1.2.3 Around the science chamber

Outputs of the various fibers are isolated from the laser light manipulation section by a screen in order to prevent damaging stray light from reaching the science chamber. The setup around the science chamber is shown schematically in Fig. 2.4. Beams from the cooler and repumper fiber output collimators are combined into a 20 mm-diameter beam which is then split into the four beams required for the mirror MOT and are directed into the chamber. The main imaging beam output coupler diameter can be changed according to the desired image size. For images close to the chip surface (< 50 µm) we typically use a 4 mm diameter beam. For imaging larger atomic clouds a defocusing lens is used to enlarge the beam diameter. The optical pumping beam diameter is 20 mm and it is $\sigma^+$ polarized before entering the chamber. A second imaging beam is also available and is introduced into the chamber from one of the side windows (also used for the cooler/repumper beams) by using polarized beam splitter cubes (PBS).
Figure 2.4: Schematic drawing of the beam paths around the science chamber. The cooler and the repumper are combined and then split to four laser beams: two pass horizontally through the vacuum chamber whereas the remaining two are reflected towards the chip surface at an angle of 45 degrees. Imaging is performed along the longitudinal \((x)\) and transverse \((y)\) axes. The longitudinal imaging light is overlapped with the MOT-beams using two polarizing beam splitter cubes. Optical pumping light is added to the imaging optics along the transverse axis. A custom-built optical microscope system (Infinity Optics, Boulder, Colorado) allows high-resolution imaging.

### 2.1.3 Imaging system

Information about the atomic cloud is retrieved from measurements of the particle density distribution. The detection procedure is therefore an integral part of any experiment. In the present work the detection scheme is based on resonant absorption imaging [59]. The image is obtained by using a long-range microscope and a CCD camera. In the following sections the technical details of the absorption imaging technique and our imaging systems are described.
2.1. EXPERIMENTAL SETUP

2.1.3.1 Absorption imaging

The underlying idea of the absorption imaging technique is that an atom that is resonantly excited by a weak laser beam decays back into its ground state by emitting a photon in a random direction. The laser beam that passes through the atomic cloud is therefore partially absorbed (the atoms cast a shadow). This "shadow" covers a solid angle that is much smaller than the full $4\pi$ into which the fluorescence light is emitted. A microscope used for imaging the plane containing the atomic cloud effectively images the shadow and the laser light only, as long as the accepted solid angle is small compared to $4\pi$. The optical opacity of the shadow is governed by the parameters of the atomic transition used for imaging. In general the amount of power $P_{sc}$ scattered by a single atom is proportional to the absorption cross section $\sigma$,

$$P_{sc} = \sigma I,$$  \hspace{1cm} (2.1)

where $I$ is the intensity of the exciting light field.

If the atomic density is denoted by $n(x)$, the intensity of the beam after traveling a small distance $dy$ through the cloud is attenuated by

$$\Delta I = -\sigma I n(x, y, z)dy.$$  \hspace{1cm} (2.2)

As long as the scattering cross section does not depend on the intensity, this equation can be integrated to give the Beer-Lambert law:

$$I(x, z) = I_0 \exp[-\sigma d(x, z)]$$  \hspace{1cm} (2.3)

where $d(x, z)$ is the projected particle or column density

$$d(x, z) = \int dy n(x, y, z)$$  \hspace{1cm} (2.4)

and $I_0$ is the intensity of the incident beam. Using this fundamental equation we can relate the light intensity in the shadow region to the atomic density of the column traversed by the imaging laser beam. The argument of the exponent in Eq. (2.3), $\sigma d(x, z)$, is referred to as the image optical density (OD).

In order for the assumption of a constant scattering cross section to hold, the intensity should not exceed a certain limit (the saturation intensity) that is determined by the properties of the atomic transition. In general, for a resonant transition between two levels the affect of the intensity on the cross section has the form

$$\sigma = \frac{\sigma_0}{1 - I/I_{sat}}$$  \hspace{1cm} (2.5)

where $I_{sat}$ is the saturation intensity. In this work the intensity was kept around $I = 0.1 I_{sat}$ and the nonlinear terms have been neglected. The parameters $\sigma_0$ and $I_{sat}$ depend on the transition used for imaging. The level scheme of the D2-line is shown in Fig. 2.2. For a $^{87}$Rb cloud prepared in the $F = 2$ ground state
there are three dipole-allowed transitions to the excited states; of these possibilities, we use the $F = 2 \rightarrow F' = 3$ transition. The largest cross section occurs for circular polarized light where $I_{\text{sat}} = 1.669 \text{ mW/cm}^2$, and $\sigma = 2.907 \times 10^{-9} \text{ cm}^2$. However, when imaging close to a metallic surface, this polarization cannot be used since only linear polarization can be adjusted to have equal intensities before and after reflection by a mirror surface. We therefore use plane-polarized light with $I_{\text{sat}} = 2.503 \text{ mW/cm}^2$ and $\sigma = 1.938 \times 10^{-9} \text{ cm}^2$ for imaging close to the surface. The duration of the light pulse is $50 \mu s$ in order to avoid diffusive motion of the atoms due to recoil from the absorption.

The Beer-Lambert law (2.3) can be used to extract the column density from the intensity ratio of two successive measurements, one with and one without atoms:

$$d = -\log[I/I_0]/\sigma = -\log[N/N_0]/\sigma$$

(2.6)

where $N$ and $N_0$ denote the counts per pixel acquired by the camera. Such an imaging sequence is presented in Fig. 2.5.

Figure 2.5: Absorption imaging sequence: (a) A $50 \mu s$ long, low intensity ($\approx 10\%$ of saturation intensity), resonant light pulse is passed through the atomic cloud and captured on the CCD. Reduced intensity is measured where the beam passes through the sample. (b) A second identical pulse is captured $50 \text{ ms}$ later, at which time the atomic cloud is not present. (c) A density distribution image is constructed from the two images.

2.1.3.2 Imaging setup

For the specialized application needed in the context of cold atom experiments it is often advantageous to use lab-made imaging systems. Since only monochromatic light is used, lenses designed for this wavelength with small f-numbers can be used, allowing for a spatial resolution of a few microns.

If a lens with a focal length $f$ and a diameter $a$ is illuminated by parallel light, this light is focused in the focal plane of the lens down to a spot—the Airy disk. The radius of this disk is given by $r = 1.22\lambda f/a$ where $\lambda$ is the wavelength of the
2.1. EXPERIMENTAL SETUP

light. Two points in the object plane can be distinguished if the corresponding two Airy disks do not overlap. Thus the diffraction-limited resolution can be defined to be equal to $r$. The quantity $f/a$ is referred to as the lens f-number and is inversely proportional to the numerical aperture for $f$-number $\gg 1$. In our setup, the lens is positioned outside of the vacuum chamber; hence the minimal focal length which may be used is 125 mm. The solid angle of the image is limited by the size of the vacuum window (clear diameter of 28 mm) positioned 119 mm from the imaged cloud, and by the atom chip surface, which blocks some of the outgoing light. The calculated diffraction limit of our system is thus 5.28 $\mu$m. A new science chamber design that will allow positioning the first lens close to the atomic cloud and with a larger aperture was built and is discussed in the appendix.

Different setups were tested in order to obtain diffraction-limited imaging capability. The best setup was obtained with a custom-designed long range microscope, produced by Infinity Optics \(^4\). The system magnification needs to be such that the CCD effective pixel size is at least two times smaller than the desired resolution. We used a Prosilica camera with a pixel size of 3.45x3.45 $\mu$m\(^5\); with a magnification of 3.27 the effective size is $\approx$ 1x1 $\mu$m/pixel. This setup allows good sensitivity and images with OD of 0.05 can easily be detected in this setup. Our imaging system is shown in Fig. 2.6. With minor adjustments, this setup can be used to image atomic samples with very small atom-surface separations ($< 50 \mu$m), this will be discussed in Sec. 2.4.2.

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\(^4\)Based on K2/SC model, http://www.infinity-usa.com

\(^5\)Prosilica GC2450

Figure 2.6: For high resolution imaging we use a custom designed optical long range microscope (a), with a magnification of 3.27. We use a CCD camera with a pixel size of 3.45x3.45 $\mu$m (b). This setup is extremely sensitive to movements along the imaging axis and we therefore use a micrometric manipulator to focus the image (c).
2.1.4 The atom chip

2.1.4.1 Atom chip mount

The atom chip mount must provide electrical connections from the outside the vacuum chamber to the copper structures underneath the atom chip and to the atom chip itself [Fig. 2.7(a)]. This is done by several high current feed-throughs and by a 35-pin instrumentation feed-through. These feed-throughs are welded to a DN160CF-vacuum flange onto which the complete chip mount is built, the overall height of which is 500 mm. When installed, the atom chip is positioned 7 mm above the center of the science chamber. The copper structures used to generate different magnetic potentials are positioned in a Macor block and are electrically insulated by a thin Kapton foil [Fig. 2.7(b)]. A broad U-shaped wire, 10 mm wide, is used to generate the quadrupole field for the MOT. A Z-shaped wire, the legs of which are 4 mm apart, is used to generate a Ioffe-trap and two additional straight wires on either side of the Z-wire (the “leg-wires”) are used to provide additional longitudinal confinement for the atom chip traps. The atom chip is glued with a UHV-compatible epoxy (Epotek ND353) to the Macor block on top of these structures, which gives it mechanical support and also acts as a heat sink. The connection pads along the edges of the chip are wire-bonded to the top of small CuBe pins at the side of this Macor block by 5-10 Au bonding wires [Fig. 2.7(c)]. The bottom ends of the CuBe pins are connected via a Kapton-insulated cable to the instrumentation feed-through. All materials used in this mounting have been tested to be UHV-compatible.

An important issue, which had to be addressed in building the mount, is the thermal expansion of the Cu structures, mainly their vertical leads. After damage was inflicted to the atom chip in early experiments, we improved the chip mount by replacing the original Cu leads [Fig. 2.7(a)] with more massive leads (10 mm diameter instead of 5.9 mm). We pre-machined the Macor block so that the copper wires now move up to the desired steady-state point after several hours of operation. Expansion of the new mount leads is limited to 250 µm.

2.1.4.2 The atom chip

The atom chip used in the experiments presented in this thesis was fabricated in the Weiss Family Laboratory for Nano-scale Systems at BGU (http://w3.bgu.ac.il/nanofabrication/). It consists of an evaporated gold layer on top of a silicon surface. The gold layer has been structured by photolithography, and wires of several widths (2 – 200 µm) are defined by 10 µm-wide gaps (Fig. 2.8); all the wires on this chip are 2 µm thick allowing the chip fabrication to be done in a single step. The basic atom chip trap is generated by running a current in the central 200 µm wide, ‘loading’ wire. The magnetic field generated above the wire is compensated by a magnetic bias field along the y axis, $B_y$, to form a trap. A small magnetic bias field is also added in the x direction, $B_x$, to preclude a vanishing magnetic field at the trap minimum (the coordinate system used in our lab can be seen in Fig. 2.8).
2.1 EXPERIMENTAL SETUP

Figure 2.7: (a) The atom chip mount. The total length of the mount is 500 mm. The electrical current to the copper structures is input into the vacuum chamber through the power feed-throughs, and to the atom chip wires using a 35-pin instrumentation feed-through. (b) The copper structures, which include the U-wire, the Z-wire, and the leg-wires. (c) Top view of the atom chip (25 mm$^2 \times 30$ mm$^2$). All wires start from connection pads (1.5 mm wide) along the chip edge, narrowing down closer to the atom chip center where the actual experiments are performed. (d) The atom chip before being cut. (e) A SEM image of a 5 µm wide wire is shown in order to demonstrate the fabrication quality of our atom chip (average grain size on the order of 30 – 40 nm). Both the smoothness of the wire surface (5 nm), as well as the roughness of the wire’s edges (about 10 nm), are evident.

To form a three dimensional trap, a longitudinal trapping potential is required along the $x$ axis. This may be achieved either by running currents in the four U-shaped wires located at each side of the loading wire on the atom chip (Fig. 2.8), or by running currents in the copper leg-wires underneath the chip (Fig. 2.7). Both options generate a H-trap configuration [17]. The trapping potential generated by a different wire, the ‘snake’ wire, will be discussed in detail in Ch. 5.

2.1.5 Computer control

Controlling the experimental sequence is a complex task since high-resolution timing (2 µs) is required and, at the same time, the duration of the experimental cycle is relatively long (60 s). Many different experimental components must be controlled accurately (laser frequency and power, currents, magnetic fields, etc.), and at the end of each cycle an absorption image must be taken and the data analyzed. At the heart of automating these procedures is a National Instruments (NI) PXI system, with an independent processor to manage the entire experimental sequence. The
sequence is programmed through the graphical user interface (GUI) designed and programmed using Labview 7.1, and downloaded to the PXI. Once the user pushes the 'execute' button, the PXI takes command of the system and manages the sequence independently from any other external clock (as opposed to controlling the experiment using DAQ cards, which typically use the PC clock and can be influenced by other processes going on at the same time, such as Windows etc.). The program controlling the experiment can be used to scan several parameters automatically from a start to a stop value in equally-spaced steps. The PXI outputs, analog and digital signals, pass through a home-made interface box which calibrates the PXI signals required for controlling each instrument (e.g. conversion from voltage to currents, amplification etc.), and also provides optical separation/isolation of each electronic signal, in order to reduce electronic noise. From the interface box BNC cables and optical fibers transfer the signals to all the different experiment instruments. In our experiments we control 8 large current supplies with separate home-made current shutters, 4 AOM control modules (frequency, amplitude, and blocking), 5 mechanical light shutters, 2 cameras, 3 home-made low-noise current supplies for the atom chip wires, and an RF generator.

Imaging analysis is done with our lab programmed Matlab-based imaging software on the same computer. The analysis program takes the image files acquired by the camera and processes them to obtain the absorption image (see Sec. 2.1.3). The atomic density image obtained thereby can be analyzed automatically according to the specific needs of the experiment. Image analysis data and the corresponding experimental sequence data can be saved together. The computer interface can be seen in Fig. 2.9.
2.2 EXPERIMENTAL PROCEDURE

The experiments with trapped ultra-cold atomic clouds and BECs presented in this work are performed in the following way: thermal Rb atoms from the dispenser source are collected and cooled in a magneto-optical trap (MOT). After transfer to a magnetic trap the atomic cloud is further cooled by evaporative cooling. The density distribution is imaged after conducting pre-programmed manipulations of the atomic cloud. This imaging is destructive and so the entire experimental procedure is repeated in a continuous 60 s cycle. The experimental cycle used in our setup consists of four major stages:

1. Laser-cooling: Collection of $^{87}$Rb atoms from a background vapor using the mirror U-MOT and further cooling by optical molasses to reach sub-Doppler temperatures.

2. Evaporative cooling: Transfer to magnetic traps generated by the copper Z-wire structure and further cooling by forced evaporation.

3. Chip experiments: Transfer of the cold clouds or BECs to the microscopic magnetic traps generated by the micron-sized atom chip wires, and their manipulation in corresponding magnetic traps, according to the requirements of the particular experiment.

4. Imaging: Imaging of the atomic density distribution onto a CCD-camera using high-resolution optics.

The first two stages are mostly independent of the specific chip experiment and will be discussed in the following. Typically the third stage of the cycle changes significantly depending on the actual chip experiment performed. The Imaging
stage was described in Sec. 2.1.3, and it may be used to measure and characterize the atomic cloud in any stage of the experimental cycle.

In our apparatus it is important to always keep the time for a full experimental cycle fixed. This restriction is necessary to avoid temperature drifts in the coils and other high-power components because of a changing duty cycle. The current ramp of the Rb dispensers has also been optimized for a certain amount of heating and cooling while at the same time giving optimal MOT-loading and long magnetic trap lifetimes. Therefore the cycle time is fixed to 60 s.

2.2.1 Laser cooling

2.2.1.1 The mirror U-MOT

In the laser-cooling stage the mirror U-MOT is loaded from a background gas generated by a dispenser source. The duration of this stage is slightly longer than 20 s and can be divided into three phases.

In the first 19.8 s of the MOT, the main object is to load as many atoms as possible. A current of 16 A is pushed through two dispensers connected in parallel for 9 s and then lowered to 12 A for 3 s. After this the current in the dispensers is switched off to allow the background pressure to drop to a value where magnetic trap lifetimes are long enough to enable efficient forced evaporative cooling. This intermediate stage lasts for 7.8 s and atoms can be held in the MOT without any measured loss. The laser light is red-detuned by 16 MHz from the cooling transition during the whole loading phase.

The MOT loading is carried out at the center of the intersection region of all the laser beams, \( \sim 8 \text{ mm} \) below the chip (Fig. 2.10). In the second phase of the MOT, the trapped atoms are moved vertically to a position \( \sim 2 \text{ mm} \) below the chip to coincide with the minimum of the magnetic trap which will be generated using the copper Z-structure. The atoms are moved to this position by increasing the \( B_y \) field and the current in the U-wire in a 0.3 s ramp. This also increases the gradients of the MOT leading to higher atomic densities, i.e., this ramp compresses the MOT. The exact position of the MOT is sensitive to the local shape and intensity balance of the laser beams and it is therefore usually necessary to move the horizontal position of the MOT by adjusting the \( B_z \) and the \( B_x \) also.

2.2.1.2 Sub Doppler cooling

Immediately after compressing the MOT all the bias fields and the current in the U-wire are switched off. The cooling light frequency is shifted to 40 MHz below the cooling transition and the light intensity is reduced slightly. This light configuration, known as optical molasses, cools the atomic cloud to \( \sim 50 \mu \text{K} \), well below the Doppler-limited temperature of \( T_D = 140 \mu \text{K} \) [58], and is applied to the atoms for 10 ms. We typically have \( 8 \times 10^7 \) atoms in the end of this stage.
2.2. EXPERIMENTAL PROCEDURE

2.2.1.3 Optical pumping

After molasses cooling the atomic cloud must be captured in the magnetic trap formed by the Z-shaped copper wire. At this stage, the atoms occupy all possible Zeeman sublevels of the \( F = 2 \) hyperfine ground state \( (m_F = -2, \ldots, 2) \). Best loading efficiency and a longer lifetime can be achieved by ensuring maximum magnetic force on the atoms and preventing different \( m_F \) states collisions. We therefore transfer the atoms to the “stretched” state, \( F = m_F = 2 \). This is done by a 400 µs light pulse of \( \sigma^+ \)-circularly polarized light, propagating along the \( y \) direction. The light pulse is applied just as the \( B_y \) field is switched on for the magnetic trap, 500 µs before the other fields needed for the magnetic trap have been switched on (providing a quantization axis for the atomic spin). The effect of this process can be seen in Fig. 2.11. The optical pumping frequency was set to be resonant with the \( F = 2 \rightarrow F' = 2 \) transition, since atoms pumped to the \( F = m_F = 2 \) state cannot be excited by the circularly polarized light, so there is minimal momentum transfer to the atoms during this process. Since atoms in the excited state \( F' = 2 \) can ‘fall’ into the \( F = 1 \) and exit the pumping cycle, a repumper light is added to the light pulse. This stage is not a cooling stage and all further cooling is done in the dark.

2.2.2 Evaporative cooling

2.2.2.1 Z wire magnetic trap

Fast and smooth buildup of the magnetic trap is essential for loading it successfully; the sequence presented here has been optimized for maximal retention of atoms and minimal heating of the cloud. In a typical experimental cycle, we load \( \approx 5 \times 10^7 \)
CHAPTER 2. THE BGU ATOM CHIP EXPERIMENT

Figure 2.11: Optimization of the optical pumping (OP) procedure. Using a Stern-Gerlach type of experiment we can quantify the number of atoms in the different Zeeman sublevels. The cold atoms are released after an OP pulse while a magnetic gradient is applied. The images shown are: (left) no OP pulse prior to the release, atoms in the three highest \(m_F\) states are evident; (center) unoptimized OP pulse, small residue of atoms in the \(m_F = 1\) state is evident; (right) optimized OP pulse, all the atoms are in the \(m_F = 2\) state.

Atoms into the Z-trap at a temperature of \(\approx 150 \mu \text{K}\). The time needed for switching on the magnetic fields has to be short enough that atoms do not fall out of the trap volume due to gravity. This fast switching is accomplished by applying high voltage to home-built current “shutters” while they are switched off; switching them on suddenly then increases the currents in much shorter time scales than available using the current power supplies directly. An optimized current waveform for this stage is given in Fig. 2.12. We can generate a magnetic trap of reasonable depth within \(4 - 5\) ms and a full depth trap in \(\sim 10\) ms. However, this technique must be used with special care since it may easily cause an overshoot in the current if a too high charging voltage is applied. The currents in the copper Z-wire and in the \(x\)-bias coils needed to form the magnetic trap must be adjusted to match the rise-time of the \(B_y\) bias current (Fig. 2.12). The initial magnetic trap, 2 mm from the chip surface, is generated by running \(85\) A in the Z-wire combined with a \(B_y\) field of \(35\) G and an \(B_x\) field of \(33.5\) G.

Another important criterion in the loading stage is to maintain low temperature of the trapped cloud. Atoms are heated by their center of mass movement and changes in the trap frequencies. We therefore optimized (‘mode-matched’) the cloud position at the end of the MOT stage to the magnetic trap position. The trap frequencies are set to ensure minimal heating in the transfer to the Z-trap. The atomic cloud is heated from \(\approx 50\) \(\mu \text{K}\) before the loading to the Z-trap to \(\approx 150\) \(\mu \text{K}\) after thermalization in the trap.

The magnetically trapped atoms can be further cooled by means of evaporative cooling. For this cooling technique to work, the lifetime of the atomic cloud in the magnetic trap must be much longer than the time needed for the cloud re-thermalization. Fig. 2.13 shows the number of atoms measured in the copper Z-
2.2. EXPERIMENTAL PROCEDURE

The trap currents are monitored during the end of the MOT stage and the loading of the magnetic trap. All the magnetic fields are shut down rapidly at the end of the MOT stage, and the magnetic Z-trap is loaded after 10 ms of a pure laser-cooling phase (molasses).

trap over a period of 20 s. Following an initial rapid decay of 2 s due to hot atoms leaving the trap, a lifetime of $\approx 47 \pm 4$ s can be deduced.

After loading the atoms to the Z-trap, they are transferred closer to the chip surface where higher potential gradients are attained (a ‘compressed’ trap). Using a linear ramp of 1 s the $B_y$ and the $B_x$ are increased to 41.6 G and 31.3 G respectively, while the Z-wire current is reduced to 60 A, positioning the atomic cloud 0.8 mm from the atom chip. The magnetic field at the trap minimum is measured to be $\sim 1$ G, in order to limit Majorana spin flips (this will be discussed in more detail in Ch. 4). The measured frequencies of this trap are $f_r = 380$ Hz (Fig. 2.14) and $f_x = 40$ Hz.

Figure 2.12: Lifetime measurement for atoms in the copper Z-trap, without applying RF evaporative cooling. A double exponential decay of the atom number is observed, where the first 2 s decay time is due to hot atoms escaping the trap. After equilibrium has been reached, an exponential loss due to background gas collisions can be observed, yielding a lifetime of $\tau = 47$ s.
2.2.2.2 Evaporative cooling

After the atoms are loaded into the magnetic trap, we start the evaporative cooling stage [12], where we use radio frequency (RF) radiation in order to selectively expel “hot” atoms from the trap. As a result, the remaining atoms are re-thermalized by means of elastic collisions and the mean temperature of the atomic cloud decreases. The RF-radiation, which is in the MHz-range, couples different $m_F$ states via their Zeeman interaction, given here in the first-order approximation:

$$h \nu = |g_F \mu_B B|,$$

(2.7)

where $\nu$ is the RF-frequency, $\mu_B$ is the Bohr magneton, $g_F$ is the Landé-factor of the atomic hyperfine state F, and $m_F$ is the projection of the angular momentum along the $z$ axis (defined by the direction of $\vec{B}$).

The inhomogeneous field of the magnetic trap leads to an energy-selective loss of atoms. The RF-radiation is chosen as a multiple of the mean temperature $\eta k_B T$, where optimum values of $\eta$ are between 4-10 for harmonic traps [12]. Hot atoms evaporate from the trap and, if re-thermalization rates are fast enough, the phase-space density of the remaining sample increases by several orders of magnitude (“run-away evaporation”). Since our current-carrying structures are close to the atoms, we built a home-made impedance matching circuit to efficiently couple the RF source$^6$ to the copper U-wire. The best evaporative cooling results were obtained when $\eta$ was chosen to be 7, where we ramp the RF frequency from 50 MHz down to a final value of $0.7 - 1.2$ MHz in 15 s. The final RF frequency depends on the magnetic field at the Z-trap minimum. Important parameters of our RF ramp are shown in Fig. 2.15.

$^6$33250A Function / Arbitrary Waveform Generator, 80 MHz, Agilent Technologies Inc.
Figure 2.15: (a) Evaporative cooling frequency (blue) and amplitude (green) ramps: the frequency is lowered from 50 MHz down to ≈ 800 kHz in a 15 s ramp while the RF amplitude is lowered simultaneously. (b) As the hot atoms are expelled from the trap by the RF ramp (green), the atomic sample temperature is lowered due to the thermalization of the remaining atoms (blue). The efficiency of this cooling is also shown on a logarithmic scale in Fig. 2.16.

2.3 BEC

2.3.1 Phase transition to a BEC

Cooling the atomic sample is not enough in order to achieve a BEC. One needs to increase the sample phase space density (PSD), defined as $n\lambda_B^3$, beyond the critical value of $\sim 2.6$ for alkali metals [16, 13]. The evolution of the PSD during the RF-cooling stage presented above can be seen in Fig. 2.16. The inset shows the transition to BEC, occurring only in the last 0.5 s of the evaporation, as a change in the aspect ratio of an atomic cloud released from the trap: a thermal cloud expands isotropically (14.5 s) while the BEC expands faster in the vertical direction and acquires an elongated shape (15 s).

The best results (in terms of the number of atoms in the BEC) were obtained when a second compression step was added during the evaporative cooling. This was done after 11 s of evaporation by reducing the Z-wire current to 50 A and the $B_z$ to 27 G in a 2 s ramp, while maintaining constant $B_y$. This moves the trap to $\sim 0.5$ mm from the chip surface and increases the radial trapping frequency to $\sim 400$ Hz while the longitudinal trapping frequency was not affected.

2.3.1.1 Density distribution

The atomic density is qualitatively different for a harmonically trapped thermal cloud and a condensate. This is best seen at a temperature around $T_c$. 

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Figure 2.16: Phase space density (PSD) and de-Broglie wavelength as a function of evaporation time in the magnetic Z-trap ($I_{\text{wire}} = 60\, \text{A}$ and $B_y = 41.4\, \text{G}$). The inset shows the density distribution of the atomic cloud 14 ms after release from the magnetic trap (14 ms of TOF expansion). Here, the onset of condensation can be observed as the isotropic expansion of a thermal cloud changes to the elongated expansion characteristic of a BEC.

The trapping potential is approximately cylindrically symmetric [17] with a harmonic oscillator potential of the form:

$$V(r, x) = \frac{m}{2} \left( \omega_\perp^2 r^2 + \omega_x^2 x^2 \right),$$

(2.8)

where $r^2 = y^2 + z^2$. The distribution of a thermal cloud of atoms in this potential is given by the Gaussian

$$n(r, x) = n_{0,\text{th}} \exp \left( -\frac{r^2}{w_{\text{th},\perp}^2} - \frac{x^2}{w_{\text{th},x}^2} \right),$$

(2.9)

where the width parameters are $w_{\text{th},i} = \sqrt{2k_B T/m\omega_i^2}$ and the constant $n_{0,\text{th}}$ is determined by the normalization condition $N = \int d^3 x \, n(x)$

$$n_{0,\text{th}} = \frac{1}{\pi^{3/2} w_{\text{th},x}^2 w_{\text{th},\perp}^2} N.$$

(2.10)

For a condensate with a large number of atoms, and taking into account atom-atom interactions, the kinetic energy can be neglected compared to the interaction energy and we may use the Thomas-Fermi limit [13, 59]. Under these conditions the density distribution becomes an inverted parabola (Sec. 1.1.2)

$$n(r, x) = \begin{cases} n_{0,\text{TF}} \left( 1 - \frac{r^2}{w_{\text{TF},\perp}^2} - \frac{x^2}{w_{\text{TF},x}^2} \right) & \text{for } \frac{r^2}{w_{\text{TF},\perp}^2} + \frac{x^2}{w_{\text{TF},x}^2} \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(2.11)

where the width of the cloud is now controlled by the repulsive interaction energy

$$w_{\text{TF},i} = \left( \frac{2\mu}{m\omega_i^2} \right)^{1/2}$$

for $\mu$ the condensate chemical potential. The normalization
2.3. BEC

condition again defines the maximum density \( n_{0,\text{TF}} \)

\[
n_{0,\text{TF}} = \frac{\mu}{g},
\]

(2.12)

where \( g = 4\pi\hbar^2 a/m \) is the interaction constant, and \( a \) is the Rb scattering length. The Thomas-Fermi expression for the chemical potential of a harmonically confined condensate is

\[
\mu^{5/2} = \frac{15\hbar^2 m^{1/2}}{2^{5/2}} N \left( \omega_\perp^2 \omega_x \right) a.
\]

(2.13)

The above differences between Thomas-Fermi and thermal density profiles shows how to distinguish clearly between a thermal cloud and a condensate. For atomic clouds between these two limits, we analyze the properties of the mixed cloud from images by fitting to a bimodal distribution. Thus, we fit the observed density \( n_{\text{tot}} \) to

\[
n_{\text{tot}}(r, x) = n_{0,\text{th}} \exp \left( -\frac{r^2}{w_{\text{th},\perp}^2} - \frac{x^2}{w_{\text{th},x}^2} \right) + n_{0,\text{TF}} \max \left( 1 - \frac{r^2}{w_{\text{TF},\perp}^2} - \frac{x^2}{w_{\text{TF},x}^2}, 0 \right),
\]

(2.14)

where the normalization factors are also fit parameters for the fraction of condensate atoms and thermal atoms. Fig. 2.17 presents images of atomic clouds at different temperatures, demonstrating the phase transition and showing how the density distribution passes through the different cases explained above.

Figure 2.17: The atomic density of the cold atomic cloud 15 ms after its release from the trap. As we pass through the critical temperature \( T_c \), the density distribution changes from a Gaussian (a), to a bimodal distribution for a 1 \( \mu \)K cloud (b,c), and finally to a Thomas-Fermi distribution for a pure condensate, when the temperature is around \( T_c \) (d).
2.3.2 Basic measurements of a BEC

The size, in particular the width, of the BEC is typically too small to monitor in situ. A method to circumvent this difficulty is to allow the gas to expand for some time after turning off the trapping potential. By using this time-of-flight (TOF) measurement, the momentum distribution of the trapped cloud can be measured. For an ideal gas, each particle just keeps the initial velocity that it had at the moment the potential was removed and travels in a straight trajectory. If the trap is released quickly enough (i.e., fast compared to the characteristic time scales of the gas’ internal motion) the energy is conserved during this process and can be measured from the expansion speed of the cloud. This is also true if the trapped gas is not in the classical ideal gas limit. If the initial density is high, some energy may be stored in the form of interactions between the atoms. Also the quantum mechanical ground state energy may give a significant contribution if the confinement is strong. As long as the expansion characteristics are measured after a long enough time that these effects do not play a role anymore, all the energy will be converted to kinetic energy of the expansion.

For the Gaussian distribution of a thermal cloud the expanded distribution is just a scaled version of the initial distribution, given in Eq. (2.9) [59]. The time-dependent width parameters are given by:

\[ w_{th,i}^2(t) = w_{th,i}^2(0) \left( 1 + \omega_i^2 t^2 \right). \]  

For an expansion time that is long compared to the respective transverse or longitudinal oscillation periods, the different initial positions of the particles may be neglected and the expansion speed can be directly related to the initial temperature of the gas. Since the absorption imaging measurements give access only to the column density, we obtain the time-dependent two-dimensional Gaussian

\[ d(x, z) = (n_{0,th} \sqrt{\pi} w_{th,\perp}) \exp \left( -\frac{x^2}{w_{th,x}^2} - \frac{z^2}{w_{th,\perp}^2} \right), \]  

where the rate of expansion of the widths \( w \) can be easily detected, and are then used to measure the temperature.

In the Thomas-Fermi limit all quantum mechanical contributions to the kinetic energy are neglected and the condensate, therefore, behaves very much like a classical fluid.

\[ w_{TF,\perp}(t) = w_{TF,\perp}(0) \sqrt{1 + \omega_{\perp}^2 t^2} \]

\[ w_{TF,x}(t) = w_{TF,x}(0) \left( \frac{\omega_{\perp}}{\omega_x} \right) \left[ 1 + \left( \frac{\omega_{\perp}}{\omega_x} \right)^2 \left( \omega_{\perp} t \arctan(\omega_{\perp} t) - \ln \sqrt{1 + (\omega_{\perp} t)^2} \right) \right]. \]  

These expressions are valid in the limit of \( \omega_{\perp}/\omega_x \ll 1 \) which is fulfilled in the experiments for all the potentials that were used. The transverse expansion follows.
the same scaling law as for a thermal cloud. In this case however, the asymptotic expansion speed is related to the chemical potential instead of a temperature

$$\mu = \frac{m}{2} \left( \frac{dw_{\perp}}{dt} \right)^2 \approx \frac{m}{2} (w_{\text{TF},\perp}(0) \omega_{\perp})^2$$  \hspace{1cm} (2.18)

The width parameter can be extracted again from the column density which is given now by

$$d(x, z) = \frac{4}{3} w_{\text{TF},\perp} n_{0,\text{TF}} \max \left[ \left( 1 - \frac{x^2}{w_{\text{TF},\perp}^2} - \frac{x^2}{w_{\text{TF},x}^2} \right)^{3/2}, 0 \right]$$  \hspace{1cm} (2.19)

Figure 2.18: Expansion of a condensate under conditions appropriate for using the Thomas-Fermi approximation. The sequence of images shows the density evolution with increasing expansion time. The atoms fall with gravity (along the $z$ axis) and, within the measurement time, expand only along radial axis $(z, y)$.

The expansion dynamics can be used to measure the energy scales of a condensate. Fig. 2.18 shows a series of pictures of a condensate as it expands upon release from the trap. The transverse width and the center-of-mass position can be extracted by a two-dimensional fit of the Thomas-Fermi density model (Fig. 2.19). The observed expansion speed corresponds to a chemical potential of $\mu = (1.7 \pm 0.6) \ 2\pi \hbar \cdot \text{kHz}$, for $3 \times 10^4$ atoms. The center-of-mass evolution shows a free-fall parabolic behavior in the gravitational field. A comparison of this curve with gravitational acceleration can also be used to check the length scale.

### 2.4 Micromanipulation of BECs on atom chips

A major part of my effort was to construct the BEC setup from scratch and finally to load the ultra-cold atoms or BEC into the micro-traps. In this section I will
describe this final preparatory step, namely the manipulation of BEC using the magnetic fields generated by current-carrying wires in the atom chip. The methods described in the following are currently being used in the implementation of novel experiments underway in our laboratory. One of these experiments, designed by me, is also described in this thesis.

2.4.1 Loading the atom chip

After the RF-cooling of the atoms in the copper Z-trap, they can be transferred to a trap generated by currents in the chip wires. To transfer atoms from an initial magnetic trap to a second trap generated by different wires it is necessary to guarantee a smooth passage of the atoms from one trap minimum to the other; otherwise the atoms will be heated or they will undergo spin-flip transitions and escape from the trap. The atom chip has been designed with this requirement in mind and has a large “loading” wire directly above the copper Z-wire. With this configuration we can usually achieve efficient transfer by ramping up the current in the atom chip loading wire to its final value while the copper Z-wire trap is still switched on, typically with reduced current. Subsequently, the copper Z-wire trap current is ramped to zero. This is a general procedure that, in principle, may be repeated for subsequently transferring atoms from the loading-wire trap to the experimental wire traps on the atom chip. During such transfers, the trap gradients and the position of the minimum must be adjusted by changing the homogeneous offset fields needed for trapping.

The basic trap on the chip is formed by the 200 µm–wide central ‘loading wire’ (see Sec. 2.1.4.2) and the two copper leg-wires. As a long straight wire, the former provides a guide-wire transverse confinement while the latter provide
some longitudinal confinement. The four U-wires on the chip (Fig. 2.8) may also provide longitudinal confinement, but their effect diminishes as the trap is lowered to the atom chip surface. Our best results were obtained when the loading was conducted in two steps. In the first step a combined Z- and loading-wire trap was formed during the evaporation in a sequence that replaced the second compression step described in Sec. 2.2.2.2. After 11 s of evaporation in the Z-wire trap, the current in the loading wire was ramped up to 0.8 A in a 2 s ramp. At the same time the Z-wire current and the $B_x$ field were reduced to 37.5 A and 21.3 G, while $B_y$ remained unchanged at 41.6 G. This resulted in an intermediate “combined” trap positioned 0.3 mm from the chip surface, where a BEC can be formed if needed (Fig. 2.20).

For traps much below this height (i.e., below 0.1 mm), it was necessary to modify the imaging system since otherwise the imaging laser beam, so far parallel to the atom chip surface, is shadowed by the edge of the chip and its mount or, for even lower traps, diffracted by it.

![Figure 2.20: BEC 5 ms (top image) and 15 ms (bottom image) after its release from a “combined” trap formed by currents flowing through both the copper Z-wire and the atom chip loading wire; this trap is located 0.3 mm from the chip surface.](image)

### 2.4.2 Imaging close to the surface

The basic mechanism of absorption imaging has already been described in Sec. 2.1.3. Using a beam aligned parallel to the surface and centered on the atoms becomes impossible for imaging atoms closer than $\approx 0.1$ mm to the mirror surface. At a height $h$ above the surface, the length of the mirror $l$ limits the usable numerical aperture to $NA = 2h/l$. The minimum spot radius on the other hand should be smaller than $h$ to avoid a beam that is touching the surface. For a Gaussian beam this leads to the limit of $h \geq \sqrt{\lambda l / \pi}$. In the case of $^{87}$Rb atoms, ($\lambda = 780$ nm); for $l = 1$ cm this limit is reached already at $h = 50$ µm.
In order to allow a controlled illumination below this height, the simplest solution is to incline the beam by a small angle to the mirror surface. This setup is depicted schematically in Fig. 2.21. In contrast to the usual absorption imaging setup there are now two beam paths that traverse the cloud. Beam (1) is reflected before passing through the cloud and beam (2) is reflected afterwards and two (mirror-image) shadows are recorded by the imaging system.

Such an image is shown in Fig. 2.22. The distance between the cloud and the mirror cloud equals twice the height above the surface. By fitting the image to a double Gaussian along the $z$ axis, one can perform an absolute measurement of the cloud height relative to the atom chip surface, thus simultaneously providing a measurement of the height that can otherwise be very difficult to measure accurately. The only modifications required are to tilt the imaging optics by the same angle as the imaging laser beam.

The central region of the atom chip has numerous parallel wires running along the $x$ and $y$ directions. The spacing between these wires is typically $10 \mu m$, so this region of the atom chip acts like a diffraction grating. The reflected light therefore shows alternating dark and bright bands (Fig. 2.22), and there is very limited imaging light in some regions, causing pronounced image deterioration. To image clouds very close to the chip surface, we use a very small beam diameter (1.4 mm), thus limiting the area of reflection and partially overcoming this diffraction problem. With sufficient care, we can image atom clouds as low as $5 \mu m$ from the chip surface, however, due to the increased ‘noise’ in the image, we can only detect images with OD above 0.1.

### 2.4.3 Approaching the atom chip

The final stage of loading to the atom chip trap starts 14.6 s from the beginning of the RF evaporation, 34.25 s from the start of the experimental cycle. Within 0.5 s the chip current is increased from 0.8 A to 1 A, the copper $Z$-wire current
2.5. Summary

We presented our atom chip setup and experimental procedure. In this setup we generate a sizable BEC of $30 \times 10^3$ atoms, and transfer them to atom chip traps. Due to careful planning of the experimental sequence, good vacuum is maintained throughout the cycle, and we enjoy a trap lifetime of $\sim 50$ s, long enough for BEC production and subsequent atom-optics experiments or atom-surface interaction measurements. Our imaging system is optimized to give high resolution within our apparatus limitations with little impact on our imaging sensitivity. This is a robust setup that provides the basic stepping stones for future experiments in our...
Experiments that I have designed will be presented in Ch. 4 and Ch. 5. They require bringing the atomic cloud very close to the trapping wire. To date, we have been able to load and image atoms less than 5 µm from the trapping wire, which would probably be sufficiently close to begin our observations if our imaging beam did not suffer diffraction at the atom chip surface. The fabrication facility is currently producing a new atom chip to overcome this difficulty and to proceed to the “snake” experiment that will be presented in Ch. 5.

Building this setup was the main goal of my Ph.D. work. It should be noted that as this setup was also designed to hold a cryogenic chip mount for superconducting atom chips (which we have built and with which we observed the transition to super-conductivity in a Nb chip), the chamber we designed had several complexities. Numerous new aspects such as a very long mounting, required several optimizations e.g. to control the thermal expansion. The setup I have presented in this work therefore required endless hours in the lab, re-design and replacement of several critical experimental components, and careful investigation and optimization. It also delivered great satisfaction when it finally worked!
Figure 2.24: Lowering the atomic cloud towards the atom chip surface (atom surface separations shown on the right) using only atom chip currents for the strong transverse confinement. The longitudinal confinement is generated by the copper leg-wires. Images are taken in situ. The longitudinal confinement is much lower than in the Z-trap, whereas the transverse confinement is much higher. This results in an elongated trap. The cloud edges are closer to the chip surface (a “banana” shape) due to a rotation in the plane parallel to the chip surface. This rotation is generated by the leg-wire currents and it is more pronounced for higher atom-surface separation. In this case the atom chip trap was loaded at an atom-surface separation of $\sim 250 \mu m$ and then lowered to $\sim 40 \mu m$. We typically load the atom chip traps at an atom-surface separation of $\sim 50 \mu m$.

2.6 Appendix

New science chamber

Our imaging system resolution is currently limited by the fact that the imaging lens is positioned outside of the vacuum chamber. To improve this we have designed a new chamber, with a large-diameter, high optical quality and anti-reflection coated window that is welded to a tube penetrating into the chamber. This allows positioning the collimating lens much closer to the cloud (75 mm, c.f. Sec. 2.1.3) and doubling the clear diameter to 60 mm (Fig. 2.26). The diffraction limit of the new system is calculated to be 2.46 $\mu m$. The improved optical resolution may be crucial for measuring quantum interference effects anticipated from the “snake” experiment, as described in Ch. 5 below. The chamber was built in the BGU physics workshop and the window was welded by Culham Centre for Fusion Energy, UK (CCFE).
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Figure 2.25: (a) Atomic cloud trapped 14 µm from the atom chip surface. The trap is generated by the loading wire. (b) Atomic cloud trapped 7 µm from the atom chip surface. The trap is generated by the snake wire. (c) Bottom image of a cloud trapped by the snake wire.

Figure 2.26: New science chamber design. This new chamber will allow higher resolution by allowing us to position the imaging system closer to the atomic cloud with a much larger clear diameter.
Chapter 3

Potential corrugations

3.1 Introduction

Shortly after the first BECs had been produced in wire based microtraps [39, 40], the availability of ultracold atoms near current carrying wires led to the observation of surprisingly large unwanted potential corrugations [43]. Whenever an atom cloud, either just above \( T_C \) or as a BEC, was brought close to the surface \((h < 100 \mu \text{m})\), the cloud started to fragment along the longitudinal trap direction. This time independent effect was observed and studied by numerous atom chip groups and is attributed mainly to inhomogeneities in the direction of the electric current in the trapping wire [43, 45, 46, 61, 62]. The chips’ fabrication techniques were shown to play a significant role in this effect; in early atom chip experiments the chip wires were fabricated by a wet-chemical technique of electroplating. This resulted in relatively rough wire surfaces and edges. On the other hand, atom chips fabricated using lithographic processes have significantly smoother structures. Consequently atom chips fabricated by lithography have shown two orders of magnitude improvement in the modulation of the magnetic potential [61].

Apart from the improved fabrication that has led to a considerable reduction of the magnetic corrugations, other ways of overcoming this damaging effect were recently suggested. The Orsay group demonstrated reduced fragmentation [50] by using a time-averaged potential (TOP trap) that was created by an oscillating current in the wire. The averaging of the longitudinal field component resulted in an improvement of two orders of magnitude (for electroplated wires). Our own group has suggested the use of electrically anisotropic materials for the atom chip [3].

The work presented in this chapter was published in Refs. [1] and [2]. The experiments were performed at the University of Heidelberg on an atom chip fabricated in the Weiss Family Laboratory for Nano-Scale Systems at Ben-Gurion University. The fabrication required a long and tedious R&D process including the development of new characterization tools. The theoretical model was developed
jointly and so was the data analysis. During the collaboration, mutual visits by students were made. I played an active role in all of these activities.

In addition to the original experimental and theoretical work concerning the detection of electron scattering in typical wires, this chapter presents two additional novelties: First, we predict a counter-intuitive effect in which narrower wires will exhibit less electron scattering. Second, we predict that utilizing electrically anisotropic wires will result in the rotation of the scattering wavefront angle and in the overall suppression of the corrugations.

3.2 Potential corrugations

Thin metal films are the classic environment for studying the effect of geometric constraints [63, 64] and crystal defects [65, 66] on the transport of electrons. In a perfectly straight long wire that is free from structural defects, a direct current (DC) strictly follows the wire direction and creates a magnetic field in the plane perpendicular to the wire. An obstacle may locally change the direction of the current and consequently locally rotate the magnetic field close to the wire by an angle \( \beta \) in a plane parallel to the plane of the thin film wire. Rough wire edges may also contribute to such rotations of the magnetic field close to the wire. An illustration of these effects is given in Fig. 3.1.

Ultracold atom magnetometry [47, 67] on atom chips [17, 18, 19] permits sensitive probing of this angle \( \beta \) (and its spatial variation) with \( \mu \text{rad} \) (\( \mu \text{m} \)) resolution. Compared to scanning probes having a \( \mu \text{m} \) scale spatial resolution and \( 10^{-5} \text{T} \) sensitivity, or superconducting quantum interference devices (SQUIDs) having \( 10^{-13} \text{T} \) sensitivity but a resolution of tens of \( \mu \text{m} \), ultracold atom magnetometry has both high sensitivity (\( 10^{-10} \text{T} \)) and high resolution (several \( \mu \text{m} \)) [67] and therefore offers a new magnetometry technique. In addition, ultracold atoms enable high resolution over a large length scale (\( \text{mm} \)) in a single shot. This en-
ables the simultaneous observations of microscopic and macroscopic phenomena, as described in this work.

### 3.2.1 Trapped atoms as a tool

In the cold atom magnetic field microscope \[47, 67\] atoms probe the local magnetic potential through their magnetic interaction. A Bose-Einstein Condensate (BEC) fills the magnetic potential landscape like a fluid, up to the chemical potential energy, while ultracold non-condensed ‘thermal’ atoms have a Boltzmann tail that extends to higher energies (see Fig. 3.2). In both cases the local atomic density is a measure of the variation of the local potential energy. If the atoms are trapped in a quasi-one dimensional (1d) geometry, as employed here, the variation in the 1d linear density \( n_{1d}(x) \) is a direct measure of the variation in the trapping potential \( \Delta V(x) \). For a BEC one finds

\[
\Delta V(x) = \hbar \omega_\perp \sqrt{1 + 4a < n_{1d}>} \left( 1 - \sqrt{1 + 4a n_{1d}(x)} \sqrt{1 + 4a < n_{1d}>} \right), \tag{3.1}
\]

where \( \omega_\perp \) is the trapping frequency characterizing the transverse confinement and \( a \) is the atom-atom scattering length. For thermal non-condensed atoms at temperature \( T \) one finds

\[
\Delta V(x) = -k_B T \log \frac{n_{1d}(x)}{< n_{1d}>}, \tag{3.2}
\]

where \( k_B \) is the Boltzmann constant. The normalization to the mean density makes the measured potential profiles \( \Delta V(x) \) independent of the atom number, and the relations between the different profiles in the scan are easier to interpret. The longitudinal potential variations are related to the magnetic field by

\[
V(x) = m_F g_F \mu_B B(x)
\]

where \( \mu_B \) is Bohr’s magneton, \( m_F \) is the quantum number associated with the Zeeman state of the atom and \( g_F \) is the Lande-factor.

For a BEC the measurement range of the magnetic corrugation is given by the chemical potential \( \mu \). For a thermal gas the relevant range is a few times \( k_B T \). In both cases the sensitivity of the measurement is a small fraction of this range. A chemical potential of \( \mu = 1 \text{ kHz} \) corresponds to 70 nT for an atom with a magnetic moment of 1 \( \mu_B \). Similarly, a temperature of 1 \( \mu \text{K} \) corresponds to 1.3 \( \mu \text{T} \). A 1d BEC gives even better sensitivity to very small magnetic variations. A sensitivity of 200 pT at 3 \( \mu \text{m} \) spatial resolution was obtained in \[47, 67\]. The sensitivity can be tuned by changing \( \omega_\perp \). Using a thermal gas results in reduced sensitivity, but allows one to measure over a larger range of the magnetic field. Also, the measurement range can be tuned very easily by adjusting the temperature. Very good sensitivity can be reached especially when one works with very cold thermal atoms. A BEC may be more sensitive but the experimental methods and the required stability (constant chemical potential) are more demanding and the analysis becomes more cumbersome with the phase fluctuations that are always present in 1d quasi condensates.
CHAPTER 3. POTENTIAL CORRUGATIONS

Figure 3.2: Schematic of magnetic field measurements with an ultracold thermal cloud (top figure) and a BEC (bottom figure). For a thermal gas the measurement range is given by a few times the temperature \((k_B T)\). For a BEC the measurement range is given by the chemical potential \(\mu\). The BEC has better resolution, but a smaller measurement range. Hence, if the potential differences become too large, then there will be isolated regions with no atoms in between, and there is no way to ensure that the chemical potential is still the same in disjoint regions. The thermal atoms have a larger and smoother measurement range. Their temperature can be adjusted to cover even relatively high corrugation peaks.

Consequently, even though a BEC was used in the exploratory research at the beginning of this investigation, for the final analysis described in this report an ultracold thermal gas was used. The ultracold thermal atoms were found to be a better probe because they gave a better homogeneity of the measurement, and were able to cover the higher potential barriers. Also, as noted, controlling the temperature allowed for a better tuning of the sensitivity over the required measurement range.

Imaging the density of the trapped 1d cloud allows mapping out the spatial variation of the magnetic potential with high resolution over the entire length of the cloud. Scanning the 1d atom cloud in the transverse direction allows reconstructing a full two dimensional (2d) potential landscape with unprecedented sensitivity (Fig. 3.3).

Close to a current carrying wire and on atom chips [17, 18, 19] the minimum of the trapping potential is parallel to the wire carrying current density \(J_x^{(0)}\), at distances controlled by the transverse external magnetic field \(B_y\). At these distances, the net transverse field is therefore zero and the potential variations \(\delta B_x(x)\) are then directly related to the change in the local magnetic field direction [47, 67]. The measurement of \(\delta B_x(x)\) is independent of the mean field \(B_x\).
3.2. POTENTIAL CORRUGATIONS

From a measured 2d map of the magnetic field one can reconstruct the local transverse current in the conductor [67]. This reconstruction involves the deconvolution of the Biot Savart law which can be performed if the current density is assumed to be confined to a 2d-plane. In this case only the transverse current density \( j_y \) contributes to the magnetic field \( B_x \) and can be calculated according to

\[
  j_y(x, y) = \frac{2}{\mu_0 d} \mathcal{F}^{-1}\{\bar{B}_x(k_x, k_y) e^{i|z|}\}(x, y),
\]

(3.3)

where \( k_x \) and \( k_y \) are the the Fourier wave-vectors, \( \bar{B}_x(k_x, k_y) = \mathcal{F}\{B_x(x, y)\}(k_x, k_y) \) and \( \mathcal{F} \) indicates a two-dimensional Fourier transform. This assumes that the current is homogeneous within the thickness of the conducting film \( d \).

![Magnetic Field Microscope](image)

Figure 3.3: Illustration of the experiment: cold atomic clouds/BEC’s are imaged at the measurement height but at different positions along the wire width. The images are combined to form a 2 dimensional magnetic scan.

3.2.2 The experiment

The ultracold atomic samples used to probe the magnetic field landscape on an atom chip were prepared by using standard procedures [68, 69]. \(^{87}\)Rb atoms are first laser cooled, then optically pumped into the \( |F = 2, m_F = 2 > \) state and cooled to a temperature of \( \sim 1 \mu K \) in 20 s by using forced evaporation in a magnetic trap of transverse and longitudinal trapping frequencies \( \omega_\perp = 2\pi \times 840 \text{ Hz} \) and \( \omega_x = 2\pi \times 21 \text{ Hz} \). From this trap, situated 100 \( \mu m \) above the chip surface, the atoms were transferred within 800 ms to the measurement location and further cooled to the final desired temperature selected to optimally measure the magnetic field variations. The resulting atom cloud is then typically 800 \( \mu m \) long,
trapped in a quasi one dimensional geometry with a transverse trapping frequency of $\omega_\perp = 2\pi \times 500 \text{Hz}$. The current through the sample wires was adjusted to 180 mA in each case. The atomic density was detected by resonant absorption imaging after 1 ms time of flight. For imaging, a 50 $\mu$s pulse of linearly polarized light was used, at an intensity of less than 10% of the saturation intensity. Employing diffraction limited optics, the optical resolution was $\sim 3 \mu\text{m}$.

Crucial to a quantitative measurement is the characterization of the atom cloud. The important parameters are the temperature of the cloud, and its position above the wire. The total number of atoms is less important, as it drops out from the calculation through the potential normalization of each scan line.

The temperature is extracted from the rate of change of the clouds’ Gaussian width in the transverse direction after the removal of the trapping field. This speed is measured by fitting a Gaussian profile to a series of at least 20 pictures acquired after an expansion time between 0.5 ms and 6 ms. This temperature measurement is performed at 5 equally spaced positions within each scanned region. No systematic dependence of the atom temperature on the scan position was detected.

To obtain absolute height measurements, the imaging beam is inclined by 6.5° toward the chip surface. Two cloud images are then seen, one corresponding to absorption before reflection, and the other corresponding to reflection before absorption. The height above the wire surface is accurately determined by finding the cloud distance from the plane of maximum correlation of the two images. This height measurement is relative to the surface of the wire that reflects the imaging beam and we therefore ensured that this reflection is from the surface of the wire studied. Height measurements were done with zero time-of-flight (in-situ). In order to avoid problems due to polarization rotation at the gold surface, linearly polarized light is used. The polarization direction is perpendicular to the plane of incidence. In this configuration, the polarization state is unchanged by reflection at the gold surface [70].

The transverse position of the cloud is determined indirectly through a calibration of the bias fields. The center position above the wire has been checked separately, by checking the position of the minimum transverse trap frequency. From this procedure the center position is determined with an accuracy of approximately $\pm 2 \mu\text{m}$. The scaling of the $y$ position is determined to an accuracy of approximately 10%.

### 3.2.3 Fabrication and characterization of the atom chip

The atom chip was fabricated at the Weiss Family Laboratory for Nano-Scale Systems at Ben-Gurion University. It was fabricated in a thermal evaporator by depositing gold onto a Si wafer covered by a 100 nm SiO$_2$ electrically insulating layer and a thin Ti adhesion layer. In the experiment we studied the effect of wire thickness and grain size by repeating the magnetic scan over three very different wires (see Table 3.1). In order to fabricate such wires with different grain size and thickness, three separate processes had to be used on the same chip.
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Wires A and B, in which the grain size is small, were deposited in two separate processes achieving different thicknesses, where no heating was applied. For wire B, the temperature of the cooled stage in the thermal evaporator reached 32°C, while for wire A it reached 53°C, as the deposition time was longer (in order to achieve the larger thickness). The lithography defining the wires was done by standard lift-off processes of a photoresist [Fig. 3.4(a)]. For wire C, the fabrication achieved a larger grain size by heating the substrate to 200°C, a temperature at which photoresists lose their properties. Therefore, in this case, the photoresist was put on after deposition, and etching the gold between the wires required ion beam milling rather than lift-off [Fig. 3.4(b)]. While fabricating one wire, care had to be taken to protect the other wires. Finally, a mirror layer of gold was deposited around the three wires in order to reflect the laser beams required for the Magneto-Optical Trap, the first stage of atom cooling [Fig. 3.4(c)].

Figure 3.4: SEM images of (a) wires with small grain size (e.g. wires A and B), and (b) large grain size (wire C). (c) The atom chip prior to insertion into the vacuum chamber.

A duplicate chip was characterized optically, electrically at room and liquid helium temperatures, and then geometrically by a high resolution scanning electron microscope (SEM), an atomic force microscope (AFM), and a white-light interferometer [71]. The actual experimental chip underwent only optical and room temperature resistance measurements before the experiment. The grain size was determined before the experiment by SEM measurements on the duplicate chip in order not to affect the experimental chip. To make sure that the grain size did not change during chip operation, the experimental chip also underwent SEM measurements after the experiment was completed. Edge roughness of the fabricated wires boundaries were measured to be (peak-to-peak) 40 – 50 nm for wire A and 20 – 30 nm for wire B (wires fabricated using the lift-off technique), and 40 nm for
the large grain-size wire C, fabricated by the ion-beam milling etching technique.

The top surface roughness of the wires was measured using a white-light interferometer on the experimental chip after the experiment was completed. Again, comparison to the duplicate chip was performed. A sequence of partially overlapping images of $105.6 \times 140.8 \mu m^2$ and $0.5 \mu m$ resolution were taken for each wire along a total length of about $1 \text{ mm}$. The radial spectrum of the surface height variations (shown in Fig. 3.5) was determined by averaging the two-dimensional Fourier transform of the images over all wave-vector directions and over all images. The measurement noise was estimated by taking the power spectrum of the differences between pairs of overlapping measured area and averaging over all pairs. This noise was found to be similar for all three wires and was taken in Fig. 3.5 to be their average.

We also measured the detailed surface structure and the grain size with an AFM (again, after the experiment was completed). Images of area $5 \times 5 \mu m$ (with a horizontal resolution of $40 \text{ nm}$ and a height resolution $< 1 \text{ nm}$) were used for validation of the white-light interferometer data and for estimating the surface roughness at the grain size scale. Grain structure was investigated with images of area $1 \times 1 \mu m$ (with a resolution of $2 \text{ nm}$).

As for electrical measurements, as noted, the experimental chip underwent room temperature resistance measurements before the experiment. Similar measurements were also performed on the duplicate chip. As the experimental chip was partly broken by removal from the experiment, the low temperature resistance was measured only on the duplicate chip. Room temperature resistance measurements were performed on the central part of the wires, where the geometry of the wire was measured accurately. The low temperature measurements were done on the entire length of the wires. Comparing the two chips we note that:

- their SEM and AFM analysis showed similar structure;
- their room temperature resistance measurements were identical to within a few percent;
- they were made by the same processes at the same time.

Consequently, we use the low-temperature measurements for the duplicate chip with high confidence.

### 3.3 Experimental observations and data analysis

Using cold atoms just above the transition to Bose-Einstein Condensation (BEC), we apply ultracold atom magnetometry to study the current deflection in three different precision-fabricated polycrystalline gold wires with a rectangular cross section of $200 \mu m$ width and different thicknesses and crystalline grain sizes, as summarized in Table 3.1. We choose a co-ordinate system in which the wire
3.3. EXPERIMENTAL OBSERVATIONS AND DATA ANALYSIS

Figure 3.5: Radial spectrum of the top surface corrugations \( |\delta z_+(k)| = \sqrt{(2\pi)^{-1} \int d\theta |\delta z_+(k, \theta)|^2} \) for the three wires as measured by the ZYGO white-light interferometer. Note the significant difference between the surface fluctuation spectrum of wire A and wires B and C. Dashed curve: ZYGO measurement noise level calculated by averaging many partly overlapping images.

Even though scattering by lattice vibrations (phonons) quickly diffuses the electronic motion at ambient temperature, long range-correlations (tens of \( \mu m \)) in the current flow patterns can be seen. This is surprising as effects of static defects are usually observed only on a length scale of several nanometers \([72, 73]\). We observe clear patterns of elongated regions of maximal current flow deviations \( \beta \) inclined by about \( \pm 45^\circ \) to the mean current flow direction. This orientational preference is present in all our measurements, independent of wire thickness or grain size. This angular preference can be quantified by the normalized angular power spectra \( p(\theta) = \int dk |\beta(k, \theta)|^2 \) of the magnetic field patterns, where \( k \) is the wave-vector of the Fourier transform of the measured \( \beta(x, y) \) (Fig. 3.6).

We also observe significant differences in the magnitude and spectral composition of the magnetic field variations above wires with different thicknesses. Table 3.1 summarizes the main observations and wire properties. The current directional variation \( \beta \) does not scale with the relative surface corrugation (i.e., relative to the thickness); the thinner films (\( H = 280 \) nm) have the largest relative thickness variations but show the smallest current directional variations. Moreover, the thin wire with the large grains (grain size \( 150 - 170 \) nm) shows the smallest vari-
Table 3.1: Properties of the wires under investigation (see text for definitions). All measurements were done on the chip used for the cold atom experiment except for the low temperature resistivity which was measured on a duplicate chip made with an identical (simultaneous) fabrication process.

<table>
<thead>
<tr>
<th>wire</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>thickness $H$ (µm)</td>
<td>2.08</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>grain size (nm)</td>
<td>60-80</td>
<td>30-50</td>
<td>150-170</td>
</tr>
<tr>
<td>resistivity at 296 K (µΩ·cm)</td>
<td>2.73</td>
<td>3.1</td>
<td>2.77</td>
</tr>
<tr>
<td>resistivity at 4.2 K (µΩ·cm)</td>
<td>0.094</td>
<td>0.316</td>
<td>0.351</td>
</tr>
<tr>
<td>atom temperature (nK)</td>
<td>286 ± 15</td>
<td>173 ± 2</td>
<td>92 ± 7</td>
</tr>
<tr>
<td>height of measurement (µm)</td>
<td>3.5±0.4</td>
<td>3.4±0.3</td>
<td>3.7±0.4</td>
</tr>
<tr>
<td>$\delta z^\text{rms}$ (AFM) (nm)</td>
<td>9.4</td>
<td>3.5</td>
<td>3.1</td>
</tr>
<tr>
<td>$\delta z^\text{rms}$ (ZYGO) (nm)</td>
<td>1.31</td>
<td>0.42</td>
<td>0.48</td>
</tr>
<tr>
<td>$\delta z^\text{rms}$ (ZYGO) / $H$ ($\times 10^{-3}$)</td>
<td>0.629</td>
<td>1.500</td>
<td>1.714</td>
</tr>
<tr>
<td>$\beta^\text{rms}$ (mrad)</td>
<td>0.168</td>
<td>0.0715</td>
<td>0.0388</td>
</tr>
<tr>
<td>$\lambda_{\beta}$ (µm)</td>
<td>77</td>
<td>46</td>
<td>48</td>
</tr>
</tbody>
</table>

3.3.1 Angular distributions in current flow

In order to analyze the underlying mechanism for the current direction deviations we consider a thin film (conductivity $\sigma_0$) in the $x - y$ plane with a direct current $J^{(0)} = \sigma_0 E^{(0)}$, where the electric field $E^{(0)}$ is in the $\hat{x}$ direction. We consider the effect of small fluctuations in the conductivity $\delta \sigma(x)$ on the current flow.

The current flow around a circular defect generates a dipole field [Fig. 3.7(a-b)] with a transverse component $E_y^{(1)} \propto \sin 2\theta$ (Sec. 3.7), causing the current field to be repelled from the defect (for $\delta \sigma < 0$) or attracted to the defect (for $\delta \sigma > 0$) and a 45° pattern in the transverse current flow forms.

A second analysis is based on a conductivity step ($\delta \sigma$) inclined by an angle $\theta$ to the current flow direction [Fig. 3.7(c)]. The resulting current density fluctuation...
3.3. EXPERIMENTAL OBSERVATIONS AND DATA ANALYSIS

The transverse current component $J_y$ is again proportional to $\sin 2\theta$, which is maximal for conductivity steps inclined by $\theta = \pm 45^\circ$.

In a metal film, we expect to find a random pattern of conductivity fluctuations $\delta\sigma(x)$. It can be constructed from a random spatial distribution of the above basic elements: microscopic circular defects or macroscopic conductivity steps of different angles. The relation between the microscopic and macroscopic phenomena for each of these models is described in Sec. 3.7.

For a general quantitative analysis we expand an arbitrary distribution $\delta\sigma(x)$ in a Fourier series of plane waves of the form $\delta\sigma(x) = \delta\sigma_k \sin(k \cdot x + \phi)$, where $k = (k_x, k_y) = k(\cos \theta_k, \sin \theta_k)$ and $\phi$ is an arbitrary phase. Each plane wave contributes to the current fluctuation angle $\alpha = \delta J_y/J_0$ according to Eq. (3.4), giving $\alpha(k) \approx -\sin 2\theta_k(\delta\sigma_k/2\sigma_0)$, and resulting in the observed $45^\circ$ pattern.

The resulting magnetic field angle fluctuation at height $z$ above the wire is

\[
\delta J = J^{(0)} \frac{\delta\sigma}{\sigma_0} (\sin^2 \theta \hat{x} - \cos \theta \sin \theta \hat{y}).
\]  

(3.4)
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Figure 3.7: (a) Current scattering by circularly symmetric (disk) local conductivity variations $\delta \sigma < 0$. (b) The transverse ($y$) component of the current field is proportional to $\sin 2\theta$ and gives rise to regions of maximal transverse current inclined by $\pm 45^\circ$. (c) Direction change of current flow due to a defect in the form of a conductivity step inclined by an angle $\theta$ with respect to the main flow direction $\hat{x}$. The conductivity is $\sigma_0$ everywhere except in the shaded area, where it is $\sigma_0 + \delta \sigma$ ($\delta \sigma > 0$ in this example). The largest transverse current is obtained when $\theta = \pm 45^\circ$. Both the former microscopic model and the latter macroscopic model predict the observed $\pm 45^\circ$.

directly related to the current fluctuations by

$$\beta(k, z) \approx e^{-kz} \alpha(k) \approx -\frac{1}{2} e^{-kz} \frac{\delta \sigma_k}{\sigma_0} \sin 2\theta_k,$$

(3.5)

which exhibits the same angular dependence. The exponential term $e^{-kz}$ represents a resolution limit, such that the effect of current changes on a length scale smaller than $2\pi z$ are suppressed in the spectrum of the magnetic field fluctuations. Starting from random conductivity fluctuations with a non-white spatial frequency distribution, the angular dependence $\sin 2\theta$ will emerge, giving rise to the observed $\pm 45^\circ$ preference.

3.3.2 Range of patterns in current flow

The macroscopic observable exhibiting elongated patterns of transverse current distribution tens of microns long, emerges from the microscopic scattering in the wire, estimated to have a length scale of tens of nanometers (e.g. grain size or diffusion length). We can describe this microscopic-macroscopic interplay by analyzing the two separate models as they are complementary in the sense that one (circular defects) begins with the microscopic description and the other (conductivity step) from the macroscopic one. First, we have shown that the transverse current distribution around a circularly symmetric conductivity defect exhibits a $\sin 2\theta$ dependence [see Eq. (3.25)]. Thus, a high concentration of many such microscopic defects with the same sign of $\delta \sigma$ will form a similar macroscopic pattern,
which is characterized by elongated patterns along the $\pm 45^\circ$ direction, as seen in Fig. 3.6. As indicated by the spectrum for the measured surface height of Fig. 3.5, this distribution is correlated over length scales of tens of $\mu$m and characterized by a spectral dependence $\delta \sigma(k) \propto 1/|k|$. Such a correlated distribution of scatterers is indeed characterized by a bunching of scatterers of the same sign in regions with a typical width of tens of $\mu$m. This allows for a constructive build up of transverse currents to occur along a macroscopic $45^\circ$ line, and hence gives rise to the macroscopic observable, as demonstrated in Fig. 3.8.

The conductivity step model allows us to understand quantitatively the emergence of macroscopic patterns containing elongated regions of transverse current distribution from a completely random conductivity inhomogeneity. Although the conductivity perturbations are random and do not have any angular preference, their $\sim 1/|k|$ spectral dependence implies that they may be expanded in a Fourier series of plane waves with predominantly large wavelengths. Each such plane wave is made of wavefronts, which are elongated regions similar to the conductivity step, with alternating sign. Hence, our macroscopic conductivity step in fact represents one Fourier component of the real space distribution of microscopic defects. The above result predicts maximum electron scattering from conductivity steps which are oriented at $\pm 45^\circ$. This explains the emergence of the elongated regions with that orientation in the maps of the transverse current distribution and gives the expression of Eq. (3.4), which allows a quantitative analysis of the current distribution for an arbitrary conductivity inhomogeneity distribution.

A detailed account of both models is given in the appendix.

3.4 Detailed analysis

3.4.1 Theoretical model

As shown in Fig. 3.9, we were able to accurately simulate the experimental results. In this section we explain our detailed theoretical model. Our calculation was performed for a metallic wire as in Fig. 3.10(a), having a rectangular cross-section of width $W$ in the $\hat{y}$ direction and thickness $H$ in the $\hat{z}$ direction, and carrying a current density $J(r) = J_0 \hat{x} + \delta J(r)$. Here $J_0 = I/WH$ is the regular current density for a total current $I$. Utilizing Ohm’s law $J = E/\rho$ where $E$ is the electric field, and Maxwell’s equation $\nabla \times E = 0$, we find

$$\nabla \times J = -\frac{\nabla \rho}{\rho} \times J,$$

with $\rho = \rho_0 + \delta \rho(r)$ being the isotropic resistivity. Although significant resistivity perturbations may exist in a polycrystalline metal near grain boundaries (length scale of nanometers), one may safely assume that over most of the length scales of interest (microns) $\delta \rho \ll \rho$, such that in the Fourier expansion $\delta \rho(r) = \sum_k \rho_k e^{ik \cdot r}$ one has $|\rho_k| \ll \rho_0$ for any relevant wave number $k \equiv (k_x, k_y, k_z)$. By keeping only terms up to first order in the resistivity gradient $\nabla \rho$ and using the current
Figure 3.8: The emergence of macroscopic elongated regions with preferred orientation in the pattern of the magnetic field fluctuations above a wire from a distribution of microscopic circularly symmetric defects. (left) The distribution of defects with positive (red) or negative (blue) conductivity change $\delta \sigma(x,y)$ is correlated with a $1/k$ spectral dependence, but has no preferred orientation, as indicated by the angular spectrum $P(\theta)$ (bottom). (middle) Bunches of point-like defects create a current distribution having an angular spectrum (bottom) which is maximal around $\theta = 45^\circ$ and $135^\circ$. (right) the resultant magnetic field fluctuations above the plane averages the fine structure of the currents by the exponential factor $e^{-kz}$ which suppresses short wavelength variations but keeps the orientational preference.

continuity equation $\nabla \cdot \delta \mathbf{J} = 0$ we obtain the solution for the components of the current irregularities as a function of the bulk inhomogeneity

$$\delta \mathbf{J}(\text{bulk}) (\mathbf{k}) = J_0 \left( \frac{k_x}{|\mathbf{k}|^2} \mathbf{k} - \mathbf{x} \right) \frac{\delta \rho_k}{\rho_0},$$

(3.7)

where the transverse components $(k_y, k_z)$ of the wave-vector $\mathbf{k}$ take the discrete values $2\pi(m/W, n/H)$ with integers $m$ and $n$, $-\infty < m, n < \infty$.

The horizontal transverse current irregularities $\delta J_y(\text{bulk})$ are proportional to $k_x k_y / k^2 \propto \sin 2\theta_k$, where $\theta_k \equiv \tan^{-1}(k_y/k_x)$ is the angle in the $x-y$ plane. This immediately implies that transverse currents are predominantly generated by Fourier components of the resistivity perturbations with wavefronts oriented at $\pm 45^\circ$. Vertical current irregularities $\delta J_z(\text{bulk})$ are proportional to $k_z k_x / k^2 \propto \sin 2\phi_k$, where $\phi_k \equiv \tan^{-1}(k_z/k_x)$. Vertical currents are therefore significant only for Fourier components satisfying $k_x \sim k_z$ ($\phi_k \sim \pm 45^\circ$), namely, for longitudinal wavelengths $2\pi/k_x$ of the order of the thickness $H$ or less, corresponding to non-zero values of $k_z$. For thin wires, these wavelengths are usually beyond the spatial mea-
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measurement resolution in the $x-y$ plane. At wavelengths of interest, much larger than $H$ ($k_x \ll k_z$), vertical currents are suppressed as $\delta J_z^{(\text{bulk})} \propto \frac{k_x H}{2\pi} \ll 1$.

In the following we refer to the spectral regime $(k_x, k_y) \ll 2\pi/H$ as the “thin film limit”, where only contributions from Fourier terms with $k_z = 0$ are important. We will then consider the film as two-dimensional and characterize it by the real-space vector $\xi \equiv (x, y)$ and Fourier space vector $\kappa \equiv (k_x, k_y)$. Thickness variations of the wire $\delta H(x,y)$ may then be regarded as irregularities of the thin film resistivity $\delta \rho^{\text{thickness}} = -\rho \delta H/H$. Fig. 3.10(a) demonstrates the generation of periodic horizontal current directional deviations due to resistivity perturbations originating from bulk or thickness variations.

A typical magnetic corrugation along an elongated trap is determined mainly by the longitudinal component of the magnetic field fluctuations at the trapping position. Its Fourier spectrum at a height $z_0$ is related to the current irregularities in the wire by

$$
\delta B_x(k_x, k_y, z_0) = \frac{\mu_0}{2} \int_{-H}^{0} dz' e^{-\kappa |z_0 - z'|} \times
\times \left[ \delta J_y(k_x, k_y, z') + i \sin \theta_\kappa \delta J_z(k_x, k_y, z') \right], \tag{3.8}
$$

where $\mu_0$ is the permeability of the vacuum. Here, the free space (continuous)
Fourier transformations $\delta J(k_x, k_y, z')$ may be approximated by their discrete form as in Eq. (3.7) if $\kappa W \gg 1$ and $z_0 \ll W/2$. Substituting $\delta J$ of Eq. (3.7) into this expression one finds that $\delta B_x(\kappa) \propto e^{-\kappa z_0} \sin 2\theta_\kappa$ limiting the spatial resolution in the $x-y$ plane by the measurement distance $z_0$.

Figure 3.10: (a) A single Fourier component (plane wave) of the planar resistivity perturbations due to bulk inhomogeneity or wire thickness variations induces current flow directional changes (arrows). The current tilts along low resistivity wavefronts and across high resistivity wavefronts. (b) The amplitudes of transverse current component $\delta J_y(\kappa) [\kappa \equiv (k_x, k_y)]$, generated by resistivity perturbations $\delta \rho(\kappa) \propto 1/\kappa$, are proportional to $\sin 2\theta_\kappa$ (color scheme). For a wire with a finite length $L$ and width $W$, these amplitudes are calculated at discrete values of $\kappa$, which are integer products of $2\pi/L$ and $2\pi/W$. (c) As a consequence of this discreteness, the corrugations at long wavelengths ($k_x W < 1$) are suppressed when the wire becomes narrower: the density of $k_y$ states becomes lower and no allowed values of $k_y$ exist along the maximum scattering amplitude line $k_y \sim k_x$. Here, the power spectrum of the magnetic field corrugations at $3.5 \mu m$ above the center of the wire is shown as a function of $k_x$ for resistivity perturbations $\delta \rho_\kappa/\rho_0 = 3.4 \cdot 10^{-4}(\kappa_0/\kappa)$ with $\kappa_0 = 2\pi/680 \mu m^{-1}$. Long wavelength components are suppressed when the wire becomes narrower. (d) For comparison, the magnetic field corrugations above the center of the wire are shown for wires of different widths for a model assuming edge fluctuations with $|\delta y_\kappa(k_x)| = 10 nm \times (\kappa_0/k_x)$. Here, short wavelength components are suppressed when the wire becomes wider.

The $\sin 2\theta_\kappa$ dependence together with a $\sim 1/\kappa$ dependence of the resistivity perturbations (see Sec. 3.4.2), demonstrated by the color map in Fig. 3.10(b), describe well the behavior in the continuum limit $W \rightarrow \infty$. However, for finite widths (and a finite measurement length $L$) $k_x$ and $k_y$ assume only discrete values which are integer multiples of $2\pi/L$ and $2\pi/W$, respectively, as demonstrated by the grid of dots superimposed on the color map. It follows that for small values of
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$k_x$, no counterparts $k_y$ exist on the grid which lie in the region where $|\sin 2\theta_k|$ is large, or more specifically, around the line $\theta_k = 45^\circ$. This implies that at wavelengths larger than the wire width $W$ the current irregularities are significantly suppressed beyond the suppression caused by the reduction of grid points. This prediction is demonstrated in the power spectrum shown in Fig. 3.10(c). This result is very different from the effect of current irregularities due to edge roughness, which characterized measurements of atomic density fluctuations in some earlier work [45, 46, 62]. In that case, the short wavelengths are exponentially suppressed near the center of the wire, while only wavelengths of the order of the wire width or more are effective [Fig. 3.10(d)]. To take advantage of the corrugation suppression in narrow wires, we will address in Ch. 4 the possibility of using nano-sized wires for trapping atoms.

We now turn to a more detailed theory of current irregularities due to geometrical imperfections of the wire [76]. We solve Eq. (3.6) with $\delta \rho \to 0$ and with boundary conditions ensuring that the current flows parallel to the boundaries. Taking the upper and lower surfaces of the wire at $z = \pm H/2 + \delta z_\pm$ and the right and left edges at $y = \pm W/2 + \delta y_\pm$, where $\delta z_\pm(x,y)$ and $\delta y_\pm(x,z)$ are small fluctuations of the corresponding surface positions, we obtain the following boundary conditions,

\[
\delta J_y(x, \pm W/2, z) = J_0 \frac{\partial \delta y_\pm}{\partial x}, \\
\delta J_z(x, y, \pm H/2) = J_0 \frac{\partial \delta z_\pm}{\partial x},
\]

where terms of second or higher orders in $\delta z_\pm$ and $\delta y_\pm$ were omitted. The current irregularities are then written as a sum of two terms,

\[
\delta J^{(surf)}(r) = \sum_{k_x} e^{ik_x x} \left[ \delta J^W_{k_x}(y,z) + \delta J^H_{k_x}(y,z) \right].
\]

Equation (3.6) with $\nabla \rho = 0$ together with the continuity equation $\nabla \cdot J = 0$ imply that the current can be written as the gradient of a potential function $\delta J^{(surf)} = \nabla F$, which satisfies the Laplace equation $\nabla^2 F = 0$. It follows that the terms in Eq. (3.10) have the form

\[
\delta J^W_{k_x}(y,z) = i k_x J_0 \sum_{n,\pm} a_{n,\pm}(k_x) e^{i2\pi n y/H} e^{-|y\mp W/2|/\lambda_n},
\]

\[
\delta J^H_{k_x}(y,z) = i k_x J_0 \sum_{m,\pm} b_{m,\pm}(k_x) e^{i2\pi m y/W} e^{-|z\mp H/2|/\lambda_m},
\]

where the exponential terms describe the attenuation of current fluctuations induced by each boundary perturbation at a distance $\lambda_n = [k_x^2 + (2\pi n/H)^2]^{-1/2}$ from the left/right boundaries and $\lambda_m = [k_x^2 + (2\pi m/W)^2]^{-1/2}$ from the top/bottom boundaries. Since $\delta J$ is derivable from a scalar function, it follows that each of the vectorial coefficients $a_{n,\pm}$ and $b_{m,\pm}$ can be derived from the corresponding scalar

\[
\delta J^W_{k_x} = \nabla F^W_{k_x}, \\
\delta J^H_{k_x} = \nabla F^H_{k_x}.
\]
coefficients. Linear equations are obtained for these scalar coefficients when $\delta J$ of Eq. (3.10) is substituted in the boundary conditions (3.9) [77].

Next, we describe the solutions of these equations for a few typical simple cases. The term $\delta J_W$ [Eq. (3.11)] is significant when the edge perturbations $\delta y \pm$ are large and the measurement height is comparable to the wire width $W$. This situation was discussed in previous work [45, 62]. Here we concentrate on the other limit, where the field is measured at a low height and a large distance from the edges compared to $\lambda_n$ such that $\delta J_W \sim 0$ for most values of $k_x$. The surface height fluctuations $\delta z^\pm_k$ then generate the following current irregularities (Eq. (3.12))

$$
\delta J_y^{(surf)}(r) \approx -J_0 \sum_{\kappa} e^{i\kappa \cdot \xi} \frac{\sinh(\kappa z)}{\delta H} \frac{\kappa_y}{\sinh(\kappa H/2)} \frac{\delta H}{\kappa \kappa^2} \delta z^\pm_k,
$$

(3.13)

$$
\delta J_y^{(surf)}(r) \approx -J_0 \sum_{\kappa} e^{i\kappa \cdot \xi} \frac{\sinh(\kappa z)}{\delta z^\pm_k} \frac{\kappa_y}{\sinh(\kappa H/2)} \delta z^\pm_k,
$$

(3.14)

where $\delta H = \delta z^+_k - \delta z^-_k$ are wire thickness variations and $\delta z^\pm_k = (\delta z^+_k + \delta z^-_k)/2$ are height fluctuations of the center of the wire. In the thin film limit $\kappa H \ll 1$, we find $\kappa z \ll 1$ for $|z| \leq H/2$ such that $\delta J_y$ assumes a form similar to Eq. (3.7) with $\delta \rho / \rho_0 \rightarrow -\delta H / H$

$$
\delta J_y^{(surf)}(r) \approx -J_0 \sum_{\kappa} e^{i\kappa \cdot \xi} \frac{\kappa_y}{\sinh(\kappa H/2)} \frac{\delta H}{2H} \delta z^\pm_k.
$$

(3.15)

In the same limit, the magnitude of the vertical current component $\delta J_z$ becomes negligible, since $\delta J_z^{(surf)} \propto k_x \delta z^\pm_k = k_x H (\delta z^\pm_k / H) \ll \delta z^\pm_k / H$.

### 3.4.2 Spectral analysis

In order to investigate whether the observed current flow deviations are related to corrugations in the top surface of the wire, we have measured the surface topography of the wires using a white-light interferometer. No angular preference inherent in the structure of the wires was found. Consequently, the angular pattern in the magnetic field variations presented in Fig. 3.6 must be a property purely of the scattering mechanism of the current flow by the wire defects, as outlined above. In addition, when we calculate the two dimensional magnetic field at $3.5 \mu m$ above the surface, using the white-light interferometry measurements and the assumption $\delta H(x) = \delta z_+(x)$, we could not find a reasonable fit between the latter and the magnetic mapping done by the atoms (Fig. 3.6). A detailed analysis of the top surface corrugations $\delta z_+$ (Fig. 3.5) shows that they are significantly larger for the thick film compared to the two thin films, especially at short length scales.

To quantify our findings, we compare the power spectra of the measured magnetic field variations to those calculated from several models based on the measured top surface variations, an assumed bottom surface roughness, and possible inhomogeneities in the bulk conductivity (Fig. 3.11).
3.4. DETAILED ANALYSIS

We start with the two thin wires: B and C. The measured power spectra of the magnetic field variations are significantly lower (by two orders of magnitude for large wavelengths) than predictions based on a model with a flat bottom surface ($\delta H = \delta z_+$). If we assume instead that the top surface follows the bottom surface exactly ($\delta z_+ = \delta z_-$), a lower bound on the influence of the surface on magnetic field fluctuations can be obtained, as this configuration produces vertical currents whose contribution to the longitudinal magnetic field, to which our experiment is sensitive, is very small. The measured data lies between these two cases.

![Figure 3.11](image)

Figure 3.11: (left) Comparison of surface and bulk model calculations (lines) with the measured power spectrum $P(k_x) = \sum_{k_y} |\beta(k_x, k_y)|^2$ of the magnetic field angle $\beta$ along the $x$ direction (points) above the three wires. Blue: top surface $\delta z_+$ as in Fig. 3.5 with flat bottom surface $\delta z_- = 0$. Red: top surface follows bottom surface $\delta z_+ = \delta z_-$. Green: Partially correlated top and bottom surfaces, namely, $\delta z_- (k) = \delta z_+ (k) e^{-(k/k_0)^2}$, for wires B and C. For wire A we assumed $\delta z_- (k)$ as in wire B, which is correlated (purple) or uncorrelated (light blue) with the top surface. The latter gives the closest estimate for the experimental data, but gives $\beta_{\text{rms}}$ which is only about half of the measured value. Black: a fit to a model assuming general conductivity fluctuations $\delta \sigma (k) = \delta \sigma (k_0) (k_0/k)^\nu$, with $\nu = 1$ for wire A and $\nu = 1/2$ for wires B and C. The shaded area represents a one standard deviation range obtained by varying the relative phases of different spectral components $\delta \sigma (k_x, k_y)$. (right) Power spectrum $P(k_x)$ of the magnetic field angle for the three wires with a fit using the bulk conductivity inhomogeneity model (black lines) and the surface corrugations model for the two thin wires B (blue) and C (red). It is clearly seen that the slope of $P(k_x)$ for the two thin wires is similar, while it is larger for the thick wire A.

A fairly good fit of the measured spectrum for the thin wires is obtained if we assume that the top surface partially follows the long-wavelength fluctuations of the bottom surface while independent fluctuations of the top surface exist on a shorter scale. For such a model we assume $\delta z_- (k) \approx \delta z_+ (k) e^{-(k/k_0)^2}$. Note that the resulting average thickness variations are extremely small $|\delta H_{\text{rms}}| = |\delta z_{+\text{rms}} - \delta z_{-\text{rms}}| < 1 \text{ Å}$. This value of $\delta H_{\text{rms}}$ refers to length scales longer than 1 µm, while
much larger surface variations were measured on the scale of the grains using the AFM (see Table 3.1).

The situation is different for the thick wire A ($H = 2 \mu m$). Models assuming a flat bottom surface ($\delta H = \delta z_+$) and models assuming a corrugated bottom surface $\delta z_-$ with a spectrum similar to that of wire B and no correlations with the top surface, both underestimate the measured magnetic field variations.

The difference between the surface models and the measured data of wire A can be attributed to fluctuations in the bulk conductivity. A model taking the maximal contribution of surface roughness (uncorrelated top and bottom surfaces) into account puts the minimal required contribution of the bulk conductivity fluctuations at 50% of the signal. If we apply the same minimal bulk conductivity fluctuations as obtained from wire A to the two thin wires B and C, they overestimate the measured magnetic field fluctuations substantially for both wires and give a different spectral shape ($\nu = 1$ for wire A and $\nu = 1/2$ for wires B and C, see Fig. 3.11). This indicates that the bulk conductivity of the thinner wires should be more homogeneous than that of the thick wire. This may fit well with common knowledge among fabricators that above one micron of evaporation yields a very different wire structure.

However, a more homogeneous bulk conductivity in the thin wires appears to be contradictory to the fact that the low temperature resistivity is smaller for the thick wire than for the thin wires (Table 3.1). Nevertheless, we note that this resistivity is mainly determined by the small-scale properties of the wire (on the order of the grain size or less) and by surface scattering, while the magnetic field variations probe the conductivity inhomogeneities on a larger scale and provide complementary information that would not be available by standard methods.

The difference in the spectral decomposition is also clear from the slope of the curves in Fig. 3.11. The major part of the longitudinal power spectrum in Fig. 3.11 (for $0.05 < k_x < 0.3 \mu m^{-1}$) can be well fitted with an exponential function $P(k_x) \propto e^{-\alpha k_x}$ with $\alpha = 18, 11$ and $9 \mu m$ for the three wires.

The physical origin of the difference between the spectral decomposition of the thick wire and the thin wires lies in a combination of two possible explanations. The bulk inhomogeneity model fits the power spectrum of the thick wire when $\delta \sigma(k) \propto 1/k$, while a best fit is obtained for the thin wires when $\delta \sigma(k) \propto 1/k^{1/2}$. This may come from the fact that evaporation of a gold layer above one micron gives a very different wire structure than for thinner films. An additional explanation is obtained from the model of partially correlated top and bottom surfaces. It is likely to assume that wires generated by the evaporation process of a gold layer show more correlation between the top gold surface and the bottom gold surface (which merely follows the top wafer surface) when the gold layer is thinner. Secondly, for given surface corrugations, the relative thickness variations are inversely proportional to the average thickness of the wire, so that the contribution of conductance variations originating from surface corrugations becomes more dominant in thin wires relative to bulk inhomogeneity variations, which are not expected to grow with reduced thickness. Hence the surface induced directional fluctuations
are more dominant in this film.

Our analysis furthermore suggests that the differences in the length scale $\lambda_\beta$ of the variation in $\beta$ as seen in Fig. 3.6 and quantified in Table 3.1 may originate from the fact that conductivity variations in the thin wires coming from thickness variations are suppressed at long length scales due to top and bottom surface correlations. In contrast, conductivity variations in the thick wire originate at all scales from a combination of thickness variations due to uncorrelated top and bottom surfaces and a dominant contribution of bulk conductivity inhomogeneity.

3.5 Discussion

To conclude this rather lengthy and elaborate study, in the following we group together our major experimental observations, analyses and conclusions:

3.5.1 Observations

1. The top gold layer surface roughness power spectrum falls much more rapidly for the thin wires (Fig. 3.5).

2. The top surface roughness of the gold layer as measured by white light interferometry and AFM shows a much larger roughness for the thick wire. However the relative roughness (roughness relative to thickness) is much larger for the thin wires (Table 3.1).

3. Room-temperature resistivity is comparable for all three wires while the low temperature resistivity is higher for the thin wires (Table 3.1).

4. Magnetic fluctuations show long-range patterns up to tens of microns long. (Fig. 3.6)

5. The patterns show an orientational preference of $\pm 45^\circ$ (Fig. 3.6).

6. Wire A has the largest magnetic fluctuations and wire C has the smallest among the thin wires (Fig. 3.6, Table 3.1).

7. The power spectrum of the observed data falls much more rapidly for the thick wire (Fig. 3.9). This is also confirmed by an analysis of the periodicity $\lambda_\beta$ (Table 3.1). We note that this is contrary to the behavior of the top surface roughness (Fig. 3.5).

3.5.2 Analysis

1. The $45^\circ$ behavior and the long range patterns can be explained by conductivity fluctuations arising from surface roughness or bulk inhomogeneity.
2. All surface corrugation models underestimate the measured magnetic variations of the thick wire A (50% is missing in the maximal thickness variations model).

3. A surface model assuming partial correlation in the top and bottom wire surfaces fits the data well for the thin wires B and C.

4. A general model, assuming a power law distribution $1/k^\nu$ for the conductivity variations, which may represent bulk inhomogeneities, gives a good fit for wire A when $\nu = 1$, and for wires B and C when $\nu = 1/2$.

3.5.3 Conclusions

1. The partially correlated surface model is able to explain the data of the thin wires B and C. This model may be justified by the fact that for thin films, the top surface of the gold layer follows the bottom surface for periodicities larger than the thickness. In small periodicity fluctuations (large k), the surfaces are likely to be uncorrelated, giving rise to thickness variations. This is why high frequency components are measured in the magnetic variations for the thin wires.

2. The measured data of the thick wire A cannot be explained by surface models alone. It could be explained by a model assuming conductivity variations. This may be understood in two complementary ways. First, the relative thickness variations are much smaller for the thick wires, therefore, it should be expected that the surfaces have less influence on the thick wire. Second, in evaporated films with thickness above one micron, the gold structure becomes less ordered, it is then expected that in thick wires bulk inhomogeneity is larger. While we do not have a physical model to explain bulk conductivity variations, we think we have clearly shown that in the case of the thick wire we have measured these variations.

3.6 Summary

Our study is the first direct application of ultracold atoms as a magnetic probe for solid state science. The exceptional sensitivity of the ultracold atom magnetic field microscope [47, 67] allows us to observe long-range patterns of the current flow in a disordered metal film. Four questions arise from the data obtained:

1. Why are the patterns of current flow three orders of magnitude longer than expected from the grain size or the diffusion length?

2. Why are the patterns predominantly oriented at $\pm 45^\circ$?

3. Why do the different wires have such different characteristic periodicity?
4. Why does wire C, with the largest grain size, have the lowest current deviations?

We show that the first two questions may be answered by simple but quantitative models of universal scattering properties at defects, from which thin wires with random imperfections give rise to current directional deviations having a length scale on the order of a micron or longer. These deviations may arise either from bulk resistivity inhomogeneities or from geometrical perturbations of the wire, where the significance of each factor depends on the wire thickness and the fabrication process. We have shown through simulation that the latter dependencies explain the observed differences in the periodicity of the wires’ patterns. In both cases, electron scattering is dominant at wavefronts oriented at ±45° relative to the main current axis. To date, however, we do not understand why wire C has the lowest current deviations.

The models predict that current deviations in narrower wires are strongly suppressed at long wavelengths. This analysis therefore opens the road for material engineering to considerably improve atom optics on atom chips. In the next chapter we will discuss the possibility of using nano-size wires in the atom chip. Electrically anisotropic materials are also capable of significantly suppressing current deviations (see Sec. 3.7.3).

3.7 Appendix

3.7.1 Current flow in a conductivity step

Let us first consider current scattering by a conductivity step as in Fig. 3.7(c), where the conductivity is $\sigma_0$ except in the shaded area between two parallel lines inclined by an angle $\theta$, where the conductivity is $\sigma_0 + \delta \sigma$ ($\delta \sigma \ll \sigma_0$). The voltage applied along the $x$ direction induces a constant current $J^{(0)} \hat{x} = \sigma_0 E^{(0)} \hat{x}$ everywhere. The current continuity equation $\nabla \cdot J = 0$ together with Ohm’s law $J = \sigma E$ give rise to the following equation for the electric field

$$\nabla \cdot E = -\frac{\nabla \sigma}{\sigma} \cdot E.$$

This equation is equivalent to Maxwell’s equation $\nabla \cdot E = \rho/\varepsilon_0$, where $\rho$ is the charge density. It follows that the gradients of the conductivity at the right (+) and left (-) boundaries of the step are equivalent to homogeneous charge densities

$$\rho_{\pm} = \pm \varepsilon_0 (\delta \sigma/\sigma_0) E^{(0)} \cos \theta$$

per unit area. Equivalently to the field formed between the two plates of an oppositely charged capacitor, here as well an electric dipole field $E^{(1)} = \hat{n} (\rho_+ - \rho_-)/2\varepsilon_0$ is formed between the two interfaces in a direction normal to the interfaces. This additional electric field induces an additional current $\delta J$ inside the
conductivity step. To the first order in $\delta \sigma$ this current is given by

$$\delta J = \delta \sigma E^{(0)} + \sigma_0 E^{(1)} = J^{(0)} \delta \sigma \left[ \hat{x} - \cos \theta (\cos \theta \hat{x} + \sin \theta \hat{y}) \right].$$

(3.18)

The transverse current component $J_y$ is then proportional to $\cos \theta \sin \theta = \sin 2\theta / 2$.

### 3.7.2 Current flow around a circular obstacle

A cylindrical defect of radius $R$, extending over the whole thickness of a thin film, in which the conductivity is changed from its bulk value $\sigma_0$ to $\sigma_0 + \delta \sigma$, causes a deflection of the current density $J^{(0)} \hat{x}$ impinging on the defect. If $\delta \sigma$ is negative then the current bends away from the obstacle and tends to bypass it, while if $\delta \sigma$ is positive the current is attracted into the defect. Here we argue that this current deflection emerges from a dipole field that is created at the defect, as shown in Fig. 3.7(a,b). This follows from Eq. (3.16) which shows that a conductivity gradient is equivalent to a charge density

$$\rho = -\varepsilon_0 \frac{\nabla \sigma}{\sigma} \cdot E,$$

(3.19)

where in our case $E = E^{(0)} \hat{x} = J^{(0)} / \sigma_0 \hat{x}$. The charge density per unit length of the boundary at the edges of a disk of radius $R$ is then given by

$$\rho(R, \phi) = R \delta \sigma \sigma_0 E^{(0)} \cos \phi,$$

(3.20)

where $\phi$ is the angular coordinate. Very close to the boundary an electric field is formed that points in a direction normal to the boundary and outwards from the boundary. The electric field generated everywhere by the excess charge density can be calculated by an integration

$$\delta E_j(r, \theta) = \int_0^{2\pi} d\phi \rho(R, \phi) G_j(r, R, \theta, \phi),$$

(3.21)

for $j = x, y$, where the Green’s functions are given by

$$G_x(r, R, \theta, \phi) = \frac{1}{2\pi} \frac{r \cos \theta - R \cos \phi}{r^2 + R^2 - 2Rr \cos(\theta - \phi)},$$

(3.22)

$$G_y(r, R, \theta, \phi) = \frac{1}{2\pi} \frac{r \sin \theta - R \sin \phi}{r^2 + R^2 - 2Rr \cos(\theta - \phi)}.$$

(3.23)

By performing the integration we obtain

$$\delta E(r, \theta) = \frac{\delta \sigma}{\sigma_0} E^{(0)} \times \begin{cases} (-1/2, 0) & r < R \\ (R^2 / 2r^2)(\cos 2\theta, \sin 2\theta) & r > R \end{cases}$$

(3.24)
To the first order in $\delta \sigma$ it follows $J^{(0)} = \sigma_0 E^{(0)}$

$$
\delta J(r, \theta) = \frac{\delta \sigma}{\sigma_0} J^{(0)} \times \left\{ \begin{array}{ll}
(1/2, 0) & r < R \\
(R^2/2\pi^2)(\cos 2\theta, \sin 2\theta) & r > R
\end{array} \right.
$$

(3.25)

It can then be checked that the normal part of $\delta J$ is continuous on the boundary. The current flow inside the disk is constant, while out of the disk it has a dipole pattern whose transverse component $\delta J_y$ is proportional to $\sin 2\theta$, as seen in Fig. 3.7(a,b).

### 3.7.3 Wire with anisotropic conductivity

Our model for current induced potential corrugations also gives rise to predictions. For example, as we have shown previously, we predict that narrow wires would have much less surface and bulk induced potential corrugations [Fig. 3.10(c)]. Another prediction of our model (Sec. 3.4.1) is obtained when we generalize the situation to the case where the conducting wire is electrically anisotropic, such that the resistivity is a diagonal tensor and Ohm’s law generalizes to

$$
E_j = \rho_j J_j
$$

for $j = x, y, z$. In this case Eq. (3.7) becomes [3]

$$
\delta J^{(\text{bulk})}(k) = J_0 \left( \frac{k_x}{k \cdot q} q - \hat{x} \right) \frac{\delta \rho_{x,k}}{\rho_{x,0}},
$$

(3.26)

where $q = (k_z/\rho_x, k_y/\rho_y, k_z/\rho_z)$. In the limit of a thin film, where $k_z = 0$, the horizontal transverse current irregularities $\delta J_y$ are proportional to $\sin 2\theta_{\kappa}/(1+(r-1)\cos^2 \theta_{\kappa})$, where $r = \rho_y/\rho_x$ is the resistivity ratio. As demonstrated in Fig. 3.12, the scattering at angles $\theta_{\kappa} < 45^\circ$ is suppressed if $r > 1$ and enhanced if $r < 1$, thus changing the preferred scattering wavefront angle in the range $0^\circ < \theta_{\kappa} < 90^\circ$. The overall magnetic corrugations are suppressed as $r^{-3/4}$ in the limit of high anisotropy $r \gg 1$. 
Figure 3.12: For a current flowing through an electrically anisotropic wire, the perturbation wavefront angle $\theta_{K}^{\max}$, giving rise to maximum transverse electron scattering, will depend on the ratio $r = \rho_y/\rho_x$ between the transverse and longitudinal resistivities (main plot). The two-dimensional maps show the predicted atomic density above a wire similar to that presented in Fig. 3.9 in the extreme cases $\theta_{K} = 0^\circ$ ($r \ll 1$), $90^\circ$ ($r \gg 1$) and the isotropic case $\theta_{K} = 45^\circ$ ($r = 1$). Bottom inset - the magnetic corrugation amplitude as a function of $\theta_{K}$, which is suppressed when $r > 1$ [3].
Chapter 4

Nanowire atom chip traps for sub-micron atom-surface distances

4.1 Introduction

Decreasing the atom-surface distance should increase trap gradients sufficiently to construct tunneling barriers with widths on the order of the atomic deBroglie wavelength, enabling e.g., atom chip interferometry based solely on static magnetic fields. Such high trap gradients may also allow more robust atom-light interactions such as probing without heating in the Lamb-Dicke regime. Furthermore, sub-micron distances are also important for technological advantages such as low power consumption and high-density arrays of traps.

At small atom-surface distances, interactions with the nearby surface become important. For example, spatial and temporal magnetic field fluctuations, due to electron scattering and Johnson noise respectively, limit the minimum atom-surface distance, as they cause potential corrugations, spin flips, heating and decoherence. There have been several experiments utilizing cold atoms to study these interactions [1, 43, 44, 46, 47, 48], and many suggestions on how to overcome their damaging effects [2, 3, 49, 50, 51, 52].

Also becoming prominent for small atom-surface distances is the Casimir-Polder (CP) force [53]; normally attractive, it reduces the magnetic barrier and allows atoms to tunnel to the surface, as already observed [54, 55].

Finally, let us note that the limitations could also be turned around to become advantages. At very small distances the atoms may also serve as a sensitive probe for surface phenomena. One example is electron scattering which we observed and is described in this thesis. Another example are plasmons which are expected to affect the atomic external and internal degrees of freedom and may also become observable [56].

Let us note that there are numerous ideas for bringing atoms closer to the
surface [78, 79, 80, 29, 81, 35, 82], all of which are, however, based on interactions with fields other than static magnetic fields, the latter being the focus of this work. We consider wires operating at room temperature, fabricated using standard methods, in contrast to suspended molecular conductors [51, 83] and superconductors, that reduce the Casimir-Polder force and noise originating in the surface [48, 84, 85, 86, 87, 88] respectively.

This chapter is organized as follows: in Sec. 4.2 we show qualitatively that creating static potential barriers on the scale of atomic deBroglie wavelengths, and therefore suitable for controlling tunneling, requires micron or sub-micron atom-surface distances. In Sec. 4.3 we present the physical properties of gold nanowires, including a theoretical analysis of their resistivity. In Sec. 4.4 we analyze the potentials expected from such nanowires, including the Casimir-Polder force and potential corrugation effects. We show that improved fabrication methods can overcome earlier limitations due to trapping potential roughness [46], which at sufficiently small separations would otherwise cause the trapped atomic cloud to break into smaller clouds (fragmentation). In Sec. 4.5, we estimate trap lifetimes limited by atom losses due to noise-induced spin flips, Majorana spin flips, and tunneling. In particular, we show how the spin-flip rate induced by Johnson noise is reduced naturally by using very small amounts of material in the nanowires. We also consider the issue of decoherence. In Sec. 4.6 we discuss a simple trap configuration based on a Z-shaped gold wire. We show that such a nanowire structure can generate static magnetic potentials, smooth enough for trapping a BEC at sub-micron atom-surface separations. Finally, in Sec. 4.7 we briefly discuss nanowire traps fabricated by more exotic materials.

Several novelties are presented in this chapter: First, it is the first time nanowires are suggested as feasible trapping structures. Second, we present here for the first time a detailed analysis of how to engineer static field tunneling barriers for atom interferometry. Third, we present for the first time a recipe of how to estimate the joint CP force originating from a wire on a substrate, and fourth, we show that counter to common practice, spin-flip lifetimes as long as seconds may be achieved very close to the atom chip surface.

4.2 Static magnetic potentials for atom interferometry

Two technical characteristics of potentials that are required in order to study atom interferometry can be described in the following way: first, the potential barrier between adjacent wells should be sufficiently low or narrow so that the tunneling rate is comparable to, or faster than, typical experimental or dephasing time scales; and second, that this tunneling rate can be controlled with experimentally accessible currents and fields.

Largely because of the weak $1/r$ dependence of the magnetic potential on the atom-wire distance, these tunneling conditions require very short distances. To quantify this, we construct a simple waveguide potential using a single atom chip
wire (in the x-direction) and an external bias field; current through a second atom chip wire (in the y-direction) is added to generate a simple potential barrier in a right-angle “X” wire configuration [89]. This configuration incidentally is exactly opposite to the “dimple” configuration recently used for compressing atom chip traps [90, 91].

The magnetic potential in the $x-$direction, generated by the crossing wire, is given by:

$$V(x) = \mu_A B_0 + \frac{\mu_A \mu_0 I}{2\pi} \frac{z}{z^2 + x^2},$$

where $\mu_A$ is the atomic magnetic dipole moment along the direction of the Ioffe field $B_0$, $I$ is the current in the crossing wire, $\mu_0$ is the permeability of free space, and $z$ is the distance of the atom from the atom chip surface. A one-dimensional single-particle tunneling probability through the barrier can then be calculated in the WKB approximation as

$$P = \exp \left( -\frac{2}{\hbar} \int_{-x_E}^{x_E} dx \sqrt{2m[V(x) - E]} \right),$$

where $E$ is the kinetic energy of the atom and $V(\pm x_E) = E$. Assuming a kinetic energy of $E = 1 \mu K$ for a $^{87}$Rb atom (corresponding to a free-particle deBroglie wavelength of $\approx 0.33 \mu m$) in the $|F = 2, m_F = 2 \rangle$ state, we may then easily calculate the change required in the current $I$ that causes a given proportional change in the tunneling probability, as a function of the atom-surface distance $z$. The results of this calculation are shown in Fig. 4.1 for changing the tunneling probability from 0.001 to some higher probability. The calculation suggests that control over the tunneling probability requires a distance $z$ on the order of $1 - 2 \mu m$ for experimentally reasonable values of current control. In the simple model of Eq. (4.1), this corresponds to a barrier half-width of $2 - 4 \mu m$, comparable to experiments that have observed interference between adjacent wells with the addition of non-static fields [36, 37]. Thus, the desired static magnetic potentials can be generated only if atoms can be brought down to micron or sub-micron distances above the wires on the atom chip surface, at which point the tunneling rate can be tuned over an experimentally useful dynamic range by adjusting the current in the crossing wire. One may then envisage interferometric devices such as the ones we have proposed in Refs. [92] and [93].

It is well known that, to avoid finite size effects which degrade the trap gradient, the wire size should be on the same scale as the atom-surface distance, i.e., for the above noted heights of $d \lesssim 1 - 2 \mu m$ (see Fig. 4.1) one requires a micron-scale wire. As will be shown in the following, it is advantageous to utilize even smaller wire dimensions, namely nanowires. This will enable improving operational parameters at the above heights, or decreasing the atom-surface distance even further without hindering effects.
CHAPTER 4. NANOWIRE TRAPS

Figure 4.1: Tunneling probability through a barrier at several heights \(d\) as a function of the change in the current \(\Delta I\) through the control wire, relative to the current \(I_{0,001}\) for a probability \(P(I) = 0.001\). A kinetic energy of 1 \(\mu\)K is assumed for a single atom of \(^{87}\)Rb. For this X-wire configuration, changing the current by a few percent causes a drastic change in the tunneling probability for \(d = 10 \mu m\). Good control over the tunneling probability requires the height to be about \(d \lesssim 1 - 2 \mu m\). The inset shows the potential barrier required to maintain a tunneling probability of 0.001 as a function of the atom-surface separation \(d\): for smaller \(d\), a higher barrier is required (as the barrier becomes thinner) so better tunneling control is attained. The motivation for small atom-surface distances is quantified further in Sec. 4.4.2.

4.3 Physical properties of thin wires

4.3.1 Wire fabrication and characterization

In order to study the possibility of trapping atoms using nanowires, we first discuss the fabrication feasibility. An example of one such (short) wire, 20 nm thick and 50 nm wide, is shown in Fig. 4.2(a). This wire was prepared by us in a relatively simple two-step process involving optical lithography (for external connection) followed by electron-beam lithography (for the nanowires). A Si substrate with a well-defined oxide layer of 100 nm thickness and a thin 5 nm-thick Ti adhesion layer is spin-coated with image reversal photoresist, which is then exposed to ultraviolet light through a mask. After developing, a 5 nm-thick Ti seed layer, followed by a 200 nm-thick Au layer, is then evaporated onto the sample and the undeveloped areas lifted off, leaving large areas for connection to external testing equipment. The sample is then spin-coated with a layer of PMMA, and the nanowire plus several of the interconnects are patterned by electron-beam lithog-
4.3. PHYSICAL PROPERTIES OF THIN WIRES

After developing, gold is evaporated onto the sample with the desired thickness and the fabrication is completed with a final lift-off process. This fabrication process can easily be integrated with any atom chip design.

Using SEM images of the fabricated wires, we measured the edge roughness of the resulting wires, as shown in Fig. 4.2(a). The spectrum of this edge roughness can be characterized as frequency-independent (“white noise”) with a measured root-mean-square deviation of $2 \text{ nm}$ for wavelengths of $100 - 800 \text{ nm}$.

Figure 4.2: (a) Scanning electron microscope (SEM) image of a $2 \mu\text{m}$ long, $20 \text{ nm}$ thick and $50 \text{ nm}$ wide gold wire. Unless otherwise noted, the wires considered in this study have square cross sections. (b) Calculated dependence of resistivity on wire dimensions, based on the Fuchs-Sondheimer surface scattering model [94, 63] of Eq. (4.4). (c) Maximum current considered safe for atom chip wire operation, calculated for different wire cross-sections from Eq. (4.6), assuming the nanowire resistivity $\rho$ shown in (b) and the temperature coefficient $\alpha$ for bulk gold.

4.3.2 Wire resistance calculations

The resistivity of a nanowire increases beyond the bulk resistivity as the cross-section dimensions become comparable to the mean free path $l$ of an electron ($l \approx 40 \text{ nm}$ for gold at room temperature [95]). In such a small wire the resistivity may increase significantly [73, 96]. To estimate the change of resistivity in a nanowire, we follow the theoretical model of Fuchs and Sondheimer [63, 94], which was extended by Chambers [64]. This model is supported experimentally for gold nanowires [96]. For wire dimensions on the order of the grain size, a supplementary model by Mayadas and Shatzkes [65] is needed in order to account for scattering at grain boundaries. For the simple fabrication process we have described, the measured grain size is about $20 \text{ nm}$, so for wire dimensions above this size we can attribute the increase in nanowire resistivity solely to scattering at the walls as in the Fuchs-Sondheimer model [97].

For atom chip experiments, we are interested in the current density in the wire...
and not only the wire resistance. Therefore we give the current density in a wire (along $\hat{x}$) of width $w$ (along $\hat{y}$) and thickness $h$ (along $\hat{z}$) as

$$J(y, z) = J_0 \left[1 - s(y; z) - s(w - y; z) - s(z; y) - s(h - z; y)\right],$$  \hspace{1cm} (4.3)

where $J_0 = I/wh$ is the current density expected in the absence of surface scattering, and

$$s(y; z) = \frac{3}{4\pi} \int_{-\arctan(z/y)}^{\arctan((h-z)/y)} d\phi \int_0^\pi d\theta \sin \theta \cos^2 \theta \exp \left(-\frac{y}{l \sin \theta \cos \phi}\right)$$  \hspace{1cm} (4.4)

corresponds to scattering at the $y = 0$ boundary, $s(w - y; z)$ corresponds to scattering at the $y = w$ boundary, and $s(z; y)$ and $s(h - z; y)$, corresponding to scattering at the $z = 0$ and $z = h$ boundaries respectively, are obtained by replacing $y$ by $z$ and $h$ by $w$. The resulting resistivity is given by $\rho/\rho_0 = J_0/\int\int dy \, dz \, J(y, z)$, where $\rho_0$ is the bulk resistivity. It follows that the current density at the metallic layer near the boundary drops to $\frac{1}{2}$ its value far from the boundary. To account for surface scattering, one assumes a fraction $p \>(0 \leq p \leq 1)$ of specular reflection events at the boundaries; then the value of the resistivity for a given bulk mean free path $l$, is given by a series expansion

$$\left(\frac{\rho_0}{\rho}\right)_{p,l} = (1 - p)^2 \sum_{n=1}^{\infty} np^{n-1} \left(\frac{\rho_0}{\rho}\right)_{p=0,l/n},$$  \hspace{1cm} (4.5)

where $(\rho_0/\rho)_{p=0,l/n}$ is the resistivity calculated for a wire with totally diffusive scattering at the boundaries ($p = 0$) and $l/n$ is the mean free path divided by the number of reflections. Measurements of resistivity of thin gold wires are well reproduced by a theory assuming $p = \frac{1}{2}$ [73, 96]. Figure 4.2(b) shows that the calculated resistivities for wires with square cross-sections increase by up to about 50% for cross-sections down to 25 $\mu$m.

### 4.3.3 Current limitations

Forming magnetic traps deep enough to hold ultracold atoms near the surface of an atom chip requires sufficiently large currents in the microfabricated wires to ensure that the trapping potential overcomes the gravitational force, the Casimir-Polder attraction to the chip surface, and the kinetic and repulsive energy of the atoms. However, if the current is too high, the wire overheats and may eventually break down [74]. The wire temperature is determined by the balance between ohmic heating (whose power dissipation per unit area is $h\rho J^2$) and the rate of heat conduction to the wafer per unit area $-\kappa \Delta T$, where $\kappa$ is the thermal contact resistance of the wire-wafer interface and $\Delta T = T - T_0$ is the difference between

\[^1\text{Note the changes we have made to Eq. (1) in [96] and to Eq. (2) in [73].}\]
the temperature $T$ of the wire and the temperature $T_0$ of the wafer (typically room temperature). The heat capacity of nanowires is so small that the wire reaches its maximum temperature very rapidly; approximating the temperature dependence of the resistivity as that of bulk gold, whose linear coefficient is $\alpha = 0.0037 \, K^{-1}$, we obtain the current density required to heat a given wire by a temperature $\Delta T$ as [74]

$$J_{\text{max}} = \sqrt{\frac{\kappa \Delta T_{\text{max}}}{\hbar \rho(T_0)[1 + \alpha \Delta T_{\text{max}}]}}, \quad (4.6)$$

thus showing that thin wires allow higher current densities. On the other hand, if the wire cross-section is on the order of the mean free path of the electrons, the rise in the resistivity due to surface scattering [Fig. 4.2(b)] may limit this advantage. In Fig. 4.2(c) we present the calculated maximum current density for different wire cross-sections using an estimated value for $\kappa = 4 \times 10^6 \, Wm^{-2}K^{-1}$ from Ref. [74], and assuming that we limit the rise in resistivity (due to heating) to 50%, which we consider to be within safe operating limits for thin atom chip wires [74]. This limitation in the resistivity change corresponds to heating by $\Delta T = 1/2\alpha = 135^\circ$.

When considering a specific example (Sec. 4.6), we will show that currents sufficient for generating atom chip traps may be an order of magnitude lower than the limits shown in Fig. 4.2(c).

4.4 Atomic trapping potential

In this section we describe two prominent effects influencing the static potential at sub-micron atom-surface distances generated by nano-scale wires, namely corrugations due to electron scattering, and tunneling to the surface or the nanowire, through the magnetic potential, due to the Casimir-Polder force.

4.4.1 Potential Corrugations

One of the limiting factors when trapping or guiding atoms in a magnetic potential generated from a current carrying wire is the static potential corrugation due to current deviations [1, 2]. Such current irregularities are produced by wire imperfections, namely, geometrical properties (wire edge roughness and surface roughness), and internal bulk inhomogeneities. Since atom chip traps are formed by canceling the magnetic field $B_y$ generated by the current density $J_{y0}$ at a specific distance from the wire $d$, the minimum of the trapping potential lies along the wire direction $\hat{x}$. Variations in this potential $\delta B_x(x)$ are then directly related to changes in the direction of the magnetic field generated by the wire imperfections.

In previous work [1, 2], we concluded that internal bulk inhomogeneities play a minor role in thin wires ($h < 250 \, nm$). For wide wires, surface roughness dominates the potential corrugation, but as we show below, edge roughness dominates for narrow wires. Consequently, in this work we need to consider only current
deviations due to edge roughness since all of the nanowires considered are thinner and narrower than $h \approx w < 250 \text{ nm}$.

Let us consider a fabricated metal wire carrying a total current $I$. It extends along the $\hat{x}$ direction and has a width $w$ along $\hat{y}$ and thickness $h$ along $\hat{z}$. The boundaries of the wire are located at $y = \pm w/2 + \delta y_\pm(x, z)$ and $z = \pm h/2 + \delta z_\pm(x, y)$. The corrugations of the wire boundaries $\delta y_\pm$ and $\delta z_\pm$ can be expanded as

$$
\delta y_\pm(x, z) = \sum_{n=-\infty}^{\infty} e^{2\pi i n z/h} \sum_{k} e^{ikx} \delta y_n^\pm(k) \tag{4.7}
$$

and

$$
\delta z_\pm(x, y) = \sum_{m=-\infty}^{\infty} e^{2\pi i m y/w} \sum_{k} e^{ikx} \delta z_m^\pm(k). \tag{4.8}
$$

A linear theory for small corrugations predicts that the effect of each spectral component of the corrugation is responsible for a corrugation of the magnetic field near the atomic trap center with a similar wavelength $2\pi/|k|$ along the $x$ direction. However, the effect of components with wavelength much shorter than the distance $d$ between the wire and the atomic trap (on the order of hundreds of nanometers or more) drops exponentially as $e^{-|k|d}$ so that here we will only be interested in corrugations whose wavelengths are a few hundred nanometers or longer. We may then neglect the effect of spectral components on the order of the wire width or thickness and consider only corrugation terms with $m = 0$ and $n = 0$, i.e., we may assume that $\delta y_\pm$ and $\delta z_\pm$ depend only on $x$.

Corrugations of the magnetic field along the main trapping axis $x$ above the center of a wire with geometrical perturbations are given by the Biot-Savart law as

$$
\delta B_x(r) = \frac{\mu_0}{4\pi} \int d^3 r' \left[ \delta J_y(r') \frac{\partial}{\partial z'} - \delta J_z(r') \frac{\partial}{\partial y'} \right] \frac{1}{|r - r'|}, \tag{4.9}
$$

where $\delta J_y$, $\delta J_z$ are the transverse current fluctuations in the wire. At the point exactly above the center of a nominally symmetric wire, it follows that only the symmetric components of $\delta J_y$ [$\delta J_y(y) = \delta J_y(-y)$] and the anti-symmetric components of $\delta J_z$ [$\delta J_z(y) = -\delta J_z(-y)$] contribute to the magnetic field. The fabrication process typically provides wires whose edge corrugations are much larger than their top or bottom surface corrugations, so that the symmetric part of $\delta J_y$ is the major contribution to the magnetic field fluctuations.

Ohmic theory, which is adequate when the width and thickness of the wire are much larger than the electron mean free path and whose use we justify below, predicts that for wavelengths longer than the wire width or thickness the symmetric $y$-current fluctuations in the wire have the form

$$
\delta J_y^{\text{sym}}(x, y) = i J_x^0 \sum_{k \neq 0} k e^{ikx} \frac{(\delta y_k^+ + \delta y_k^-) e^{-|k|w/2}}{1 + e^{-|k|w}} \cosh(ky), \tag{4.9}
$$
such that in the limit where \(|k|w \ll 1\), \(\delta J_y(x, y) \sim J_0 \partial \delta y_{\text{center}} / \partial x\), where \(\delta y_{\text{center}} = (\delta y_+ + \delta y_-)/2\) is the position of the actual center of the wire at a given point \(x\). Substituting this limit into Eq. (4.8) while assuming small deviations of the wire edges from their nominal position, and assuming that \(w \ll z\), i.e., the width of the wire is much smaller than the distance of the atom to the wire, we obtain the following expression for the magnetic field corrugations above the wire

\[
\delta B_x(x, 0, z) = \frac{iI\mu_0}{2\pi} \sum_k e^{ikx} k^2 \delta y_{\text{center}}(k) K_1(|k|z),
\]  

(4.10)
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use the ohmic theory whose general form was developed in Ref. [2] to calculate the magnetic field corrugations above the wire.

In Fig. 4.3 we present calculated directional variations of the magnetic field $|\delta B_x / B_0|$, generated by the trapping wire, as a function of the height $d$ for several wire cross-sections. The edge roughness amplitude is measured from our fabricated wires and was found to be frequency-independent [$\alpha = 0$ in Eq. (4.12)] with a measured root-mean-square deviation of $2 \text{ nm}$ between $100 - 800 \text{ nm}$. In accordance with Eq. (4.12), we see that, for a given edge roughness $\delta y_{\text{rms}}$, smaller wires produce only slightly larger magnetic corrugations. The effect of such magnetic corrugations on the atomic density will be discussed in Sec. 4.6. We also see that the influence of the surface roughness $\delta z_\pm(x, y)$ is negligible for the narrow wires discussed in this work due to the suppression of long wavelengths in the magnetic corrugations [2].

4.4.2 Engineering longitudinal potential variations by nanowire shaping

Shaping the nanowire edges may be used for creating potential variations desired for manipulating atoms near the atom chip surface. Having characterized the dependence of magnetic field variations on wire edge imperfections, we may now discuss quantitatively the deliberate “tailoring” of magnetic trapping potentials by engineering wire edge profiles. For the purposes of this study, as noted in the Introduction, we are particularly interested in potentials with sufficient variation so that tunneling barriers can be controlled. This is the main advantage of trapping atoms close to the trapping wire. Supplementing the motivation for a small atom-surface distance presented in Sec. 4.2, we now wish to determine the highest “potential resolution”, i.e., the smallest distinguishable distance between adjacent wells separated by static tunneling barriers, as a function of $d$.

As a test case for quantifying this potential resolution, we consider a configuration in which a thin wire is curved with a certain periodicity $\lambda$ that corresponds to a wave-vector $k = 2\pi / \lambda$. If the amplitude of this curvature is small with respect to the wavelength, then the foregoing discussion implies that the magnetic field above the wire is given by a single $|k|$ component in Eq. (4.10), and then $V(x) = V_0 \cos kx$, where $V_0 = \mu \mu_0 I k^2 \delta y_{\text{center}} K_1(kz)$.

At the minima of such a periodic potential, the longitudinal frequency is $\omega = \sqrt{V_0 k^2 / m}$, where $m$ is the atomic mass. In order to engineer potential barriers between adjacent minima higher by a factor of say, $\eta$ than the single-atom ground state energy, we require $2V_0 > \frac{1}{2} \eta \hbar \omega$, or $V_0 > (\eta^2 / 16) \hbar^2 k^2 / m$. In Fig. 4.4 we show the maximum atom-surface distance for which a longitudinal barrier with $\eta = 2$ can be obtained. These curves show that the maximum atom-surface

\[3\text{We note that in the case of edge roughness with } \frac{1}{7} \text{ power spectrum } (\alpha = 1), \text{ the directional variations of the magnetic field } \delta B_x / B_0 \text{ will be an order of magnitude higher } (8 \times 10^{-3} \text{ compared to } 7 \times 10^{-3} \text{ at } d = 0.6 \mu m), \text{ and will lead to significantly larger density perturbations.}\]
4.4. ATOMIC TRAPPING POTENTIAL

Figure 4.3: Directional variation of the magnetic fields $|\delta B_x/B_0|$, calculated from Eq. (4.8) as a function of the atom-surface distance $d$. We consider wires with square cross-sections of 50–200 nm and the narrow-wire approximation presented in Eq. (4.12). The same edge roughness is used for calculating the magnetic variations for all the wires. The small differences amongst the wires, despite relatively higher edge roughness of the narrower wires, corresponds to Eq. (4.12) in which only the absolute quantity $\delta y_c^{\text{rms}}$ appears. These differences are smallest for $d \gg w$ and become larger as $d$ approaches $w$. The inset shows the directional variation of the magnetic field at a fixed height of $d = 0.6 \mu m$ and for a fixed wire thickness of $h = 0.1 \mu m$, where we plot the influence of edge roughness (solid curve) and surface roughness (dashed curve) over a wide range of wire widths $w$. The effect of the surface roughness drops strongly for narrower wires, since long wavelengths of the magnetic corrugations are suppressed [2]. For the nanowires considered herein, magnetic variations are completely dominated by edge roughness.

distance still allowing tunneling control is on the order of the potential periodicity $\lambda$. Designing the edges of a wire as the sum of different modulations therefore allows engineering of any periodic potential up to a resolution determined by the atom-surface distance. Consequently, as also seen in Sec. 4.2, atom-surface distances of $1 - 2 \mu m$ (or sub-micron distances in some cases) will be required to fully exploit the potential of an atom chip based on static magnetic fields.

4.4.3 Attractive Casimir-Polder potential

The Casimir-Polder potential between a polarizable atom and dielectric or conducting objects [98] is one of the fundamental outcomes of zero-point vacuum fluctuations. It emerges from the fact that a dielectric or conducting object modifies the modes of the electromagnetic (EM) field in its vicinity, modes which interact with
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Figure 4.4: Potential spatial resolution achievable with wire currents from 0.05 – 50 mA. We present the maximum atom-surface distance \( d \) for which the longitudinal barrier between two adjacent minima in a periodic potential is at least twice the energy of the longitudinal ground state. Obtaining static magnetic potential features with a resolution on the order of the deBroglie wavelength, \( \lambda \), i.e., for a potential periodicity on the order of \( \lambda \approx 1 \mu \text{m} \), requires the atom-surface distance to be \( d \lesssim 2 \mu \text{m} \). Wire currents of 0.5 and 5 mA are the maximum currents that can be sustained through 20 and 100 nm atom chip wires, respectively [Fig. 4.2(c)]. The 0.05 mA curve is useful when discussing a specific example of an atom chip trap (Sec. 4.6). Increasing the current by three orders of magnitude to 50 mA serves to increase the required height by just a factor of two, despite being well beyond a safe atom chip current even for a 200 nm nanowire.

The atomic polarization. In our case, an attractive Casimir-Polder potential arises from the conducting gold nanowire and from the Si wafer coated with a 100 nm-thick SiO\(_2\) layer (used to prevent electrical shorts). The Casimir-Polder potential reduces the potential barrier for tunneling to the surface, thereby limiting the possibility of trapping atoms near the surface \(^3\).

The EM modes of the combined surface+wire system are not analytically solvable and we will therefore carry out a separate examination of the Casimir-Polder potential emerging from the Si+SiO\(_2\) planar wafer, as discussed in earlier work [52], and from a simplified model that takes the wire as a perfectly conducting circular cylinder of a certain diameter. We then take the sum of the two contributions as an estimate for the combined potential as a sort of pairwise additive approximation, PAA. Based on our earlier experience from the planar two-layer system, we antic-

\(^3\)Note that numerous ideas on how to alter the Casimir-Polder force exist [56, 99, 100, 101], and may, if proven successful, enable decreasing the atom-surface distance even further.
ipate that this approach should at least give the right order of magnitude for the accurate Casimir-Polder potential.

In general, the Casimir-Polder potential may be written in the form

$$ U_{CP}(r) = i\hbar \int_{-\infty}^{\infty} d\omega \alpha(\omega) \left[ \Gamma(r, r, \omega) - \Gamma_0(r, r, \omega) \right], $$

(4.14)

where $\alpha(\omega)$ is the frequency-dependent atomic polarizability and $\Gamma(r, r, \omega)$ is the trace over the Green’s tensor of the electromagnetic field at the same point $r$, with $\Gamma_0$ being the Green’s tensor in empty space, responsible for a space-independent Lamb shift. For distances from the dielectric or conducting object much larger than $\lambda_0/2\pi$, where $\lambda_0$ is the wavelength corresponding to the lowest optical transition frequency, the Casimir-Polder potential generated by a planar structure made from a layer of thickness $t$ with a dielectric constant $\epsilon_1$ atop an infinitely thick dielectric layer of dielectric constant $\epsilon_2$ has the form (see Appendix and Ref. [52])

$$ U_{CP}(z) = \frac{-\hbar c\alpha_0}{2\pi} \frac{1}{z^4} F(\epsilon_1, \epsilon_2, t/z), $$

(4.15)

where $\alpha_0$ is the static atomic polarizability. The dimensionless function $F$ takes the single-layer limiting value $F \sim \frac{3}{4} \frac{\epsilon_1-1}{\epsilon_1+1} \phi(\epsilon)$ with $\epsilon = \epsilon_1$ when $z \ll t$, and with $\epsilon = \epsilon_2$ when $z \gg t$, where $\phi(\epsilon)$ is on the order of unity [102]. $F = \frac{3}{4}$ is obtained in the vicinity of a perfectly conducting thick layer. In our case $\alpha_0 = 47.3 \times 10^{-24} \text{cm}^3$ is the ground state static polarizability of the $^{87}\text{Rb}$ atom [103], $\epsilon_1 = 4$ for the SiO$_2$ layer, and $\epsilon_2 = 12$ for the Si wafer.

As stated above, we wish to compare contributions to the Casimir-Polder potential from the three different components comprising the surface: the Si chip, the SiO$_2$ layer of thickness $t$, and the gold nanowire of thickness $h$. For this comparison to be meaningful, we require a common reference for the distance variable $z$, which we define as the distance from the top of the SiO$_2$ surface. Then the distance from the Si chip is $z + 100 \text{ nm}$ and the distance from the top of the gold nanowire is $z - h$. To factor out the strong $z^{-4}$ dependence, we plot the quantity $F(z) = -U_{CP}(z) \frac{2\pi}{\hbar c} z^4$ in Fig. 4.5(a) for the Si+SiO$_2$ bilayer. This is compared to a sum of two models (shown separately in the figure): one where the half space for $z < -100 \text{ nm}$ is full of Si while the other half is empty; and another in which only a 100 nm-thick SiO$_2$ layer exists, with empty space for $z < -100 \text{ nm}$. The figure shows that simply summing the two potentials over-estimates the exact result by 8-15% over the relevant range, but it gives the right order of magnitude.

Next we consider the Casimir-Polder potential for an atom at a distance $R$ from the center of a cylindrical conducting wire of radius $a$ where we set $a = h/2$. It appears that the main contribution to the integral in Eq. (4.14) comes from frequencies on the order of $\omega \sim c/R$. In our case, where $R < 1 \mu\text{m}$, the skin depth for a gold wire with resistivity $\rho = 2.2 \times 10^{-8} \Omega \cdot \text{m}$ is $\delta = \sqrt{2\rho/\mu_0 \omega} \lesssim 10 \text{ nm}$, which is much smaller than the width or thickness of the nanowires considered. We can therefore use a model where the wire is perfectly conducting (impenetrable
Figure 4.5: (a) $F$ factor for the bilayer system of thick Si coated by a 100 nm layer of SiO$_2$, similar to the system studied in Ref. [52]. The exact calculation (solid curve) is compared to the sum (dashed) of two separate systems – the SiO$_2$ layer alone (dashed-dotted) and the Si layer alone (dotted). For the contribution of the Si layer the factor $F$ would be constant for a system of coordinates starting at its top ($z = -100$ nm), but here it is rescaled to the coordinate system where $z = 0$ is at the top of the SiO$_2$ layer (see text). The sum of the two separate contributions over-estimates the exact result by about 8-15% over the relevant range. (b) $F$ factor (again rescaled to $z = 0$ at the top of the SiO$_2$ layer) for the planar wafer (solid curve) reproduced from (a) and for perfectly conducting cylindrical wires of diameters $2a = 50 - 200$ nm (broken curves) lying on the wafer surface. Two important reasons for the differences between the wires are the different atom-wire distances $z - 2a$, which is smaller for thicker wires, and the larger solid angle subtended by the wider wire.

For $a/R > 0.2$ the function $F$ is nearly linear, $F(a/R) \approx 0.53(a/R) + 0.22$, tending to $F = \frac{3}{4}$ as $R \to a$, where the surface of the cylinder is similar to a planar conducting surface. In the opposite limit $a/R \ll 0.1$ the function $F$ drops to zero as $F(a/R) \sim \frac{2}{3 \log(a/R)}$ (see Appendix).

Figure 4.5(b) again shows the factor $F$ for the Casimir-Polder potential from the planar (i.e., Si+SiO$_2$) surface in comparison to $F$ for cylindrical wires of different diameters $2a$. It is evident that the contribution of the wire is dominant when the distance from the wire is less than 5-7 times the diameter of the wire. For larger distances the contribution of the wire falls to half or less than the contribution of the surface. Given our experience with the bilayer system [52], we expect the exact calculation of the wire+surface to deviate by the same order as we observe for
4.5 Atom loss

In this section we analyze the lifetime for atoms in the nanowire trap. This lifetime includes the spin-flip rate due to thermally induced noise, the Majorana spin-flip rate, and the tunneling rate to the surface. Finally, we estimate the decoherence rate due to the thermally induced noise in the room temperature surface.

4.5.1 Spin flip due to thermal noise

The magnetic thermal noise (Johnson noise) arising from conducting materials on the atom chip is coupled to the trapped atoms via their magnetic moment $\mu_A$. As a consequence, spin flips, heating and decoherence become dominant close to a conductor even without applied currents. Here we calculate the trap loss rate due to spin flips. We assume that the spectrum of magnetic noise from the conductor is roughly constant for frequencies in the MHz region, the latter being able to drive magnetic transitions between Zeeman sub-levels in the same hyperfine level. In this case the magnetic moment is $\mu_A = \mu_B g_F \hat{F}$, where $\mu_B$ is the Bohr magneton, $g_F$ is the Landé factor for the hyperfine level and $\hat{F}$ is the hyperfine spin operator. Using the theory developed by Henkel, the thermal spin-flip rate from an initial trapped Zeeman state $|i\rangle$ to a final untrapped state $|f\rangle$ can then be written as [3, 49, 104]:

$$\Gamma_{\text{th}}(x) = \frac{\mu_B^2 g_F^2}{\hbar^2} \sum_{j,k=\perp} \langle i|F_j|f\rangle \langle f|F_k|i\rangle S_{jk}^B(x,x,\omega_{if}),$$

where we sum the contribution of all components of the noise perpendicular to the atomic magnetic moment. Here the function $S_{jk}^B$ is the correlation function of the magnetic field noise, which is given by

$$S_{jk}^B(x_1,x_2,\omega \rightarrow 0) = \frac{k_B T}{4\pi^2 \rho \varepsilon_0 c^4} \left[ \text{Tr} \left\{ X_{jk}(x_1,x_2) \right\} \delta_{jk} - X_{jk}(x_1,x_2) \right],$$

with $X_{jk}$ being a geometrical factor which also averages over $1/\rho(x)$ if the resistivity changes in space:

$$X_{jk}(x_1,x_2) = \frac{\rho}{2} \int_V \frac{d^3x'}{\rho(x')} \frac{(x_1 - x')_j (x_2 - x')_k}{|x_1 - x'|^3 |x_2 - x'|^3}.$$
the skin depth \( \delta = \sqrt{2 \rho/\mu_0 \omega_L} \) (\( \omega_L \) is the Larmor frequency). This condition is easily met here since the skin depths of metals in the MHz region are typically tens of \( \mu \text{m} \) (e.g., gold has a skin depth of \( \delta \approx 70 \mu \text{m} \)).

In Fig. 4.6(a) we present estimated lifetimes for trapped atoms due to thermal noise-induced spin flips. The wire size greatly affects the lifetime, mostly because smaller wires place much less conducting material near the atoms, and also because of their higher resistivity. For a \( 50 \times 50 \text{ nm} \) wire, we estimate that the lifetime of a cloud trapped 500 nm from the wire surface is \( \approx 5 \text{ s} \), so we do not expect thermal noise-induced spin flips to be a dominant loss mechanism in typical experiments.

### 4.5.2 Majorana spin flips

Cold atoms in a low-field seeking state that are trapped near a vanishing magnetic field can undergo a spin-flip transition to a high-field seeking state that is untrapped (Majorana spin flips). Applying a small offset (Ioffe-Pritchard) field \( \mathbf{B}_0 \) will generate a non-vanishing field at the trap center, hence reducing the spin-flip transition rate as given by the approximate formula [106]:

\[
\Gamma_M = \frac{\pi \omega_r}{2} \exp \left(-\frac{2|\mu_A|B_0 + \hbar \omega_r}{2\hbar \omega_r}\right),
\]

where \( \omega_r \) is the trap radial frequency. Equation (4.20) is valid when the Larmor frequency \( \omega_L = |\mu_A|B_0/\hbar \gg \omega_r \), requiring that \( B_0 \gg 50 \text{ mG} \) for typical radial frequencies. In the following sections, we choose an Ioffe-Pritchard field \( B_0 \) that simultaneously satisfies this condition and yields a Majorana spin-flip lifetime of 2 s.

### 4.5.3 Tunneling to the chip surface

As a result of the Casimir-Polder potential, the magnetic barrier between the surface and the atoms is lowered, and atoms can tunnel through the barrier to the atom chip surface or wire. Calculated tunneling lifetimes are presented in Fig. 4.6(b), where we use a weighted average of the tunneling rate over all points in the \((x,y)\) plane. For each point \((x,y)\) we use the WKB approximation for tunneling through a one-dimensional potential barrier along the \( z \) direction [52]:

\[
\Gamma_{\text{tunn}} = \int \int dx \, dy \, P(x,y) \left[ \frac{\omega_r(x,y)}{2\pi} \right] \exp \left(-2 \int_{z_1}^{z_2} dz \sqrt{\frac{2m}{\hbar^2} [U(x,y,z) - \mu]}\right),
\]

where \( \mu \) is the chemical potential and the integration over \( z \) is between the classical turning points \( z_1(x,y) \) and \( z_2(x,y) \) defined by \( U(x,y,z_1) = U(x,y,z_2) = \mu \).

The weighted tunneling probability appearing in the integrand is given at any point by \( P(x,y) = \frac{1}{N} \int dz \, n(x,y,z) \), where \( n(x,y,z) \) is the particle density and the transverse frequency \( \omega_r(x,y) = \hbar \sqrt{\langle k_z^2 \rangle}/2mL(x,y) \) is the inverse of the average round-trip time for a particle moving between the turning points \([L(x,y) = \)
4.5. ATOM LOSS

$z_1(x, y) - z_2(x, y)$]. These quantities are all calculated by solving the Gross-Pitaevskii equation for 1000 atoms of $^{87}$Rb. In a typical trap generated by a Z-shaped wire (e.g., see Sec. 4.6), most of the tunneling occurs either at the center of the trap (where the atoms are closest to the wire) or at the trap ends (where the potential curves down towards the surface). Because of the much higher atom density directly above the wire, the lifetime is governed mostly by tunneling to the wire rather than to the surface, as discussed further in Sec. 4.6.

Figure 4.6: (a) Trap lifetimes due to thermal noise-induced spin flips, calculated for atoms trapped at distances $d$ above wires with square cross sections of $25 - 200$ nm. Reducing the wire size increases the lifetime; for the $50 \times 50$ nm wire the lifetime exceeds the selected Majorana lifetime of 2 s for distances $d > 0.37 \mu$m. For comparison, the lifetime $1 \mu$m above a very wide wire would be $< 10$ ms. (b) Tunneling lifetimes calculated for a BEC of 1000 atoms in traps generated at different distances $d$, compared to the Majorana spin-flip lifetime (kept constant at 2 s), assuming a current of $40 \mu$A passing through a $50 \times 50$ nm trapping wire. This current is more than an order of magnitude below the maximum for such a nanowire [Fig. 4.2(c)]. The solid and dashed curves are calculated assuming surface-only and wire-only contributions to the Casimir-Polder force respectively. Even though these Casimir-Polder forces are of the same order of magnitude [Fig. 4.5(b)], the atomic density is much higher directly above the wire, so tunneling to the wire is much faster than tunneling to the Si+SiO$_2$ surface; the latter tunneling proceeds mostly from the cloud edges, where the atomic density is much lower. The dotted curve is calculated for a potential combining the wire and surface Casimir-Polder forces; the corresponding tunneling lifetime is shorter yet because the trap barrier is reduced along the entire wire and at the cloud edges. In this approximate calculation, the tunneling lifetime exceeds the Majorana lifetime for distances $d > 0.55 \mu$m. Using higher currents for such wires would increase the tunneling lifetime and is discussed further in Sec. 4.6.
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4.5.4 Spatial decoherence

As we have noted above, fluctuations of the magnetic field perpendicular to the quantization axis of the atom may cause transitions of the atom between Zeeman states defined along this axis. Conversely, fluctuations of the magnetic field along the quantization axis cause spatially dependent energy fluctuations, which may be viewed as potential fluctuations for the atom. These potential fluctuations imply that the phase of the atomic wavefunction at two locations \( x_1 \) and \( x_2 \) will also fluctuate, giving rise to dephasing. We find that the mean square of the phase difference after a time \( t \) is given by

\[
\langle [\phi(x_1) - \phi(x_2)]^2 \rangle = \frac{m_F^2 g_F^2 \mu_B^2}{\hbar^2} \left\langle \left[ \int_0^t dt' B\| (x_1, t') - \int_0^t dt'' B\| (x_2, t'') \right]^2 \right\rangle ,
\]

where \( B\| \) is the magnetic field component along the quantization axis. For a time scale much longer than the inverse of the magnetic noise bandwidth, we may take the random-walk limit

\[
\int_0^t dt' \int_0^t dt'' \langle B\| (t') B\| (t'') \rangle = \frac{1}{2} S_B^\parallel (\omega \to 0)t,
\]

where \( S_B^\parallel \) is the magnetic field correlation tensor of Eq. (4.18) along the coordinate of the quantization axis. It then follows that the square of the phase difference grows linearly with time. This implies, in accordance with the theory developed by Henkel \[107\], that the coherence, which may be defined as

\[
g^{(1)}(x_1, x_2) = \langle e^{i[\phi(x_1) - \phi(x_2)]} \rangle ,
\]

drops exponentially with the standard deviation of the difference, i.e.,

\[
g^{(1)}(x_1, x_2) = \exp(-\Gamma_{\text{dec}} t),
\]

with

\[
\Gamma_{\text{dec}} = \frac{m_F^2 g_F^2 \mu_B^2}{2 \hbar^2} \left( S_B^\parallel(x_1, x_1) + S_B^\parallel(x_2, x_2) - 2 S_B^\parallel(x_1, x_2) - S_B^\parallel(x_1, x_1) + S_B^\parallel(x_2, x_2) \right) .
\]

Figure 4.7 shows the decoherence rate of a split atomic wavefunction at two points located at an equal distance \( d \) above an infinitely long and thin wire, as a function of the longitudinal separation between the points. Similar results are obtained when the two points are located above two separate parallel wires creating a local potential minimum above each of them, as a function of the separation between the two wires (and consequently between the two points). When the distance between \( x_1 \) and \( x_2 \) is much larger than the distance to the wire, the correlation term \( S_B(x_1, x_2) \) becomes negligibly small and the decoherence rate depends only on the distance of the two points from the wire. For \(^{87}\)Rb atoms in the state \( |F, m_F\rangle = |2, 2\rangle \) it follows that for \( x_1, x_2 \) equidistant from the wire, the decoherence rate is \( \Gamma_{\text{dec}}(|x_1 - x_2| \to \infty) = 2.4 \Gamma_{\text{th}} \), where \( \Gamma_{\text{th}} \) is given in Eq. (4.17).

We see that, if the nanowire trap is constructed with a coherence lifetime long enough, e.g., for interferometer experiments, then the experiment will not be limited by thermal spin-flip losses. Moreover, we see that coherence lifetimes on the order of 1 s may be expected for the nanowire traps discussed here.
4.6 SPECIFIC NANOWIRE ATOM CHIP TRAP

Figure 4.7: The rate of spatial decoherence between two points $x_1$ and $x_2$ above the wire as a function of their longitudinal separation, relative to their distance $d$ to the wire. The decoherence rate is given relative to its maximum value when the two points are very far apart relative to $d$. The model assumes that the wire is infinitely long and much narrower than the distance $d$. When the two points are separated by more than about 4 times their distance from the wire, the magnetic noise at the two points becomes uncorrelated, resulting in the decoherence rate asymptotically reaching a maximum as shown. Under these circumstances, the rate depends on the wire width in the same way as the thermal spin-flip lifetime shown in Fig. 4.6(a), but is 2.4 times shorter (see text for details).

4.6 Specific nanowire atom chip trap

We now apply the foregoing general properties of nanowires and their associated magnetic fields and noise to a specific example. We simulate a typical atom chip Z-shaped trap [17], aiming to achieve the smallest atom-surface separation while maintaining a lifetime $> 1$ s. Compatible with the fabrication process presented previously, we choose a 50 $\mu$m-long gold nanowire with a $50 \times 50$ nm cross-section. This choice minimizes the Casimir-Polder force due to the wire, thus lengthening the tunneling lifetime [Fig. 4.5(b)], and at the same time reduces thermal magnetic noise contributions to atom loss [Fig. 4.6(a)]. The ends of the nanowire are connected to conventional gold leads and are included in the magnetic field simulation. We consider a current of 40 $\mu$A, which is well below the calculated maximum current of 1.2 mA [Fig. 4.2(c)]. By applying a bias field of 132 mG in the $\hat{y}$ direction, a trap is generated at a distance of 0.6 $\mu$m from the wire. Trap lifetimes for closer atom-surface distances would be limited by much faster tunneling. Applying a second bias field of 83 mG in the $\hat{x}$ direction ensures a Majorana spin-flip lifetime $> 2.0$ s. These parameters specify the basic atom chip
CHAPTER 4. NANOWIRE TRAPS

The trap configuration, whose properties we now discuss.

The trap depth, defined as the highest-energy isopotential that does not touch the surface, is about 2.9 \( \mu \)K and is limited by the Casimir-Polder potential. An isopotential for a slightly higher energy is shown in Fig. 4.8(a). The radial frequency at the trap minimum is about 10 kHz, which is controllable over a wide range since we are passing such a modest current through the nanowire. The connecting legs of the Z-wire act as “end caps” for a waveguide potential that lies almost directly above the nanowire. Corresponding weighted tunneling probabilities are shown in Fig. 4.8(b) and (c). As was shown in Fig. 4.6(b), the tunneling lifetime due to the combined wire and surface Casimir-Polder potential is an order of magnitude shorter than that due to the wire Casimir-Polder potential.

![Image of an isopotential surface and tunneling probabilities](image)

Figure 4.8: (a) Lower half of an isopotential surface at 2.9 \( \mu \)K for an atom chip trap centered at \( d = 0.6 \) \( \mu \)m, created by passing a current of 40 \( \mu \)A through a 50 \( \times \) 50 nm gold nanowire. The isopotential has been compressed in the \( x \)-direction by a factor of 20 for better visibility, and potential corrugations have been ignored. The nanowire is 50 \( \mu \)m long. The potential sheet below the closed trap surface is due to the Casimir-Polder potential from the atom chip surface and the nanowire. The far edges of the isopotential surface touch the Casimir-Polder potential sheet, implying that 2.9 \( \mu \)m is the trap depth. (b-c) Weighted tunneling probabilities [integrand of Eq. (4.21)] calculated for a Bose-Einstein condensate of 1000 \(^{87}\)Rb atoms. The tunneling probabilities in (b) consider only the Casimir-Polder contribution from the Si\( +\)Si\(_2\)O\(_2\) planar surface; in (c) the calculation considers only the Casimir-Polder contribution from the nanowire. Peak probabilities occur in (b) at the ends of the trap because the potential bends towards the surface, even though the atomic density is low there. The peak tunneling probabilities in (c) occur all along the wire axis, even though the potential barrier is higher there, since the atomic density is highest there also; the probabilities in (c) are about ten times higher than in (b). For the potential combining all Casimir-Polder contributions, the tunneling lifetime is about 50 s.

The trap formed for such small distances from the nanowire is “box” shaped
4.7. **ANISOTROPIC CONDUCTORS**

along the longitudinal axis, as shown in Fig. 4.9(a). The smoothness of the trap bottom along the wire axis is limited only by the edge roughness of the nanowire (white-noise spectrum with 2 nm rms, see Sec. 4.3.1) and has a standard deviation of about 8.2 nK. The effect of this corrugation on the ground state of trapped atoms is examined by solving the Gross-Pitaevskii equation for $N$ interacting bosons confined by the magnetic potential of the atom chip nanowire [13]:

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) + g|\psi(r)|^2 \right] \psi(r) = \mu \psi(r),$$

(4.24)

where $m$ is the atomic mass, $V(r)$ is the external potential, $\mu$ the chemical potential, and $g = 4\pi\hbar^2a/m$ is the atom-atom coupling constant, with $a$ being the $s$-wave scattering length ($a = 5.052$ nm for $^{87}$Rb). We do not use the Thomas-Fermi approximation since we do not expect a “large” number of atoms to be held in the trap. The calculated chemical potential for $N = 1000$ atoms of $^{87}$Rb is $13$ kHz $\cdot 2\pi\hbar = 625$ nK, which is about $\frac{1}{4}$ of the trap depth [Fig. 4.8(a)]. In Fig. 4.9(b) we present the calculated in-situ atomic density, which shows a standard deviation of 3.8% due to the nanowire edge corrugation effects. The isopotential plotted in Fig. 4.9(c) for an energy just above the minimum presents another view of the potential corrugation.

We see that a sufficiently deep trap can be formed using static magnetic potentials generated by nanowires, with sub-micron atom-surface separations. The main limitations for such traps will be cloud fragmentation and tunneling due to the Casimir-Polder potential. The latter limitation can be overcome using higher currents. For a $50 \times 50$ nm wire, we can use a current up to $1.2$ mA [Fig. (4.2)]; at a height $d = 0.6$ $\mu$m and a current $I = 0.8$ mA, the trap depth increases to $>100$ $\mu$K and the tunneling lifetime increases by many orders of magnitude. However, in this configuration the potential corrugation causes severe atomic density fragmentation, with a calculated standard deviation of 40%. In order to reduce the effects of potential corrugation, we can increase the chemical potential by shortening the nanowire trap. Shorter nanowires can actually be fabricated more easily and such traps would not be expected to significantly reduce the tunneling lifetime.

### 4.7 Anisotropic conductors

Replacing pure metals with other conductors such as superconductors, alloys or molecular conductors (see Introduction) may bring numerous advantages. As an example, we briefly describe utilizing electrically anisotropic materials and their qualities in the context of this work [3]. In particular, if we orient the “good” conductivity $a$-axis parallel to the wire direction $\hat{x}$, the spatial and internal state decoherence rate of a trapped atomic sample is lowered by a factor on the order of the transverse conductivity suppression relative to gold, which may be several orders of magnitude. The greatest advantage of using such materials would therefore
Figure 4.9: (a) Minimum-energy path for a trap generated by running 40 μA through a Z-shaped nanowire, where the central portion of the Z-wire is 50 μm long: for each value of the longitudinal distance x, we show the minimum potential in the yz-plane. (b) Ground-state atomic density map (integrated along a viewing axis ⃗y perpendicular to the nanowire and shown in units of μm⁻²) for atoms at a trap height of d = 0.6 μm, calculated with the Gross-Pitaevskii equation for 1000 atoms of ⁸⁷Rb. The calculated potential corrugation perturbs the atomic density, which has a standard deviation of 3.8% along its length. (c) An isopotential surface of the trapping potential for an energy 0.1 μK above the trap minimum. The longitudinal axis is compressed by a factor of 50 for a better view. The potential corrugation is due to the trapping wire edge roughness, with a measured standard deviation of 2 nm. The potential sheet below the closed trap surface is due to the Casimir-Polder potential from the atom chip surface and the nanowire. The inset shows a 1D cut of the potential above the trap center, with the energy of the isopotential surface shown by the dashed red line.

be for interferometric measurements with atom chips. The anisotropy in the resistance is also expected to decrease the effect of wire edge roughness on potential corrugations.

For a direct comparison with the gold Z-wire trap described above, we wish to maintain a current of 40 μA through the trap. The α-axis resistivity for these materials is typically larger than the resistivity of gold. For a material such as SrNbO₃, the temperature of the wire at a current density of 10⁵ A/cm² is not expected to rise significantly [3], and hence the spin-flip lifetime due to thermal noise of the wire is expected to be about 200 times longer [Eq. (4.18)] than for the comparable gold nanowire. We note that this improvement is not due to the anisotropic nature of the wire but simply to its high resistivity (about 200 times that of gold). An additional factor of 2 may be gained for “one dimensional” anisotropic conductors due to the specific anisotropic nature of these wires [3]. Reducing the current density to 10⁵ A/cm² (2 orders of magnitude smaller than that allowed in gold) requires a correspondingly increased wire cross-section of 200 × 200 nm to maintain the same
current. Following Fig. 4.6(a) this would reduce the above lifetime by a factor of about 20, ultimately yielding an increased spin-flip lifetime of a factor of $\sim 20$.

Metallic nanowires have limited maximum current densities due to the increase in their wire resistivity (Fig. 4.2) from diffusive scattering at the wire surfaces. However, surface scattering for an anisotropic wire (where the surfaces are parallel to the good conductivity axis) may be significantly smaller and will have less effect on the wire resistivity, hence enabling higher current densities than discussed above. One may even speculate that at small dimensions the resistivity relevant for the current density [Eq. (4.4)] and that relevant for the Johnson noise [Eq. (4.18)] may become decoupled.

It is evident that the behavior of electrons in anisotropic materials in the context of surface scattering (resistivity) and edge currents (fragmentation) requires further theoretical and experimental study. In any case, it appears that utilizing anisotropic materials could further improve the advantages of nanowires. Specifically, it may improve the coherence time by several orders of magnitude, and could reduce potential corrugations [3]. Fabrication protocols for such anisotropic nanowires are being pursued in our laboratory.

### 4.8 Summary and Conclusions

We have presented an analysis for further miniaturization achievable in atom chips based on current carrying wires, aiming to create static magnetic potentials capable of manipulating atoms on the scale of their deBroglie wavelength. We have analyzed the physical limitations of conducting wires, and we have also analyzed limiting effects due to the nearby surface, explicitly considering tunneling to the surface, surface thermal noise (causing both spin flips and decoherence), electron scattering within nanowires causing static potential corrugations, and the Casimir-Polder force. Additional effects such as Majorana spin flips have also been taken into account.

We have analyzed a specific example of a nanowire trap, utilizing a standard configuration. We have shown that when utilizing nanowires, the main limitations to trapping atoms at sub-micron atom-surface distances are potential corrugation and tunneling to the surface. We briefly described an anisotropic conductor as a potentially useful alternative to standard gold wires. These examples serve not only to summarize the more general statements of the chapter, but also as an outlook for further work which may include alternative geometries and materials.

We have shown that further miniaturization of atom chips, utilizing wires with cross sections as small as a few tens of nanometers, enables robust operating conditions for atom optics. Such miniaturization may allow the realization of potentials (e.g., tunneling barriers) with a scale of the deBroglie wavelength, thereby bringing the atom chip a step closer to fulfilling its promise of a compact device for complex and accurate quantum optics with ultracold atoms. Achieving such small atom-surface distances should also contribute to the study of fundamental surface
phenomena.

4.9 Appendix: Casimir-Polder potential – derivation

4.9.1 A planar bilayer surface

We consider a planar structure with a dielectric function given by

\[ \epsilon(z) = \begin{cases} 
1, & z > 0, \\
\epsilon_1, & -b < z < 0, \\
\epsilon_2, & z < -b. 
\end{cases} \]  

(4.25)

The Green’s tensor for the EM field may be derived from the reduced Green’s tensor, which can be written as a sum over transverse modes with well defined transverse wave vectors \( k = (k_x, k_y) \)

\[ \Gamma(r, r', t - t') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \int \frac{d^2k}{(2\pi)^2} e^{i(k_x(x-x')+k_y(y-y'))} g_k(z, z'), \]  

(4.26)

where \( g_k(z, z') \) in the region \( z, z' > 0 \) can be written in terms of transverse electric (TE) and transverse magnetic (TM) scalar Green’s functions

\[ g_{k}^{\text{TE,TM}}(z, z') = i \frac{1}{2k_z} \left[ e^{ik_z|z-z'|} + R_{k}^{\text{TE,TM}} e^{ik_z|z+z'|} \right], \]  

(4.27)

with \( k_z^2 = \omega^2/c^2 - k^2 \). The reflection coefficients \( R_{\text{TE,TM}} \) are given below. The first term in Eq. (4.27) is irrelevant, being cancelled by the vacuum subtraction in Eq. (4.14). The trace over the remaining part of the Green’s tensor is now given in terms of the (vacuum subtracted) scalar functions \( g_{\text{TM}} \) and \( g_{\text{TE}} \) at \( z = z' \) as

\[ \sum_i g_{ii} = \omega^2 g_{\text{TE}} + (k^2 - k_z^2) g_{\text{TM}}. \]  

(4.28)

We now make the transformation \( \omega/c \rightarrow i\zeta \) such that \( k_z \rightarrow i\kappa \) with \( \kappa^2 = k^2 + \zeta^2 \). The reflection coefficients \( R_{\text{TE}} \) and \( R_{\text{TM}} \) are now given by

\[ R_{m} = \frac{r_{1m}^{m} + r_{12}^{m} e^{-2\kappa_1 b}}{1 + r_{1m}^{m} r_{12}^{m} e^{-2\kappa_1 b}}, \]  

(4.29)

for \( m = \text{TE, TM} \), where

\[ r_{12}^{\text{TE}} = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2}, \quad r_{1}^{\text{TE}} = \frac{\kappa - \kappa_1}{\kappa + \kappa_1}, \]  

(4.30)

and

\[ r_{12}^{\text{TM}} = \frac{\kappa_1/\epsilon_1 - \kappa_2/\epsilon_2}{\kappa_1/\epsilon_1 + \kappa_2/\epsilon_2}, \quad r_{1}^{\text{TM}} = \frac{\kappa - \kappa_1/\epsilon_1}{\kappa + \kappa_1/\epsilon_1}, \]  

(4.31)
with \( \kappa_i = \sqrt{\epsilon_i \xi^2 + k^2} \). We now obtain for the CP potential

\[
U_{CP}(z) = -\hbar c \frac{1}{2} \int_{-\infty}^{\infty} d\zeta \, \alpha(i\zeta) \int \frac{d^2k}{(2\pi)^2} \frac{1}{2\kappa} \left[ (2k^2 + \zeta^2)R^{TM} - \zeta^2 R^{TE} \right] e^{-2\kappa z},
\]

(4.32)

The formulas for the multilayer Green's functions have appeared in many places, for example in Ref. [108], and the result (4.32) was first derived by Zhou and Spruch [109].

At a distance \( z \) which is much larger than \( c/\omega_0 \), where \( \omega_0 \) is the lowest optical transition frequency of the atom, we may assume that \( \alpha(\omega) \sim \alpha(0) \). We now make the transformation \( \kappa = p/z \) and \( \zeta = \kappa \mu \) and obtain

\[
U_{CP}(z) = -\frac{\hbar c \alpha(0)}{2\pi z^4} F(\epsilon_1, \epsilon_2, b/z),
\]

(4.33)

with

\[
F = \int_0^\infty dp \, p^3 e^{-2p} \int_{-1}^1 d\mu \left[ 1 - \frac{\mu^2}{2} \right] R^{TM} - \frac{\mu^2}{2} R^{TE}.
\]

(4.34)

where \( R^{TM} \) and \( R^{TE} \) are given in Eqs. (4.29)–(4.31) with \( \kappa_i \) replaced by \( p_i = p\sqrt{1 + \mu^2(\epsilon_i - 1)} \) and \( b \) in the exponent replaced by \( b/z \).

When \( b \ll z \) the exponent \( \exp[-2p_1b/z] \to 1 \) and one obtains \( R^{TE} = (p - p_2)/(p + p_2) \) and \( R^{TM} = (p - p_2/\epsilon_2)(p + p_2/\epsilon_2) \), which are the reflection coefficients for an interface between vacuum and a medium with dielectric function \( \epsilon_2 \). On the other hand, when \( b \gg z \) we obtain \( R^{TE} \to r^{TE}_1 \) and \( R^{TM} \to r^{TM}_1 \). We conclude that the CP potential becomes similar to that generated by the deeper layer when \( z \gg b \) and similar to the one generated by the outer layer when \( z \ll b \).

### 4.9.2 Cylindrical wire

The Green’s tensor in cylindrical coordinates is given by [110]

\[
\Gamma(r, r', \omega) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dk \left[ -\hat{M}^{ts} \frac{\hat{d}_m - k^2}{\omega^2} F_m(r, r') + \hat{N}^{ts} \frac{1}{\omega} G_m(r, r') \right] \chi_{mk}(\phi, z) \chi^{*}_{mk}(\phi', z'),
\]

(4.35)

where \( \hat{M} \) and \( \hat{N} \) are vectorial differential operators that generate the vector fields from the scalar fields for the TE and TM modes, respectively, \( \hat{d}_m \) is the two-dimensional Laplacian operator where the differentiation with respect to \( \phi \) is replaced by \( \partial/\partial \phi \to im \), \( F_m(r, r') \) and \( G_m(r, r') \) are the scalar radial Green’s function for the TE and TM modes, respectively, and \( \chi_{mk}(\phi, z) \) is the wave function that holds the angular and longitudinal dependence of each mode. As we are interested only in the single-point case \( \phi = \phi' \) and \( z = z' \), we have here \( \chi_{mk} \chi^{*}_{mk} = 1/2\pi \).
By substituting for the right forms of the Green’s function we obtain

\[
\sum_j \Gamma_{jj}(R, R) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{e^{i\xi k}}{2k^2} \left[ -\omega^2 m^2 \frac{J_m'(\lambda a)}{H_m'(\lambda a)} H_m^2(\lambda R) \\
+ \omega^2 \lambda^2 \frac{J_m'(\lambda a)}{H_m'(\lambda a)} H_m^2(\lambda R) \right] + \left( \frac{m^2 \xi^2}{R^2} + \lambda^4 \right) \frac{J_m(\lambda a)}{H_m(\lambda a)} H_m^2(\lambda R) - k^2 \lambda^2 \frac{J_m(\lambda a)}{H_m(\lambda a)} H_m^2(\lambda R),
\]

(4.36)

where \( J_m \) and \( H_m \) are the Bessel functions and the Hankel functions of the first kind and \( \lambda^2 = \omega^2/c^2 - k^2 \) is the wave-vector along the radial direction. As above, we now make the transformation \( \omega/c \to i\zeta \), such that \( \lambda \to ik \) with \( k^2 = k^2 + \zeta^2 \).

The Bessel function \( J_m(\lambda a) \) transforms into \( I_m(\kappa a) \), where \( I_m \) is the modified Bessel function, and \( H_m \) transforms into \( K_m \), the modified Bessel function of the second kind. We then obtain the following result:

\[
U_{CP}(R) = -\frac{\hbar c}{2\pi} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\zeta}{2\pi} \frac{1}{\kappa^2} \times \left\{ \frac{I_m'(\kappa a)}{K_m'(\kappa a)} \left( \frac{\kappa^2 m^2}{R^2} K_m^2(\kappa R) + \kappa^2 \zeta^2 K_m^2(\kappa R) \right) \right. \\
+ \frac{I_m(\kappa a)}{K_m(\kappa a)} \left[ \left( \frac{\kappa^4 + m^2 \xi^2}{R^2} \right) K_m^2(\kappa R) + k^2 \kappa K_m^2(\kappa R) \right] \left\}
\]

(4.37)

Now we take the static approximation for the polarizability \( \alpha(i\zeta) \to \alpha(0) \) and change variables to \( x = \kappa R, \zeta = (x/R) \cos \phi \) and \( k = (x/R) \sin \phi \) and find

\[
U_{CP}(R) = -\frac{\hbar c \alpha(0)}{2\pi R^4} \sum_{m=-\infty}^{\infty} \int_{0}^{\infty} dx \left\{ \frac{I_m'(x a/R)}{K_m'(x a/R)} \left( \frac{m^2 x^2}{2} K_m^2(x) + \frac{x^3}{2} K_m^2(x) \right) \right. \\
+ \frac{I_m(x a/R)}{K_m(x a/R)} \left[ \left( \frac{m^2 x^2}{2} \right) K_m^2(x) + \frac{x^3}{2} K_m^2(x) \right] \right\}
\]

(4.38)

This can be written in the form

\[
U_{CP}(R) = -\frac{\hbar c \alpha(0)}{2\pi (R - a)^2} F(a/R),
\]

(4.39)

with

\[
F(a/R) = \left( 1 - \frac{a}{R} \right)^4 \sum_{m=-\infty}^{\infty} \int dx x \left[ I_m(x a/R) \frac{x^2 K_m^2(x)}{K_m(x a/R)} \right. \\
+ \frac{1}{2} \left( \frac{I_m(x a/R)}{K_m(x a/R)} - \frac{I_m'(x a/R)}{K_m'(x a/R)} \right) \left( \frac{x^2}{2} K_m^2(x) \right) \left( \frac{x^3}{2} K_m^2(x) \right) \right]
\]

(4.40)

The result of a numerical integration of \( F \) is shown in Fig. 4.10. It is found that at \( a/R \to 1 \), such that the atom is very close to the surface relative to the radius \( a \), we obtain the same result as for a plane conductor \( F = 3/4 \). In the other limit, where \( a/R \to 0 \) one may see that \( F \sim -\frac{2}{3 \log(a/R)} \) and the contribution to \( F \) is dominated by the term \( m = 0 \) only.
Figure 4.10: The function $F(a/R)$ for a perfectly conducting cylindrical wire.
Chapter 5

Outlook: Engineered fragmentation - a magnetic lattice for interference experiments

The fields of atom optics and matter wave interferometry have made enormous progress during the last few years. Creating periodic potentials (lattices) allows the exploration of tunneling, insulating phases, Bloch oscillations, coherence and the breakdown of coherence in BECs. Numerous experiments have been performed with BECs serving as phase coherent sources of atomic matter waves [111, 112, 113, 114, 115, 116, 117, 118]. Since the scale of these systems makes them difficult to probe directly, they are studied via the interference pattern created when a BEC is released from a lattice. The implementation of similar potentials on atoms trapped in atom chips is an intriguing challenge of integrated atom optics [119].

In a recent atom chip experiment [120], a magnetic periodic potential was applied as a grating for atom diffraction. However, the potential was produced from multiple wires and the trapping potential did not allow long trapping times. Coherence measurements of a BEC were also made on atom chips using time dependent potentials to generate a “double well” potential [36, 37, 38]. Using these techniques the phase fluctuations of a 1D BEC were measured [42]. However, despite numerous attempts, to date no experiment has managed to control tunneling through a static magnetic barrier and the realization of such a task will be a significant step forward.\(^1\)

1\(^1\)as a single exception to this statement, let us note that the group of Jakob Reichel has announced in the beginning of 2010 that at a height of 10 µm and with a double well separation of 20 µm they have been able to see first signs of coherent behavior in a static magnetic double well.

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created after the release of the periodic atomic density profile is also discussed. In addition, we also discuss the experimental difficulties that may arise in performing this experiment and the means to overcome them. We present an analysis that considers not only the sudden release of the trapped cloud, but also a process in which the magnetic trap is turned off gradually. We show that the release of the cloud plays an important role in the formation and the characteristics of the interference pattern. An illustration of the experiment is presented in Fig. 5.1.

The proposed periodic potential is not only different in the manner in which it is produced, it also allows access to coherence decay measurements of a 1D quasi-condensate [121, 122]. (In very elongated condensates, axial phase fluctuations are found to manifest themselves even at temperatures far below $T_c$. The equilibrium state is a condensate with a fluctuating phase (quasi-condensate) [123, 124, 125]).

5.1 Overview of interference patterns

For an intuitive and simplified explanation of the interference patterns generated when separated BECs are overlapping, we will analyze first the simple case of two overlapping BECs.

We examine the interference pattern generated when two BECs, separated by a distance $d_s$, are released from an anisotropic trap ($\omega_r \gg \omega_z$) and overlap. We compare the case where the clouds are separated along the trap radial (strongly confined) direction, to the case where the clouds are separated along the trap longitudinal direction (Fig. 5.2).

5.1.1 A simple model

Following [16], the two BECs are assumed to be separated by distance $d_s$ and can be described by a single condensate wavefunction:

$$\psi(r, t) = \sqrt{N_1}\psi_1(r, t) + \sqrt{N_2}\psi_2(r, t)e^{i\phi}. \quad (5.1)$$
where $N_1$ and $N_2$ are the numbers of atoms per site and $\phi$ is the relative phase of the BECs. The atomic density after expansion time $t$ will be:

$$n(r, t) = |\psi(r, t)|^2 = N_1 |\psi_1|^2 + N_2 |\psi_2|^2 + 2\sqrt{N_1 N_2} \text{Re} \left[ \psi_1 \psi_2^* e^{-i\phi} \right], \quad (5.2)$$

where Re denotes the real part. The last term of this expression displays an interference pattern in the atomic density. We assume that the clouds are described initially by Gaussian wave packets of width $R_0$ and $X_0$, and we can write the widths $R_t, X_t$ of the wave packets at time $t$ as:

$$R_t^2 = R_0^2 \left(1 + \omega_r^2 t^2\right) = R_0^2 \left[1 + \left(\frac{\hbar t}{m R_0^2}\right)^2\right], \quad (5.3)$$

$$X_t^2 = X_0^2 \left(1 + \omega_x^2 t^2\right) = X_0^2 \left[1 + \left(\frac{\hbar t}{m X_0^2}\right)^2\right].$$

The interference pattern in Eq. (5.2) is:

$$2\sqrt{N_1 N_2} \text{Re} \left[ \psi_1 \psi_2^* e^{i\phi} \right] \sim A \cos \left(\frac{\hbar}{m} \frac{r \cdot d}{R_0^2 R_t^2} + \phi\right), \quad (5.4)$$

where the prefactor $A$ is the exponential envelope of the expanding condensates. The fringe separations along the radial direction $\Delta r$ for times longer than $\sqrt{m R_0^2 / \hbar}$, is given by [16]:

$$\Delta r = \frac{2\pi m R_0^2 R_t^2}{\hbar d_s} \approx \frac{2\pi \hbar}{m d_s}, \quad (5.5)$$

and similarly for the longitudinal direction $\Delta x$. The relation between the initial sizes of the clouds and their asymptotic velocities is such that the dependence of the fringe separation on the initial wave packet width is canceled at long times. (the initial width is inversely translated into velocity).
5.1.2 BEC in a lattice

Let us consider a BEC in a periodic potential \( V(x) = \sum_j V_s(x - d_s j) \), where \( j \) labels the different sites in the lattice, \( d_s \) is the distance between the lattice sites and \( V_s \) is a single well potential. Differently from the case of two separate BECs, interference patterns will appear only in the case of a fixed relative phase between condensates in consecutive wells. If we assume a fixed relative phase, \( \phi = 0 \), over all wells, we can write the order parameter, \( \psi(x) = \sum_j \psi_0(x - d_s j) \), in momentum space as [126]:

\[
\psi(p_x) = \psi_0(p_x) \sum_j e^{ip_x (jd_s)/\hbar} = (5.6)
\]

\[
\psi_0(p_x) e^{-ip x_0/\hbar} \sum_{j=0,\ldots,M-1} e^{ip_x (jd_s)/\hbar} = e^{i\theta} \psi_0(p_x) \frac{\sin [Mp_x d_s/\hbar]}{\sin (p_x d_s/\hbar)},
\]

where \( \psi_0(p_x) \) is the Fourier transform of \( \psi_0(x) \), \( M \) is the total number of sites, \( x_0 \) is the position of the first (leftmost) site, and \( e^{i\theta} \) is an arbitrary phase. Since we are interested in the density \( |\psi|^2 \), by choosing the position of the middle site appropriately we can ignore the phase \( \theta \). The momentum distribution of the entire system, \( n(p_x) = |\psi(p_x)|^2 \), is affected by the lattice structure, and exhibits distinctive interference patterns. In the case of a large number of sites, the momentum distribution has sharp peaks at \( p_x = 2\pi n \hbar/d_s \) where \( n \) is a positive or negative integer.

The width of the central peak (\( n = 0 \)) of the momentum distribution is of the order of \( \Delta p_x = 2\hbar/Md_s \), where \( Md_s/2 \) is half of the length of the whole sample along the \( x \) axis, and therefore the corresponding atomic expansion, after the release from the trap, will be slow. The peaks with \( n \neq 0 \) carry higher momentum, and their center of mass will travel according to the asymptotic law:

\[
x_n(t) = \pm n \frac{2\pi \hbar}{d_s m} t.
\]

5.1.3 BEC expansion

To fully understand the expansion of the BECs and its effect on the interference pattern, we need to take into account atom-atom interaction. The following discussion is valid in the limit where the Thomas-Fermi approximation holds for both axes (not in the quasi-1D case). Here, the initial half length size of the condensate is determined by the cloud chemical potential \( \mu \).

\[
R_0 = \sqrt{\frac{2\mu}{m\omega_r^2}},
\]

\[
X_0 = \sqrt{\frac{2\mu}{m\omega_x^2}} = \frac{\omega_x}{\omega_r} R_0,
\]

(5.8)

\( R_0 \) and \( X_0 \) do not have the same meaning as in Sec. 5.1.1. Here they denote half the length (or thickness) of the condensate and not the rms width as before.
5.2 Creating a 1D magnetic lattice

Once released, these measures evolve according to [59, 127]:

\[
R_t = R_0 \sqrt{1 + \omega_r^2 t^2} = R_0 \sqrt{1 + \frac{2\mu}{mR_0^2} t^2},
\]

\[
X_t = X_0 \left( 1 + \left( \frac{\omega_r}{\omega} \right)^2 \left[ \omega_r t \arctan(\omega_r t) - \ln \sqrt{1 + (\omega_r t)^2} \right] \right).
\]

(5.9)

Since we are interested in interference patterns generated after a relatively long expansion time \( t \gg \frac{1}{\omega_r} \), we may look at the asymptotic expansion of the half length:

\[
R_t \approx R_0 \omega_r t = \sqrt{\frac{2\mu}{mR_0^2}} t,
\]

(5.10)

\[
X_t \approx \frac{\pi}{2} X_0 \frac{\omega_r^2}{\omega} t.
\]

The anisotropy in the cloud expansion is such, that according to the lattice periodicity and trapping frequency, some parameters may lead to a fringe separation [Eq. (5.7)] evolving faster than the BEC half width. In this case no interference pattern can be formed. The BEC expansion, [Eq. (5.10)], is valid only for abrupt shutdown of the magnetic trap.

5.2 Creating a 1D magnetic lattice

5.2.1 Trapping potential

To generate the ‘magnetic lattice’ potential, we have specifically fabricated a wire (Fig. 5.3) with a 5 \( \mu \)m sinusoidal periodicity. The amplitude of perturbation is 2 \( \mu \)m and the wire width and thickness are 10 \( \mu \)m and 2 \( \mu \)m, respectively. In Sec. 4.4.2 we consider a configuration in which a thin wire is curved with a periodicity \( \lambda \) that corresponds to a wave-vector \( k = 2\pi/\lambda \). If the amplitude of this curvature is small with respect to the wavelength, the trapping potential can be approximated by:

\[
V(x, z) = \mu_A \mu_0 I k^2 \delta y_{\text{center}} \left( \frac{e^{-k z}}{k z} \right) \sqrt{1 + \frac{k z}{2} \cos kx}
\]

(5.11)

where \( \mu_A \) is the atomic magnetic dipole moment, \( I \) is the current in the wire, \( \mu_0 \) is the permeability of free space, and \( \delta y_{\text{center}} \) is the center of the wire position variation amplitude. However, this approximation is not valid for the wire parameters we are discussing here. We therefore calculated the resulting potential in two steps, the current density in a wire with this specific boundary conditions was derived numerically by using a finite element method (FEM Lab) and from this current density the magnetic potential was calculated.

Similar to the approximated potential, Eq. (5.11), the periodicity amplitude of the numerically calculated potential has a decaying exponential as a function of
the trap-wire separation $d$. For typical currents in the snake wire (e.g. 50 mA), the lattice potential is significant at heights of 5 $\mu$m or less. For example, the potential variation amplitude of a trap positioned at $d = 4.1 \mu$m will be 1.1 $\mu$K peak-to-peak, while a trap positioned at $d = 5.5 \mu$m will have a variation of 0.25 $\mu$K peak-to-peak (Fig. 5.4). The calculated trap parameters for different trap positions are given in Table 5.1. An isopotential plot for the generated trapping potential and the minimum energy path of the generated traps at different atom-surface separations are shown in Fig. 5.4.

<table>
<thead>
<tr>
<th>$d$</th>
<th>5.5 $\mu$m</th>
<th>5.0 $\mu$m</th>
<th>4.5 $\mu$m</th>
<th>4.1 $\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrier [$\mu$K peak-to-peak]</td>
<td>0.25</td>
<td>0.45</td>
<td>0.72</td>
<td>1.1</td>
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<tr>
<td>$\omega_T$ [kHz]</td>
<td>23</td>
<td>26</td>
<td>29.2</td>
<td>32.3</td>
</tr>
<tr>
<td>$\omega_\phi$ [kHz]</td>
<td>0.62</td>
<td>0.86</td>
<td>1.13</td>
<td>1.44</td>
</tr>
<tr>
<td>Chemical potential $\mu$ [$\mu$K]</td>
<td>0.312</td>
<td>0.412</td>
<td>0.519</td>
<td>0.619</td>
</tr>
<tr>
<td>Spin-flip lifetime [sec]</td>
<td>1.246</td>
<td>1.102</td>
<td>0.966</td>
<td>0.864</td>
</tr>
</tbody>
</table>

Table 5.1: Trap properties of a single site at different atom-surface separations $d$. The trap is generated by running 50 mA in the snake wire and the height is adjusted by $B_y$. A magnetic field, $B_x$, of 1 G is applied to avoid Majorana spin-flips at low magnetic fields. The chemical potential $\mu$ is calculated in a 1D approximation, for 500 atoms over a length of 40 $\mu$m. Trap lifetimes due to thermal noise-induced spin flips are calculated for atoms trapped above the snake wire (see Sec. 4.5.1). Tunneling to the surface and Majorana spin-flips at these trap parameters and atom-surface separations are negligible.
5.3. INTERFERENCE PATTERNS

The trap aspect ratio in a single lattice site depends on the current in the snake wire and the trap height. However, for typical currents and heights ($I = 50 \text{ mA}$ $d = 5 \mu\text{m}$) it is above 1:20, yielding a quasi 1D trapping potential.

Figure 5.4: (left) An isopotential surface of the trapping potential for an energy of 1 $\mu$K above the trap minimum. The longitudinal direction is compressed by a factor of 25 for a better view. The current in the snake is 50 mA and $B_y$ is adjusted so that the trap is positioned 4.1 $\mu$m from the wire. (right) Potential energy (relative to the global minimum) along the minimum-energy path, defined as the position of the local potential minimum on the transverse plane ($y - z$) for each longitudinal position $x$ for a few values of the trap height ($d$). The trap parameters are given in Table 5.1.

5.2.2 Atomic ground state

The ground state atomic density for a BEC is examined by solving the time independent Gross-Pitaevskii equation for $N$ interacting bosons confined by the magnetic potential of the snake wire [Eq. (1.7)]. We do not use the Thomas-Fermi approximation since the trapping potential of the separated lattice site is quasi 1D (aspect ratios above 1:20), and we do not expect a high atomic density. The calculated chemical potential $\mu$ for $N = 500$ atoms of $^{87}\text{Rb}$ trapped at a trap height of $d = 4.1 \mu\text{m}$ is $13 \text{kHz} \cdot 2\pi h = 0.619 \mu\text{K}$. In Fig. 5.5 we present the calculated ground state atomic density, for the potentials shown in Fig. 5.4. This shows the possible control over the BEC separation at different trap heights $d$.

5.3 Interference patterns

The main goal of this experiment, is to measure the phase evolution of the BEC in the lattice potential. Information regarding the coherent splitting of the cloud, and the phase evolution of the BECs once the clouds are separated, can be obtained by imaging the interference patterns generated after the clouds overlap (Sec. 5.1).
Figure 5.5: (left) Ground-state atomic density map (integrated along a viewing axis $\hat{y}$ perpendicular to the snake) for a BEC at a trap height of $d = 4.1 \mu m$ (similar trap parameters as in Fig. 5.4), calculated with the Gross-Pitaevskii equation for 500 atoms of $^87\text{Rb}$. (right) 1D atomic density for atoms at a trap height of $d = 4.1 \mu m$ (blue), $4.5 \mu m$ (black), $5 \mu m$ (red) and $5.5 \mu m$ (green). Calculated using the Gross-Pitaevskii equation for 500 atoms over a length of $40 \mu m$. The calculated chemical potential is given in Table 5.1. As the trap is lowered towards the snake wire, the BEC is separated to the different lattice sites.

Our imaging setup and its limitations are described in Sec. 2.1.3. The resolution of our setup is $> 5 \mu m$ and the minimal optical density $\sigma_d(x, z)$ that we can detect close to the chip surface is $\sim 0.1$. With this imaging resolution we cannot see the separated clouds in situ. This resolution can also limit the observation of the interference patterns. In this section we present the calculation of the estimated interference patterns, and show the important role of the trap release rate in the formation of these interference patterns.

The time evolution of the trapped condensates was calculated by numerically solving the time dependent Gross-Pitaevskii Eq. (1.6). We used a cylindrical approximation according to a method presented in [128], and used the Matlab ODE variable step method to solve the time evolution of the clouds. The initial ground state was the same as presented in Sec. 5.2.2, but for simplicity we limited the number of sites in the ground state.

In Fig. 5.6(a) we present the calculated column density pattern of a BEC of 100 atoms per site occupying 4 sites of the snake trapping potential, 10 ms after abruptly turning off the magnetic trap. According to Eq. (5.7) the fringe separation 10 ms after the release from the trap should be $9.2 \mu m$. However, the longitudinal ($x$) expansion velocity of a BEC from a single site is $0.667 \mu m/ms$. Therefore, the clouds are not overlapping and no interference pattern is visible. As the interference pattern center of mass velocity is $0.920 \mu m/ms$, the interference pattern will never be formed. The single site width after expansion agrees with the asymptotic width [Eq. (5.10)]. The expansion velocity along the radial ($z$) direction is
13.7 $\mu$m/ms, and consequently the peak optical density (Sec. 2.1.3) drops to 0.1
10 ms after the release.

In Fig. 5.6(b) we present the expected image after taking into account our
diffraction limit. This is done by cutting out spatial frequency components with
frequency higher than the diffraction limit. In Fig. 5.6(c) we bin the image to ac-
count for the finite size pixels ($1.5 \mu$m/pixel) of our imaging system, to obtain a
realistic image of the calculated atomic density. As can be seen from the figures,
the fine details of the atomic density cannot be detected by our system.

![Figure 5.6](image)

Figure 5.6: (a) Simulated atomic density 10 ms after the abrupt release of the BEC
from the snake trapping potential. Due to the anisotropy in the expansion of the
BEC, the clouds from different lattice sites do not overlap, and no interfe-
rence pattern is formed. (b) The image as it would be seen due to our imaging setup
diffraction limit, $\sim 6 \mu$m. (c) The approximated image as it would be captured in
our camera, taking into account the pixel size of 1.5 $\mu$m.

In typical interference experiments the barriers in the lattice are formed along
the strongly confined direction (the radial direction), and hence the clouds over-
lap rapidly. In our configuration, the clouds’ expansion along the longitudi-

al direction presents an obstacle in forming the interference pattern.

### 5.3.1 Controlled trap release rates

The main problem raised at the end of the previous section is that the expansion
in the radial (strongly confined) direction is much faster than the expansion in the
(loosely trapped) axial direction. This prevents the formation of an interfe-
rence pattern since the overlap between BEC clouds from different sites takes too long. A
way to cure this would be the reduction of the radial trapping frequency $\omega_r$, in view
of Eq. (5.10), which shows that the expansion in the radial direction is proportional
to $\omega_r$ while the expansion in the axial direction is inversely proportional to the
radial frequency. This means that reduction of the radial frequency may cure both
problems. This may be achieved by a gradual release of the trap, in which the
trapping potential is ramped from its initial value down to zero in a finite time $t_{\text{release}}$. As long as the rate of change of the trapping frequency is small enough to
maintain the adiabatic criterion

\[ \left| \frac{d\omega_{\text{trap}}}{dt} \right| \ll \omega_{\text{trap}}^2, \]  

(5.12)

the BEC ground state will evolve into a new ground state according to the change of the trapping frequency. The time in which the above criteria is no longer satisfied, may be considered as the starting point of the cloud free expansion, whereby the starting point of this expansion, is the trap existing at this point in time.

5.3.1.1 Expansion of a single BEC

We examined the time evolution of a single trapped BEC, where the trap is turned off linearly in a time \( t_{\text{release}} \). We consider a BEC of 150 atoms, with \( \omega_r = 30 \text{ kHz} \) and \( \omega_x = 1.5 \text{ kHz} \), similar to the parameters in a single site of the lattice (Sec. 5.3).

The BEC ground-state at the lattice minima are highly anisotropic (typical trap parameters are \( \omega_r \sim 2\pi \cdot 30 \text{ kHz} \) in the radial direction and \( \omega_x \sim 2\pi \cdot 1.5 \text{ kHz} \) in the longitudinal direction). According to the adiabatic condition, Eq. (5.12), we can define two characteristic times, one for the radial trapping frequency \( t_r = 2\pi/\omega_r \approx 33 \mu\text{s} \), and the other for the longitudinal trapping frequency \( t_x = 2\pi/\omega_x \approx 667 \mu\text{s} \). If the release time is larger than \( 2\pi/\omega_r \) but much shorter than \( 2\pi/\omega_x \), then the potential ramp would not significantly change the initial ground state density in the axial direction, but will change the ground state in the radial direction and the consequent expansion will be as if the initial radial frequency was lower.

In Fig. 5.7 we present the calculated evolution of the BEC half width, calculated by numerically solving the time dependent Gross-Pitaevskii equation [Eq. (1.6)]. For \( t_{\text{release}} \ll t_r \) the expansion shows anisotropic behavior corresponding exactly to Eq. (5.9). Longer \( t_{\text{release}} \) \( (t_r < t_{\text{release}} \ll t_x) \) results in faster expansion along the axial direction, and slower radial expansion. For \( t_{\text{release}} = 25 \mu\text{s} \) the radial expansion velocity is \( \sim 6 \) times lower relative to the abrupt trap release, and the longitudinal expansion is \( \sim 3.3 \) times higher. When \( t_{\text{release}} > 25 \mu\text{s} \) the longitudinal expansion becomes faster than the radial expansion.

In Fig. 5.8 we show the difference in the expansion velocity for the different axes as \( t_{\text{release}} \) is changed. The resulting change in the cloud aspect ratio after 10 ms TOF is shown in the inset. The longitudinal expansion velocity is increasing from 0.667 \( \mu\text{m}/\text{ms} \) for \( t_{\text{release}} \ll t_r \) to 3 \( \mu\text{m}/\text{ms} \) for \( t_{\text{release}} = 50 \mu\text{s} \) while the radial expansion velocity reduces from 13.7 \( \mu\text{m}/\text{ms} \) to 1 \( \mu\text{m}/\text{ms} \) for similar parameters.

5.3.1.2 Interference patterns with controlled trap release times

We have calculated the resulting atomic density, where the lattice trap was released in a linear ramp at different release times \( t_{\text{release}} \). Fig. 5.9 presents the calculated atomic density 10 ms after the release, for different trap release times \( t_{\text{release}} \). According to the expansion velocity presented in Fig. 5.8, we see that for
5.3. INTERFERENCE PATTERNS

Figure 5.7: BEC half length expansion after different release times $t_{\text{release}}$ from a single trap, calculated numerically using the time dependent Gross-Pitaevskii equation. For $t_{\text{release}} = 1 \mu s$ the expansion corresponds exactly to Eq. (5.9). Longer $t_{\text{release}}$ is equivalent to releasing the trap from a smaller radial trapping frequency $\omega_r$ while maintaining the longitudinal trapping frequency $\omega_x$. This results in faster expansion in the longitudinal direction and slower expansion in the radial direction.

Figure 5.8: Expansion velocity of a BEC along the radial (blue) and longitudinal (red) directions, for different trap release times. (inset) The change in the cloud aspect ratio after 10 ms TOF for the different trap release times.

$t_{\text{release}} > 25 \mu s$, the clouds overlap, and an interference pattern is visible with a fringe separation of $\sim 9.2 \mu m$ in accordance with Eq. (5.7). Moreover, the reduced expansion along the radial direction allows much higher optical densities, even after long TOF, and allows longer expansion times. Interference patterns generated when $t_{\text{release}}$ is larger than 50$\mu$s are easily detectable in our system.
Figure 5.9: Simulated BEC atomic density 10 ms after the controlled release from the snake trapping potential. The trap release times are: (a) 10 μs, (b) 25 μs, (c) 50 μs, and (d) 100 μs. The presented figures are estimations, taking into account the diffraction limit and the actual pixel size of our imaging system. For trap release times above 25 μs the clouds overlap, resulting in interference patterns with fringe separations of ~ 9.2 μm. The reduced expansion velocity along the radial direction allows longer TOF.

5.4 Summary

We have presented a proposal for an experiment utilizing atom chips to measure the coherence properties of a BEC. The experiment not only presents a new approach to generate the 'lattice' potential, but also paves the way to measure specific coherence properties of a quasi-1D BEC.

The presented calculations show that interference patterns could be measured if the snake potential is released in a controlled way. The typical shut down time of our current shutters (for the current supplies generating the trap) is ~ 100 μs. This is the time needed for the realization of the experiment. The experiment is currently underway in our lab.
Chapter 6

Summary and conclusions

This thesis work was focused on the generation and manipulation of a Bose-Einstein condensate in magnetic micro-traps generated by an atom chip. A new experimental setup was built from scratch with which we achieved BEC and loaded the atom chip traps, attaining atom-surface separations as low as 5 µm. By this we achieved the milestone of an operational atom chip laboratory. New experiments such as the magnetic lattice described in Ch. 5 are already underway and are designed and based on work described in this thesis.

The setup we designed is unique as it was designed and built to house a cryogenic chip mount for superconducting atom chip experiments. The cryogenic chip mount was built and tested, and transition to superconductance was observed in microfabricated Nb wires. However, this topic is beyond the scope of this thesis.

In a second project I was involved in, a similar setup was used to measure inhomogeneities in the magnetic potential of traps generated by the microfabricated wires of the atom chip. This study, described in Ch. 3, revealed new insight into how electrons scatter inside conductors. This work also further enhanced our understanding of physical processes limiting atom-surface separations, specifically, the fragmentation of the atomic cloud. Here, I was involved in the long fabrication R&D process, the data analysis, and the experiment itself that took place in Heidelberg. In addition, I took part in developing the theoretical models explaining our observations, models which also gave rise to some unique predictions. The work was published in [1, 2].

In a third project, I led our work aimed at further improving atom chips by utilizing nanowires (Ch. 4). We showed that by using such wires, sub-micron atom-surface separations can be achieved, and tunneling barriers may be constructed, making atom chip interferometry more feasible. We accounted for static corrugations of the magnetic fields (based on nanowires fabricated specifically for this purpose) and temporal variations of the magnetic field due to the Johnson (thermal) noise in the room temperature conductor. We also accounted for effects of the Casimir-Polder force on the trapping potential at such small atom-surface separations. This work has been submitted for publication [4]. As part of our work on
corrugations and noise I was also involved in studying static magnetic corrugations and surface-induced noise in electrically anisotropic conductors. This work was published in [3].

Finally, in the fourth part of my work, I led the effort to design our first experiment in which we would like to observe the many body behavior of the BEC in a 1d magnetic lattice. Based on the developments in our understanding of the potential corrugations in the atom chip, we designed a trapping potential in which we engineered the potential inhomogeneities to form a magnetic lattice. The design of this experiment required the extensive use of theory and numerical simulations, including a detailed study of the trapped BEC time evolution, once released from the trapping potential. We have learned that the trap release dynamics plays an important role in the formation of the atomic density interference patterns. This experiment is already in progress.
Bibliography


[71] The white-light interferometer used here was Zygo New View 200, with 50x objective. The atomic force microscope (AFM) was Veeco AFM/SPM Dimension 3100 with Nanoscope4 controller. For further details see www.zygo.com and www.veeco.com.


[75] The chip was fabricated at The Weiss Family Laboratory for Nano-Scale Systems at Ben-Gurion University, Israel, www.bgu.ac.il/nanofabrication.
[76] Our treatment gives a full solution for small arbitrary fluctuations in a wire with rectangular cross section. The case of top surface fluctuations was given in Ref.[62] only for symmetric fluctuations that contribute to the magnetic field above the middle of the wire.


[120] A. Günther, S. Kraft, C. Zimmermann, and J. Fortágh, “Atom interferometer based on phase coherent splitting of Bose-Einstein condensates with an


The evaporation of a metal film, however, is accompanied by a magnetic field, which can significantly affect the properties of the resulting film. The development of a model for the statistical and dynamic fluctuations of the magnetic field is a crucial step in understanding the behavior of the evaporation process. In this context, we present a model that accounts for the statistical and dynamic fluctuations of the magnetic field, which are essential for predicting the behavior of the evaporation process.

The model is based on the analysis of the magnetic field fluctuations in a metal film, which are caused by the statistical and dynamic fluctuations of the magnetic field. The model is validated by comparing the predicted and experimental results, which show a good agreement.

The model is not only applicable to metal films but can also be extended to other materials, such as magnetic and non-magnetic materials. The model can be further refined by taking into account the effects of the surrounding environment, such as the presence of other materials, on the magnetic field fluctuations.

The development of the model is not only of scientific interest but also has practical implications, such as the design of novel devices that exploit the magnetic field fluctuations. The model can be used to optimize the design of such devices, which can lead to improved performance and efficiency.

The model is not only of interest to the scientific community but also to industry, which can benefit from the improved design of devices that exploit the magnetic field fluctuations. The model can be used to optimize the design of devices, which can lead to improved performance and efficiency.

The model is not only of interest to the scientific community but also to industry, which can benefit from the improved design of devices that exploit the magnetic field fluctuations. The model can be used to optimize the design of devices, which can lead to improved performance and efficiency.
העבודה נעשתה בהדרכת פרופ' רן פולמן

במחלקה לפיזייקה

בפקולטה למדעי הטבע
העבות בוה-אינטיציינל על השבב האטומלי

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התעבות בווה-אינטשטיין על השבב האטומי

מתק 않습니다 מילוי חלקי של הדרישות לקבלת תואר ”דוקטור לפילוסופיה”

🌼 נא
🌼 סלם

הוגש לסינטס אוניברסיטט בן גוריון בנגב

שבט תש"ע
יוזר 2010

באד שבי