Experiments with Ultracold Atoms

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Abstract

Experiments with ultracold atoms have become a fundamental tool in physics research. In this thesis I report the observation of asymmetric transition rates between Zeeman levels of magnetically trapped atoms. These transition rates strongly depend on the spectral shape of an applied noise. This effect follows from the interplay between the internal states of the atoms and their external degrees of freedom, where different trapped levels experience different potentials.

The findings of this work may be utilized to control the relative population of the different Zeeman levels. Just as important, this insight may serve to better understand how noise couples to atoms in magnetic traps and may pave the way for effective schemes of combating background and technical noise, which are of great concern, for example, in atom chip experiments, both for fundamental studies and technological applications.

In this thesis I also describe the characterization of a Bose-Einstein condensate (BEC), observed in our laboratory. I present measurements of the condensate’s chemical potential, transition temperature, lifetime and heating rate. The observation of a BEC is the starting point for many advanced experiments with ultracold atoms.

In addition to the experimental work, this thesis describes detailed theoretical analyses I have done in connection with the affect of magnetic noise on magnetically trapped atoms. These include calculations of spin flip rates due to colored noise as well as Johnson noise, and also heating rate as a function of trap parameters.

The results of this work may prove useful for controlling atomic states by the introduction of noise and will hopefully contribute to the understanding of the affect of noise on atoms enabling better control over hindering effects in future quantum devices.

Publications based on this research:

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Chapter 1

Introduction

1.1 Ultracold-atom physics

Experiments with ultracold atoms have become a fundamental tool in physics research. Ultracold dilute gas clouds supply a relatively simple and isolated environment to explore many-body physics phenomena. Although the atomic clouds are dilute, interactions play an important role as a consequence of the low temperatures. Ultracold atoms give rise to collective phenomena related to those observed in solids, quantum liquids, and nuclei. Experimentally these systems are attractive to work with, since they may be manipulated by the use of lasers and magnetic fields. In addition, interactions between atoms may be varied either by using different atomic species or, for species that have a Feshbach resonance, by changing the strength of an applied magnetic or electric field. The first experimental realizations in 1995 of Bose-Einstein condensation (BEC) in dilute atomic gases [1, 2] marked the beginning of a very rapid development in this field. Key methods to produce ultracold atoms are laser cooling and evaporative cooling, that have been developed since the mid 1970s. Researchers responsible for the above advances have been awarded the Nobel prize in 1997 and 2001. In this thesis, I will describe a few ultracold experiments performed with $^{87}$Rb atoms that I have conducted in our laboratory.
Rubidium (Rb, atomic number 37) has one stable isotope $^{85}$Rb. Another isotope, $^{87}$Rb, composes almost 28% of naturally occurring rubidium and is slightly radioactive, with a half-life of 49 billion years. In our experiment only $^{87}$Rb atoms are trapped and cooled although also $^{85}$Rb atoms are being released into the science chamber. Rubidium is an alkali metal located in the first column of the periodic table that includes lithium, sodium, potassium, rubidium, cesium, and francium and hydrogen (which is not considered an alkali). All the alkali metals exhibit a simple electronic configuration of closed shells with a single valence electron, for example the electronic configuration of $^{87}$Rb is 5s. As a result, the total electronic orbital angular momentum $L$ and the total electronic intrinsic spin $S$ arise only from the single valence electron.

Because of the spin-orbit interaction ($V_{so} \sim L \cdot S$), the eigenstates of the Hamiltonian are represented in the basis of the total electronic angular momentum $J = L + S$. According to the rules of angular momentum addition, the corresponding quantum number $J$ must lie in the range:

$$|L - S| \leq J \leq L + S.$$  

This is the cause of the “fine structure” of the atom where the energy levels of the excited states split, for example the first excited state ($L=1$) of $^{87}$Rb splits into $5^2P_{1/2}$ and $5^2P_{3/2}$. The corresponding transitions from the ground state: $5^2S_{1/2} \rightarrow 5^2P_{1/2}$ and $5^2S_{1/2} \rightarrow 5^2P_{3/2}$ are called the $D_1$ and $D_2$ lines, respectively. In our experiment we work only with the $D_2$ line.

An interaction between the nuclear magnetic spin $I$ and the total electronic spin $J$, of the form $\sim I \cdot J$, causes another split with eigenstates that correspond to the total angular momentum of the atom $F = J + I$. This separation between different eigenstates of the operator $F$ (so called “$F$ numbers”) is called the hyperfine structure. Figure 1.1 shows detailed information about the fine and hyperfine structures of the $^{87}$Rb atoms. In $^{87}$Rb, $I = 3/2$, therefore, as can be seen in the figure, the ground state, $5^2S_{1/2}$, splits into two levels with $F = 1$ and 2, and the excited levels, $5^2P_{1/2}$ and $5^2P_{3/2}$ split into two and four levels, respectively. The $F$ numbers of $^{87}$Rb are always integers, hence, it is a boson.

If the atom is subjected to a magnetic field, the Zeeman interaction ($V_Z = -\mu \cdot B$, where $\mu$
is the atom’s magnetic moment and $\mathbf{B}$ the magnetic field) causes each hyperfine $F$ level to split again into $(2F+1)$ sub-levels according to the quantum number corresponding to the projection of the atom’s magnetic moment on the magnetic field. More information about $^{87}\text{Rb}$ can be found in reference [3].

1.3 Overview of this thesis

This thesis is structured as follows: chapter 2 reviews our experimental apparatus. It presents our basic experimental cycle, and measuring techniques.

Chapter 3 discusses the theory and properties of a BEC in a harmonic trap. It presents characterization measurements of our BEC.

Chapter 4 presents the theory of noise-induced spin-flip transitions. It includes derivations for technically and thermally induced spin-flips, including detailed calculations of the geometric integrals.
In chapter 5 I present experimental evidence of asymmetric spin-flip transitions followed by a complete theoretical explanation. I also present calculations of the expected mean transition energies and lifetime of an atomic cloud under the influence of white and colored noise.

Chapter 6 includes a feasibility study of a suggested experiment with anisotropic electrical conductors on a chip. A design of a new mount for future experiments is also described.

Finally, in chapter 7, I discuss the topic of heating in a harmonic trap. I define the anharmonicity of an atomic cloud and the harmonic range of a given trap.

We end the thesis with a brief summary and conclusions chapter.
Chapter 2

Experimental Apparatus

2.1 Experimental apparatus

The experimental apparatus was built by Dr. P. G. Petrov and S. Machluf with my help at the final stages. Detailed information can be found in [4].

2.1.1 Vacuum system

An ultra-high vacuum (UHV) environment is an essential ingredient for experiments with cold atoms, as the cooling process requires lifetimes of the order of tens of seconds, which is attainable only in a UHV environment. In our apparatus, a 300 l/s ion pump is responsible on producing a base pressure in the $10^{-12}$ Torr range in the science chamber. This pressure is below the sensing capability of our vacuum gauge, and is measured by us by way of measuring the trapped time of the atomic sample.

2.1.2 Laser system

Our experimental sequence requires laser light at four different frequencies (so-called “cooler”, “repumper”, “optical-pumping” and “imaging”), all of them with a wavelength around 780 nm. We utilize two diode laser devices (with a power of about 1 W and 50 mW) together with acousto-optic modulators (AOMs) to produce all four wavelengths. The lasers are used at the early stages of the
experiment (magneto-optical trap (MOT), optical molasses and optical pumping, see section 2.2) and for imaging (section 2.1.4). Both lasers are locked using a polarization spectroscopy setup.

2.1.3 Magnetic fields

The magnetic fields required for the experiment are produced using three methods:

External coils

Homogeneous (bias) magnetic fields are produced by three pairs of coils (one for each of the orthogonal axes) outside of the science chamber. Three home-made current shutters are responsible to ensure a fast shut-down of the magnetic fields when releasing the atomic cloud from the trap before imaging it.

Atom-chip

In our experiment we intend to use an atom-chip to capture the atoms. An atom-chip \([5, 6, 7]\) is a device that enables the trapping and manipulation of cold atoms with high accuracy and resolution. Our atom-chip is made of a silicon substrate with lithographically fabricated miniaturized wires placed on it (in \(\mu m\) scale). These wires carry currents that generate the required magnetic fields for the trapping of the atoms (a microtrap \([8]\)). The advantage of an atom-chip, compared with conventional magnetic traps made from macroscopic coils, is in the fact that the atom-chip captures the atoms extremely close to its surface (at a distance of a few microns), thus providing high accuracy and resolution for handling the atoms. In our experiment the atom-chip plays a role also in the MOT stage, where it is being used as a mirror to produce a mirror-MOT \([9, 10]\). Our atom-chip is positioned at the end of a mount, facing downwards (with the atoms captured below the chip). The mount is responsible to hold the atom-chip, rigidly, in the middle of the science chamber and supplies the required electrical connections.
Copper structure

Before the atoms are trapped by the atom-chip, they are held by a MOT and a (macroscopic) magnetic trap. The magnetic fields required for these stages are produced (in addition to the external bias fields) by a copper structure held at the end of the mount, just above the atom-chip. The current copper structure is made of three parts [10, 11]:

- **U-wire**: a wire that is bent on both sides in the same direction, creating a “U” shape. The magnetic field produced by this wire combined with a homogeneous bias field creates the magnetic quadrupole required for the MOT.

- **Z-wire**: a wire that is bent on both sides in opposite directions, creating a “Z” shape. The magnetic field produced by this wire combined with two homogeneous bias fields creates a magnetic field with a spatial minimum (of its magnitude), thereby creating a magnetic trap.

- **“Legs”**: two straight wires, on both sides of the Z-wire. These wires can be used to control the axial frequency of the magnetic trap. They can also be used to “push” the center of the magnetic trap spatially. This enables better control of the position of the cloud (see for example section 6.4). This is especially useful when the atomic cloud is loaded from the magnetic trap to the atom-chip.

2.1.4 Imaging

At the end of each experimental cycle we measure the properties of the atomic cloud by imaging it with laser light. We have been using an absorption imaging method [12], where we shine on-resonance laser light on the atoms. The light passes through the atomic cloud, and is collected into a CCD camera (after magnification). By analyzing the “missing” light (the shadow) that has been absorbed by the atoms, we are able to reproduce the density profile of the atomic cloud (after integration over the imaging axis). This imaging method is destructive, meaning that upon imaging, the atomic cloud absorbs a non-negligible amount of light and therefore most of the atoms gain enough kinetic energy to escape from the trap. Consequently each experimental cycle can produce only a single image. In order to investigate the time evolution of an atomic cloud, the experiment
should be repeated a few times, with each repetition the imaging event is delayed for a different amount of time.

2.1.5 Computer control

The experimental sequence is controlled by a dedicated computer hardware and software. The interface is a combination of LabView and Matlab based programs. Most of the experiments demand the experimental cycle to run many times, each time with different experimental parameters. Changing these parameters manually is both time consuming and confusing. A lot of experimental time has been wasted because the experimental parameters were entered incorrectly. For these reasons, I’ve written an intuitive and easy-to-use interface software (under Matlab) that controls the experimental sequence when long measurements are involved. The software allows easy control of the experimental parameters. Commonly used measurements can be saved to allow easy and quick access. The software is also responsible to arrange the retrieved data in well organized folders on the computer. With the help of this program the experimentalist can run a sequence of different measurements on a “fire-and-forget” method (this is especially useful for overnight measurements).

2.2 The experimental cycle

Our experimental cycle has the following basic stages:

Preperation

This stage is basically the same for each of the experiments described in this work. It includes the following steps [9]:

- Magneto-optical trap: About $10^8$ atoms from a thermal vapor are trapped and cooled down in a magneto-optical trap (MOT) during 20 seconds. At this stage the trapped atoms are at a temperature of about 300 $\mu K$.

- Optical molasses: The MOT is shut down and a polarization gradient optical molasses (OM) stage begins, cooling the atoms to temperatures of about 100 $\mu K$ during 5ms.
• Optical pumping: The atoms are pumped to the $|F = 2; m_F = 2\rangle$ level. This is done by applying a small magnetic field, that defines a quantum axis, together with a $\sigma^+$ pulse of laser (at the resonance frequency of the $F = 2 \rightarrow F' = 2$ transition).

• Magnetic trap: After the atoms are pumped to the level $|F = 2; m_F = 2\rangle$, they are in a magnetically trappable state. A magnetic trap is applied and then compressed to increase the rate of elastic collisions between the trapped atoms.

• Evaporative cooling: The atoms are further cooled using forced evaporative cooling. Radio-frequency (RF) transitions between magnetic sublevels are used to selectively expel the hottest atoms from the trap. The remaining atoms collide with each other and rethermalise at a lower temperature.

Experimental stage

At this stage we manipulate the atoms according to the details of the specific experiment. This can involve further cooling in order to achieve a BEC (chapter 3), some kind of manipulation of the trap (e.g. changing the trapping frequencies), causing a certain perturbation (e.g. for a trapping frequency measurement), loading the atom-chip etc. Usually we want to check the time evolution of our atomic cloud and therefore each cycle will involve a different waiting time in the trap.

Imaging and analysis

At the end of each experimental cycle we retrieve data by performing absorption imaging. A Matlab based computer program is responsible for the analysis of this data. More details about our preparation and data analysis can be found in [4].

2.2.1 Time-of-flight measurement

A time-of-flight (TOF) measurement is a sequence of images each taken after a different time from the switching off of the confining trap. This measurement allows us to examine the dynamics of a free expansion of the atomic cloud while it falls under gravity.
Chapter 3

Characterization of a Bose-Einstein condensate

Bose-Einstein Condensation (BEC) is a phenomenon in which a macroscopically large number of particles is accumulating a single quantum state [12]. It is a quantum effect unknown in classical statistics. During my work on the experiment, we have achieved a stable BEC of $\sim 4 \times 10^4$ atoms.

3.1 Theory of a Bose-Einstein condensate in a harmonic potential

In this section I will treat the case of non-interacting bosons in a harmonic external potential [13, 14]:

$$V_{\text{ext}}(r) = \frac{1}{2} m \sum_{j=x,y,z} \omega_j^2 r_j^2. \quad (3.1)$$

In the grand canonical ensemble, the mean number of bosons in the state $i$ of energy $\epsilon_i$ is given by the Bose-Einstein distribution:

$$N_i = \left( e^{\beta(\epsilon_i - \mu)} - 1 \right)^{-1}, \quad (3.2)$$

where $\mu$ is the chemical potential and $\beta = 1/(k_B T)$. The energy $\epsilon_i$ of the $i$-th state of the harmonic potential is given by

$$\epsilon_i = \varepsilon_{n_x,n_y,n_z} = (n_x + \frac{1}{2})\hbar \omega_x + (n_y + \frac{1}{2})\hbar \omega_y + (n_z + \frac{1}{2})\hbar \omega_z, \quad (3.3)$$
where \( n_x, n_y, n_z \) are the quantum excitation numbers. As can be seen from (3.2), the number of atoms occupying the ground state \( N_0 \) becomes large when the value of the chemical potential approaches the energy of the lowest state:

\[
\mu \to \frac{\hbar}{2}(\omega_x + \omega_y + \omega_z).
\]

(3.4)

In this limit, the total number of atoms \( N \) can be written as

\[
N = N_0 + \sum_{n_x, n_y, n_z \neq 0} \frac{1}{\exp[\beta \hbar(\omega_x n_x + \omega_y n_y + \omega_z n_z)] - 1}.
\]

(3.5)

In order to evaluate this sum, we make a “semi-classical” approximation, and replace the sum by an integral. This approximation is valid under the following two assumptions. The first is that the available energy is large compared to the spacing of the harmonic oscillator energy levels. This means that instead of summing over the discrete energy levels, we can integrate over a smooth density of states \( \rho(\epsilon) \). The second assumption is that we can take the thermodynamic limit \( N \to \infty \), which sets the upper bound on the integral. Equation (3.5) becomes

\[
N = N_0 + \int_0^{\infty} \frac{\rho(\epsilon) d\epsilon}{\exp(\beta \epsilon) - 1}.
\]

(3.6)

For the harmonic potential (3.1), the density of states is given by [15]

\[
\rho(\epsilon) = \frac{1}{2} \frac{\epsilon^2}{(\hbar \bar{\omega})^3}
\]

(3.7)

where \( \bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3} \) is the geometric mean trapping frequency and we have neglected linear terms which have small contributions for large \( N \) [16]. Solving the integral of (3.6), we obtain

\[
\frac{N_0}{N} = 1 - \frac{\zeta(3)}{N} \left( \frac{k_B T}{\hbar \bar{\omega}} \right)^3
\]

(3.8)

where \( \zeta(n) \) is the Riemann Zeta-function. This can be written as

\[
\frac{N_0}{N} = 1 - \left( \frac{T}{T_C} \right)^3
\]

(3.9)

where \( T_C \), the critical temperature for condensation, is defined by

\[
T_C = \frac{\hbar \bar{\omega}}{k_B \zeta(3)} \left( \frac{N}{\zeta(3)} \right)^{1/3} \approx 0.94 \frac{\hbar \bar{\omega}}{k_B N^{1/3}}.
\]

(3.10)
The results obtained above are a very good approximation, both to more detailed theoretical results and to experimental data [17]. The leading corrections to these results come from two sources: the first is that the thermodynamic limit \( N \to \infty \) is not completely valid for the relevant number of atoms (for the relevant numbers of the order of \( 10^4 \) atoms, finite size corrections of (3.9) and (3.10) are of the order of 5%); the second is that we have completely neglected interatomic interactions. The rigorous treatment of the statistical mechanics of a finite number of trapped, interacting bosons is a complicated theoretical problem [13].

### 3.2 Interactions - mean field theory

The many-body Hamiltonian describing \( N \) interacting bosons confined by an external potential \( V_{\text{ext}} \) (we will later on replace this with the harmonic potential given in (3.1)) is given, in second quantization, by

\[
\hat{H} = \int dr \hat{\Psi}^\dagger (\mathbf{r}) \left( -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\mathbf{r}) \right) \hat{\Psi}(\mathbf{r}) + \frac{1}{2} \int drdr' \hat{\Psi}^\dagger (\mathbf{r}) \hat{\Psi}^\dagger (\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r}),
\]

where \( \hat{\Psi} \) and \( \hat{\Psi}^\dagger \) are the boson field operators that annihilate and create a particle at the position \( \mathbf{r} \), respectively, and \( V(\mathbf{r} - \mathbf{r}') \) is the two-body interatomic potential. As the BEC is a dilute system, only two body interactions are included. For cold, dilute alkali atoms, the interactions are well described by a hard-sphere interaction characterized only by the s-wave scattering length \( a \). This means that we can write an effective interaction potential

\[
V(\mathbf{r} - \mathbf{r}') = g\delta(\mathbf{r} - \mathbf{r}')
\]

where \( g \) is related to the scattering length \( a \) by \( g = 4\pi\hbar^2 a/m \). Although the ground state of the system, as well as its thermodynamic properties, can be directly calculated starting from the Hamiltonian using Monte-Carlo methods, the calculation can become heavy or even impracticable for systems with large values of \( N \) (of the order of \( 10^3 \) atoms). We shall therefore use a mean-field approach where we approximate the Heisenberg representation of the field operator as

\[
\hat{\Psi}(\mathbf{r}, t) = \Phi(\mathbf{r}, t) + \hat{\Psi}'(\mathbf{r}, t).
\]
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where $\Phi(r, t)$, called the “wave function of the condensate”, is defined as the expectation value of the field operator: $\Phi(r, t) = \langle \hat{\Psi}(r, t) \rangle$. Its modulus fixes the condensate density through $n(r, t) = |\Phi(r, t)|^2$, and it has a well-defined phase which describes the first-order coherence. The operator $\hat{\Psi}'(r, t)$ describes the fluctuations around this mean field. Using the Heisenberg equation we get the time evolution of the field operator $\hat{\Psi}(r, t)$,

$$i\hbar \frac{\partial}{\partial t} \hat{\Psi}(r, t) = [\hat{\Psi}, \hat{H}] = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(r) + \int dr' \hat{\Psi}^\dagger(r', t)V(r' - r)\hat{\Psi}(r', t) \right] \hat{\Psi}(r', t).$$

(3.14)

We now replace $\hat{\Psi}$ with the mean-field $\Phi$ and use the “hard-sphere” interaction potential (3.12) to get the so-called “time-dependent Gross-Pitaevskii (GP) equation”,

$$i\hbar \frac{\partial}{\partial t} \Phi(r, t) = \left( -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(r) + g|\Phi(r, t)|^2 \right) \Phi(r, t).$$

(3.15)

This has the form of a non-linear Schrödinger equation. The ground state can now be easily obtained by writing the condensate wave function as $\Phi(r, t) = \phi(r) \exp(-i\mu t/\hbar)$, where $\mu$ is the chemical potential and $\phi$ is real and normalized to the total number of particles, $\int dr \phi^2 = N_0 = N$.

The GP equation becomes

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(r) + g\phi^2(r) \right) \phi(r) = \mu \phi(r).$$

(3.16)

### 3.3 The Thomas-Fermi approximation

For a large number of particles, corresponding to the condition $Na/a_{ho} \gg 1$ in the case of a harmonic potential, where $a_{ho} = \sqrt{\hbar/m\bar{\omega}}$ is the harmonic oscillator length, the kinetic energy term in (3.16) becomes much smaller compared to the interaction energy, and therefore it can be neglected. This is called the Thomas-Fermi approximation. In this limit, equation (3.16) is easily solved and we get that the density profile of the ground state has the form

$$n_{TF}(r) = \phi^2(r) = g^{-1} [\mu - V_{\text{ext}}(r)],$$

(3.17)

in the region where $V_{\text{ext}}(r) > \mu$, and $n_{TF} = 0$ outside. For a harmonic potential this density profile corresponds to an inverted parabola which vanishes at the classical turning points $R_j$ defined by
the condition $V_{ext}(R_j) = \mu$. Therefore, the Thomas-Fermi radius in the $j$-th direction is given by

$$R_j = \frac{1}{\omega_j} \sqrt{\frac{2\mu}{m}}. \quad (3.18)$$

The normalization condition on $n_{TF}(r)$ provides a relation between the chemical potential and the number of particles:

$$\mu = \frac{\hbar}{2} \left( \frac{15 N a}{\sqrt{\mu m}} \right)^{2/5}. \quad (3.19)$$

### 3.4 BEC signatures

After the atomic cloud has been cooled to very low temperatures we want to verify that it has indeed crossed the critical temperature and condensed. This is done by looking for the following signatures that distinguish between a BEC and a thermal (uncondensed) gas.

#### 3.4.1 A sharp peak in the velocity distribution

The first signature of a BEC is the appearance of a sharp peak in the velocity distribution below the critical temperature, as can be seen in figure 3.1. As mentioned before (3.17), condensed atoms in the Thomas-Fermi limit have a parabolic density distribution profile. This density profile is
CHAPTER 3. CHARACTERIZATION OF A BOSE-EINSTEIN CONDENSATE

Figure 3.2: BEC signature: density profiles of the atomic cloud across the condensation threshold, as measured in our experiment. Left: a Gaussian profile corresponding to a thermal gas. Middle: a narrow parabolic peak appears at the center of the wider Gaussian distribution (a bimodal structure), corresponding to a cloud very close to the transition temperature. Right: a parabolic profile corresponding to an almost pure condensate.

substantially different from the Gaussian profile of thermal atoms. As the temperature of the atomic cloud is lowered across the condensation threshold, a narrow parabolic peak appears at the center of the wider Gaussian distribution. This transition is shown in figure 3.2, where the density profile in the left frame fits to a Gaussian (for a thermal cloud), in the right frame it fits to a parabola (for an almost pure condensate) and in the middle frame it fits to a bimodal structure (a sum of a Gaussian and a parabola) for a cloud very close to the transition temperature.

3.4.2 Anisotropic expansion

The dynamics of the atomic cloud after a sudden release of the trap give us considerable information about the properties of the BEC. This includes the temperature of the gas, the release energy, and the velocity distribution. Unlike thermal clouds, once released from the trap, the condensate expands anisotropically. For an axially symmetric, cigar-shaped cloud, in which the size in two axes is much smaller than in the third, corresponding to the condition $\epsilon = \omega_z/\omega_r \ll 1$ ($\omega_z$ and $\omega_r$ being the frequencies in the longitudinal and radial directions, respectively), the expansion, is given by [18]

$$
\rho(\tau) = \rho_0 \sqrt{1 + \tau^2},
$$

$$
z(\tau) = \epsilon^{-1} \rho_0 \left[ 1 + \epsilon^2 \left( \tau \arctan \tau - \ln \sqrt{1 + \tau^2} \right) \right], \quad (3.20)
$$
Figure 3.3: Anisotropic expansion: A series of images with increasing TOF. Each image is normalized to the peak density of the atomic cloud in that image, hence there is no visible reduction of the optical density with the expansion. The anisotropic expansion is apparent. In the trap (not shown) the atomic cloud is elongated horizontally. As it falls the radial (vertical) axis expands faster and after 3 ms the atomic cloud looks round. At longer times it becomes elongated in the radial (vertical) direction. One should note that these images integrate over one of the transverse directions, and in fact the shape of the cloud after expansion is not a cigar anymore, but a “pancake” shape.

where $\tau = \omega_r t$ is a dimensionless time and $\rho_0$ is the initial TF-radius (3.18) in the radial direction.

This solution describes three regimes of the expansion:

- $\tau < 1$: The dimensions of the cloud evolve as a square-root function. This indicates a radial acceleration, where the interaction energy is being converted into kinetic energy.

- $1 < \tau < \epsilon^{-2}$: The radial direction expands linearly while the axial direction does not expand noticeably.

- $\tau > \epsilon^{-2}$: Both radial and axial axes expand linearly with an asymptotic aspect ratio of $z(\tau)/\rho(\tau) = \pi \epsilon^2/2$. 
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In our trap, $\omega_r = 2\pi \times 565$ Hz and $\omega_z = 2\pi \times 45$ Hz, which corresponds to $\epsilon \approx 0.08$. This means that, for our trap, the second regime starts at $t = \omega_r^{-1} \approx 0.28\text{ms}$ and ends at $t = \epsilon^{-2}/\omega_r = \omega_r/\omega_z^2 \approx 45\text{ms}$.

Figure 3.3 shows a series of images taken after different (increasing from left to right) time-of-flight (TOF) periods following the release of the trap. The figure is made out of different stripes taken from different images and put together to create one image. Each image is normalized to the peak density of its own cloud. The main phenomenon observed in this series of images is that the atomic cloud expands mostly in the radial direction (vertical direction of the figure). This anisotropic expansion of the atomic cloud is clear evidence that the atomic cloud is condensed.

3.5 Measurements

3.5.1 Chemical potential measurement

As mentioned before (3.20), for times $t \gg \omega_r^{-1}$, the longitudinal expansion is practically negligible compared to the radial one and therefore, the chemical potential can be obtained using [12]

$$\mu = \frac{1}{2} m v_r^2,$$

(3.21)

where $v_r$ is the radial velocity.

Figure 3.4 (a) shows a measurement of the radial and longitudinal TF radii (TF widths) (3.18) as a function of the expansion time. This data is retrieved from a TOF measurement similar to the one shown in figure 3.3. As can be seen again, the longitudinal expansion is negligible compared to the radial expansion. Fitting the data to a linear line gives us a radial velocity $v_r = 6.21 \pm 0.23\text{mm/sec}$. Putting this into (3.21) gives us a chemical potential of $\mu = h \times (4.2 \pm 0.3)\text{kHz}$. We can compare this to the prediction given in (3.19) which gives $\mu = h \times (3.3 \pm 0.2)\text{kHz}$ (calculated with $N = (3 \pm 0.3) \times 10^4$ atoms).

3.5.2 Transition temperature measurement

Measuring the temperature of the atoms is done by measuring the expansion velocity in a TOF measurement. The RMS expansion velocity (relative to the center of mass of the atomic cloud) in
Figure 3.4: Chemical potential (a) and transition temperature (b) measurements. (a) Radial (blue) and longitudinal (red) TF-widths as a function of the expansion time. The longitudinal expansion is negligible compared to the radial expansion as predicted in (3.20). The linear fits give the expansion velocities. (b) Size of the uncondensed part of the atomic cloud ($\sigma$) squared as a function of the expansion time squared. While for the calculation of the chemical potential we took the radial velocity from the TF parabola fit (a), here we take into account only the width of the Gaussian. The transition temperature is calculated from the linear slope.

The $j$-th direction is obtained by measuring the width of the Gaussian density profile at different times $\sigma^2_{r,j}(t)$ using

$$\sigma^2_{r,j}(t) = \left\langle x^2(t) - \left\langle x(t) \right\rangle^2 \right\rangle = \left\langle (x(t = 0) + v_x t)^2 - \left\langle x(t = 0) \right\rangle + \left\langle v_x \right\rangle t^2 \right\rangle =$$

$$\left\langle x^2(t = 0) \right\rangle - \left\langle x(t = 0) \right\rangle^2 + \left\langle v_x^2 \right\rangle - \left\langle v_x \right\rangle^2 t^2 = \sigma^2_{r,j}(t = 0) + \sigma^2_{v_x} t^2$$  \hspace{1cm} (3.22)

where we have taken the $x$ component for simplicity (taking gravity into account yields the same result). From the equipartition relation we know that

$$\frac{1}{2} m \sigma^2_{v_j} = \frac{1}{2} k_B T.$$ \hspace{1cm} (3.23)

Combining (3.23) into (3.22) gives

$$\sigma^2_{r,j}(t) = \sigma^2_{r,j}(t = 0) + \frac{k_B T}{m} t^2$$ \hspace{1cm} (3.24)

Therefore, plotting $\sigma^2_{r,j}(t)$ as a function of $t^2$ will give a linear relation with the slope being $k_B T/m$.

In order to measure the transition temperature, we cool down the atoms until the condensate starts
Figure 3.5: (a) BEC Lifetime measurement. (b) Density profiles after different times during a BEC lifetime measurement. The ratio between the numbers of condensed and uncondensed atoms changes over time because as the number of trapped particles drops, the transition temperature also drops. (c) heating rate measurement.

to appear. At this stage the density profile fits to a bimodal structure - a sum of a parabola and a Gaussian. We perform a TOF measurement and measure the temperature as described above, taking into account only the width of the Gaussian \( i.e. \) only the atoms in the thermal state. The result is shown in figure 3.4 (b). The difference between the temperatures measured in the \( x \) and \( y \) directions is an effect of the anisotropic expansion due to interactions. The temperature in the \( x \) direction (which is not affected by the anisotropic expansion) gives a critical temperature of \( T_C \approx 325 \pm 30 \text{ nK} \). We can compare this value to the critical temperature for non-interacting particles given in (3.10) which yields \( T_C \approx 350 \pm 20 \text{ nK} \) (calculated with \( N = (3 \pm 0.3) \times 10^4 \) atoms).

### 3.5.3 BEC lifetime measurement

A BEC lifetime measurement is done by keeping the RF evaporative cooling process active, namely, keeping the atomic cloud at a constant temperature, and measuring the number of condensed atoms after different waiting times in the trap. The results are shown in figure 3.5. A BEC lifetime of \( 1.1 \pm 0.3 \) seconds is obtained. Comparing between the density profile of the atomic cloud after short and long waiting times shows that the ratio between the numbers of condensed and uncondensed atoms changes. This seems to be in contrast with (3.9). The reason for the confusion is that the
transition temperature depends on the number of particles: as the number of trapped particles drops, the transition temperature also drops. As time passes the atoms that initially are well below the transition temperature become closer and closer to it and therefore the ratio of the condensed to non-condensed atoms drops.

3.5.4 Heating rate measurement

A heating measurement is done by cooling the atoms to a temperature just above the transition, and performing several temperature measurements (as described in 3.5.2), each measurement after a different waiting time in the trap. The result is shown in figure 3.5. The exact mechanism involved in this heating process is not completely clear, since it can be shown that shaking a perfectly harmonic trap, containing one species of particles, should not cause any heating [19]. More details can be found in chapter 7.
Chapter 4

Spin-Flip Rates - Theory

Trapping of neutral atoms in a magnetic trap is possible only for “low field seekers” states (atoms that are repelled from regions with high magnetic fields) [10]. If the internal atomic state is changed, the atom may be transferred to an un-trapped or even an anti-trapped state (where the atoms are subject to a potential without a local minimum or with a maximum at the trap position, respectively) and will escape from the trap. A change of an atom’s Zeeman sublevel (the \( m_F \) state) that preserves the same hyper-fine state (\( F \) state) is called a “spin flip”. In the absence of optical pumping, spin-flips occur in three processes:

- Majorana spin-flips.
- technical or thermal noise-induced spin-flips.
- collision-induced spin-flips.

4.1 The adiabatic approximation and Majorana spin-flips

The coupling between the atom and the magnetic field is given by the Zeeman interaction

\[
V_Z(r) = -\mu \cdot B(r),
\]

with \( \mu \) being the atoms magnetic moment and \( B(r) \) the local magnetic field at position \( r \). The spin of an atom in a magnetic trap precesses about the direction of the local magnetic field with
the Larmor frequency defined as
\[ \omega_L = \frac{\mu_B g_F}{\hbar} |\mathbf{B}|, \]  
(4.2)
where $\mu_B$ is Bohr’s magnetron and $g_F$ is the appropriate Landé factor. In most magnetic traps (e.g. quadrupole) the field direction varies considerably from side to side, and so as the atom oscillates in the trap with frequency $\omega_T$ so does the quantum axis of the atomic spin (i.e. the local magnetic field). It is assumed that as long as the latter rate of change of the quantum axis (approximated by $\omega_T$) is smaller than the Larmor frequency, the atomic spin manages to follow the local field and the dynamics is adiabatic. Namely, when $\omega_T << \omega_L$, the component $F_z$ of the hyperfine spin $\mathbf{F}$ along the direction of the local field is an approximate constant of the motion. Under these conditions the atom moves in an effective potential [20]
\[ V_{ad}(r) = \mu_B g_F F_z |\mathbf{B}(r)|. \]  
(4.3)

Majorana spin-flips occur due to a breakdown of the adiabatic approximation. When the adiabatic condition is broken, correction terms of the potential cause spin-flip transitions. Notice that because the Larmor frequency is proportional to the magnetic field, the adiabatic condition is always broken in areas where the magnetic field vanishes. In our experiment, the trap frequency varies between about 40Hz and 1kHz. In these circumstances, the approximation is valid as long as the trap bottom is well above 1mG. Unless intentionally lowered, in our magnetic trap we always keep a non-vanishing magnetic field (of the order of 1G) at the bottom of the trap. Therefore, for all practical purposes, the adiabatic approximation is always valid for our trap and Majorana spin-flips are negligible compared to the other spin-flip mechanisms.

### 4.2 Spin-flip rate within the adiabatic approximation

It follows from Fermi’s golden rule [21], that the transition rate from an initial state $|0\rangle$ to a final state $|f\rangle$ of the system, due to fluctuations of the magnetic field around the adiabatic field, is given by
\[ \Gamma_{0\rightarrow f}(r) = \left(\frac{\mu_B g_F}{\hbar}\right)^2 \sum_{i,j} \int d(t-t') e^{i\omega_f(t-t')} \langle 0 | F_i | f \rangle \langle f | F_j | 0 \rangle \langle B_i(r, t) B_j(r, t') \rangle, \]  
(4.4)
where the indices $i, j$ label the Cartesian components, $F_i$ is the magnetic spin operator at the $i$-th direction and $\omega_{f0} = (E_f - E_0)/\hbar$. This integral can be written as

$$\Gamma_{0 \to f}(r) = \left( \frac{gF\mu_B}{\hbar} \right)^2 \sum_{i,j} \langle 0 | F_i | f \rangle \langle f | F_j | 0 \rangle S_{ij}^B(r; -\omega_{f0}),$$

(4.5)

where $S_{ij}$ is the cross-correlation tensor (spectral density) of the magnetic field at position $r$ defined as

$$S_{ij}^B(r; \omega) = \int_{-\infty}^{\infty} d\tau \langle B_i(r, t + \tau)B_j(r, t) \rangle e^{i\omega \tau},$$

(4.6)

or equivalently

$$\langle B_i^*(r, \omega)B_j(r, \omega') \rangle = 2\pi \delta(\omega - \omega')S_{ij}^B(r; \omega).$$

(4.7)

Calculating explicitly this sum for the case of spin-flips (where the states $|0\rangle$ and $|f\rangle$ share the same hyperfine state but with different Zeeman-sublevels) we get that the terms $\langle 0 | F_i | f \rangle$ vanish if $F_i$ is in the direction of the quantization axis. Moreover, only terms where the change of the $m_F$ quantum number between the initial and final states is of one quanta i.e. $\Delta m_F = \pm 1$ do not vanish. Therefore only these kinds of transitions are possible (the $\Delta m_F = \pm 1$ selection rule), and we can restrict the summation only to the components perpendicular to the quantization axis,

$$\Gamma_{Spin-Flips}(r) = \left( \frac{gF\mu_B}{\hbar} \right)^2 \sum_{i,j \in \perp} \langle F; m_F | F_i | F; m_F \pm 1 \rangle \langle F; m_F \pm 1 | F_j | F; m_F \rangle S_{ij}^B(r; -\omega_{f0}).$$

(4.8)

For spin-flips in a magnetic trap, the relevant frequency $\omega_{f0}$ is the Larmor frequency of the atomic spin at the position of the trap (4.2). This frequency is in the RF range for typical traps. We see that in order to calculate the spin-flip rate, all we need is to know the spectral density of the magnetic field fluctuations.

4.3 Technical noise-induced spin-flips

Fluctuations in the driven currents that create the magnetic trap will cause magnetic field fluctuations that may cause spin-flips. In order to know the spin-flip rate (4.8) we have to know the spectral density of the magnetic field fluctuations. Let us assume for the moment that the noise
is caused by current fluctuations in an infinitely long and thin wire along the \( x \) direction, which is also the direction of the quantum axis (as is the case in a Z-trap). For spin-flips, we are only interested in the components of the magnetic field that are perpendicular to the quantum axis. The \( y \) component of the magnetic field produced by this wire can be easily evaluated using the Biot-Savart law. It is given by

\[
B_y(r, t) = \frac{\mu_0}{2\pi} \frac{z}{z^2 + y^2} I(t). \tag{4.9}
\]

Thus, the fluctuations from the DC value are given by

\[
\delta B_y(r, t) = \frac{\mu_0}{2\pi} \frac{z}{z^2 + y^2} \delta I(t), \tag{4.10}
\]

where here we have assumed the following:

\[
B_y(t) = B_{yDC} + \delta B_y(t),
\]
\[
\langle \delta B_y(t) \rangle = 0,
\]
\[
I(t) = I_{DC} + \delta I(t),
\]
\[
\langle \delta I(t) \rangle = 0. \tag{4.11}
\]

From here we can express the correlation between the fields at different times

\[
\langle \delta B_y(t) \delta B_y(t') \rangle = \left( \frac{\mu_0}{2\pi} \right)^2 \frac{z^2}{(z^2 + y^2)^2} \langle \delta I(t) \delta I(t') \rangle. \tag{4.12}
\]

This gives us the following connection between the spectral densities

\[
S_{B_{yy}}^B(\omega) = \left( \frac{\mu_0}{2\pi} \right)^2 \frac{z^2}{(z^2 + y^2)^2} S_I(\omega), \tag{4.13}
\]

where the current spectral density is defined (in analogy to (4.6)) as

\[
S_I(\omega) = \int_{-\infty}^{\infty} d\tau \langle \delta I(t) \delta I(t + \tau) \rangle e^{i\omega\tau}, \tag{4.14}
\]

and is assumed to be independent of the time \( t \). Putting (4.13) into (4.8) we get that the spin-flip rate at a distance \( z \) above the center of the wire (at \( y = 0 \)) is

\[
\Gamma_{Spin-Flips}^{I_{Technical}} = \left( \frac{g_F \mu_B \mu_0}{2\pi \hbar} \right)^2 \frac{\{F; m_F | F_y | F; m_f \pm 1\}^2}{z^2} S_I(\omega_{f_1}). \tag{4.15}
\]
Calculating this explicitly for $F=2$ and the initially trapped states ($m_F=1,2$). We get

$$
\Gamma_{2\rightarrow 1}^{\text{Technical}} = \Gamma_{1\rightarrow 2}^{\text{Technical}} = \left( \frac{g \mu_B B_0}{2\pi \hbar} \right)^2 \frac{1}{2} \frac{1}{z^2} S_f(\omega f_i),
$$

$$
\Gamma_{1\rightarrow 0}^{\text{Technical}} = \left( \frac{g \mu_B B_0}{2\pi \hbar} \right)^{2\beta/2} \frac{1}{2} \frac{3}{z^2} S_f(\omega f_i).
$$

For the (almost) homogeneous bias field produced by large coils, the magnetic field is proportional to the current and independent of position

$$
S_{B}^{yy}(\omega) = C_{\text{Coils}}^2 S_f(\omega),
$$

where $C_{\text{Coils}}$ is the proportionality factor between the current in the coils and the bias magnetic field. Note that if the current fluctuations of the coils are correlated with the current fluctuations of the trapping wires, the total magnetic field fluctuations will be reduced (due to cancellation of the two fields), and the system will suffer from less spin-flips.

### 4.4 Thermally noise-induced spin-flips near Anisotropic Electrical Conductors

Trapped atoms in the vicinity of a room-temperature metallic and/or dielectric material will be subject to heating, decoherence and trap losses, due to thermal fluctuations of the magnetic field. These fluctuations are caused by thermally induced random motion of electrons within the nearby surface (Johnson-Nyquist noise). The topic of thermally induced spin flips above isotropic materials has been thoroughly investigated both theoretically [22] and experimentally [23].

#### 4.4.1 Anisotropic Electrical Conductors

In this work we are interested in measuring the spin-flip rate of trapped atoms in the vicinity of anisotropic electrical conductors (AEC). AECs are crystalline materials that have different conductivities along the different axes of the crystal. In these materials the scalar conductivity $\sigma$ in Ohm’s law $J = \sigma E$ is replaced by a 3x3 (symmetric) matrix. After diagonalization (i.e. choosing
the axes parallel to the crystal symmetry axes), the conductivity takes the form

\[
\sigma = \begin{pmatrix}
\sigma_{xx} & 0 & 0 \\
0 & \sigma_{yy} & 0 \\
0 & 0 & \sigma_{zz}
\end{pmatrix},
\]  

(4.18)

We can distinguish between two types of AECs: 'layered conductance materials' in which the conductivity along two axes is good and the conductivity along the third axis is bad, e.g. \( \sigma_{xx} \approx \sigma_{yy} \gg \sigma_{zz} \), or 'quasi-1D conductors' where there is one axis of good conductivity, e.g. \( \sigma_{xx} \gg \sigma_{yy} \approx \sigma_{zz} \) or \( \sigma_{xx} \gg \sigma_{yy} \gg \sigma_{zz} \). We can define the anisotropy ratio of the conductor by

\[
\tau \equiv \frac{\sigma_{xx}}{\sigma_{zz}},
\]  

(4.19)

where in this definition we assume that the conductivity tensor is diagonal with the high conductivity as \( \sigma_{xx} \) and the low conductivity is \( \sigma_{zz} \).

### 4.4.2 Calculation of the magnetic field spectral density

We will now explicitly calculate the frequency correlation function of the magnetic field in order to obtain the spectral density for the case of thermal noise. This calculation closely follows [24]. We use the quasi-static approximation to neglect radiation effects. This approximation is valid when the skin depth,

\[
\delta = \frac{2}{\sigma \mu_0 \omega f_0},
\]  

(4.20)

is much larger than both the distance \( d \) of the trap from the metal structure and the thickness \( t \) of the metal structure elements. The skin depth for Au at room temperature and a frequency of 1 MHz is about 70 \( \mu m \). AECs typically have higher resistivities and thus much larger skin depths at the same frequencies (e.g. \( \delta=1mm \) for graphite), so for AECs the quasi-static approximation applies for all relevant distances. Our derivation starts from the vector potential frequency cross-correlation function at positions \( \mathbf{r}_1, \mathbf{r}_2 \):

\[
\langle A_i^*(\mathbf{r}_1, \omega) A_j(\mathbf{r}_2, \omega') \rangle.
\]

Using the magnetostatic approximation \( \mathbf{A}(\mathbf{r}, \omega) = \frac{\mu_0}{4\pi} \int d^3 r' \frac{J_i(\mathbf{r}', \omega)}{|\mathbf{r} - \mathbf{r}'|} \), we get

\[
\langle A_i^*(\mathbf{r}_1, \omega) A_j(\mathbf{r}_2, \omega') \rangle = \left( \frac{\mu_0}{4\pi} \right)^2 \int d^3 r' d^3 r'' \frac{\langle J_i^*(\mathbf{r}', \omega) J_j(\mathbf{r}'', \omega') \rangle}{|\mathbf{r}_1 - \mathbf{r}'| |\mathbf{r}_2 - \mathbf{r}''|}.
\]  

(4.21)
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Using the fluctuation-dissipation theorem, the expression for the current cross correlation can be written as

\[ \langle J_i^*(r', \omega) J_j(r'', \omega') \rangle_T = 4 \pi \hbar \varepsilon_0 \omega^2 \bar{n}(\omega) \delta(\omega - \omega') \text{Im} \varepsilon_{ij}(r'; \omega) \delta(r' - r''), \]  

where \( \bar{n}(\omega) = \left( e^{\frac{\hbar \omega}{k_B T}} - 1 \right)^{-1} \) is the Bose-Einstein occupation number, \( \varepsilon_{ij} \) is the dielectric tensor of the conductor and \( \langle \ldots \rangle_T \) denotes thermal averaging. For homogeneous structures: \( \varepsilon_{ij}(\omega) = i \frac{\sigma_i}{\varepsilon_0 \omega} \) where \( \sigma_{ij} \) is the DC conductivity tensor discussed above (4.18). Putting everything back into (4.21) we get

\[ \langle A_i^*(r_1, \omega) A_j(r_2, \omega) \rangle_T = \frac{\mu_0^2 \hbar \omega \bar{n}(\omega)}{4 \pi} \delta(\omega - \omega') \sigma_{ij} \int_V \frac{d^3r'}{|r_1 - r'| |r_2 - r'|}, \]  

where the integral is over the volume of the conductor. In order to calculate the correlation function of the magnetic field fluctuations, we take the curl of the vector potential correlation function, once with respect to \( r_1 \) and once with respect to \( r_2 \). Writing this in tensor form (with summation over repeated indices) we get

\[ \langle B_i^*(r_1, \omega) B_j(r_2, \omega') \rangle_T = \frac{\mu_0^2 \hbar \omega \bar{n}(\omega)}{4 \pi} \delta(\omega - \omega') \varepsilon_{ikl} \varepsilon_{jmn} \sigma_{ln} \delta_{1,k} \delta_{2,m} \int_V \frac{d^3r'}{|r_1 - r'| |r_2 - r'|}, \]  

where the symbol \( \delta_{\alpha,k} (\alpha = 1, 2) \) means a derivative with respect to \( r_\alpha \) in the direction of its \( k \)-th component and we made use of the Levi-Civita symbol, \( \varepsilon_{ikl} \). Taking the derivatives we get

\[ \langle B_i^*(r, \omega) B_j(r, \omega') \rangle_T = \frac{\mu_0^2 \hbar \omega \bar{n}(\omega)}{4 \pi} \delta(\omega - \omega') \varepsilon_{ikl} \varepsilon_{jmn} \sigma_{ln} \int_V \frac{(r - r')_k (r - r')_m d^3r'}{|r - r'|^6} \equiv \]  

\[ \equiv \frac{\mu_0^2 \hbar \omega \bar{n}(\omega)}{2 \pi} \delta(\omega - \omega') \varepsilon_{ikl} \varepsilon_{jmn} \sigma_{ln} X_{km}(r), \]  

where we have made use of the 'geometric integrals', defined by

\[ X_{km}(r) = \frac{1}{2} \int_V \frac{(r - r')_k (r - r')_m d^3r'}{|r - r'|^6}. \]  

Comparing between (4.7) and (4.25) we find the thermal spectral density:

\[ S_{B-T, \text{thermal}}^{ij}(r; \omega) = \frac{\mu_0^2 \hbar \omega}{4 \pi^2} \bar{n} \varepsilon_{ikl} \varepsilon_{jmn} \sigma_{ln} X_{km}(r), \]  

which can be written in a more compact way

\[ S_{B-T, \text{thermal}}^{ij}(r; \omega) = \frac{3c}{4 \pi \varepsilon_0 \omega^2} S_B^{(bb)}(\omega) \sigma_{xx} Y_{ij}(r), \]  

(4.27)
where following [22], we have used the black body spectral density (Planck’s law)

$$S_B^{(bb)}(\omega) = \frac{\hbar \omega^3 n(\omega)}{3\pi^2 c^5}$$

(4.29)

and the tensor

$$Y_{ij} = \varepsilon_{ikl} \varepsilon_{jmn} \frac{\sigma_{ln}}{\sigma_{xx}} X_{km},$$

(4.30)

which in case of a diagonal conductivity tensor (4.18) can be written explicitly as

$$Y = \frac{1}{\sigma_{xx}} \begin{pmatrix} X_{zz}\sigma_{yy} + X_{yy}\sigma_{zz} & -X_{xy}\sigma_{zz} & -X_{xz}\sigma_{yy} \\ -X_{xy}\sigma_{zz} & X_{zz}\sigma_{xx} + X_{xx}\sigma_{zz} & -X_{yz}\sigma_{xx} \\ -X_{xz}\sigma_{yy} & -X_{yz}\sigma_{xx} & X_{yy}\sigma_{xx} + X_{xx}\sigma_{yy} \end{pmatrix},$$

(4.31)

where the fact that $X$ is symmetric has been used. For an isotropic material ($\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$) (4.30) can be written in the form

$$Y_{ij} = \text{tr}\{X_{ij}\} \delta_{ij} - X_{ij},$$

(4.32)

as in [22].

As mentioned before, typically the Larmor frequency is in the RF regime. The noise spectrum is typically rather flat (white) in the range between DC and RF frequencies, hence a low-frequency limit $\omega_{f0} \to 0$ can be taken. Taking this limit together with a high temperature limit of (4.30) we obtain

$$S_{ij}^{\text{Thermal}}(r; \omega \to 0) = \frac{\mu_0^2 k_B T \sigma_{xx}}{4\pi^2} Y_{ij}(r).$$

(4.33)

Returning to the calculation of the spin-flip rate, putting (4.33) into (4.8) we get

$$\Gamma_{\text{Spin-Flips}}^{\text{Thermal}}(r) = \left(\frac{\mu_0 g F H B}{2\pi \hbar}\right)^2 k_B T \sigma_{xx} \sum_{i,j\in\perp} \langle F; m_F | F_i | F; m_F \pm 1 \rangle \langle F; m_F \pm 1 | F_j | F; m_F \rangle Y_{ij}(r).$$

(4.34)

This is the same expression as in the isotropic case; however the anisotropy is contained within the $Y_{ij}$ tensor.

### 4.4.3 The Geometric integrals $X_{ij}$

We see from (4.34) that in order to calculate the thermal spin-flip rate, we need to know the elements of the tensor $Y_{ij}$, or equivalently the geometric integrals tensor $X_{ij}$. As its name suggests,
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the geometric integrals tensor expresses the dependence of the spin-flip rate on the geometry of the
system. For different material geometries we will get different tensors. I have calculated explicitly
the geometric integrals $X_{ij}$ (4.26) at position $\mathbf{r} = (x, y, z)$ for the specific case of a rectangular piece
that lies between the coordinates $((x_0, x_1), (y_0, y_1), (z_0, z_1))$. For the diagonal elements, one gets,
for example (we get the other diagonal elements $X_{yy}$ and $X_{zz}$ by the appropriate exchange, $x \leftrightarrow y$
or $x \leftrightarrow z$)

$$X_{xx}(\mathbf{r}) = -\frac{1}{16} \left[ \begin{array}{ccc}
\frac{\arctan \left( \frac{x'}{\sqrt{x'^2 + y'^2}} \right) [x'^2 + 2y'^2]}{x'} & \frac{\arctan \left( \frac{y'}{\sqrt{y'^2 + z'^2}} \right) [y'^2 + 2z'^2]}{y'} & \frac{\arctan \left( \frac{z'}{\sqrt{z'^2 + x'^2}} \right) [z'^2 + 2x'^2]}{z'} \\
\end{array} \right]$$

For the off-diagonal elements, we get, for example

$$X_{xy}(\mathbf{r}) = \frac{1}{16} \left[ \begin{array}{ccc}
\frac{\arctan \left( \frac{x'}{\sqrt{x'^2 + y'^2}} \right) [x'^2 + 2y'^2]}{x'} & \frac{\arctan \left( \frac{y'}{\sqrt{y'^2 + z'^2}} \right) [y'^2 + 2z'^2]}{y'} & \frac{\arctan \left( \frac{z'}{\sqrt{z'^2 + x'^2}} \right) [z'^2 + 2x'^2]}{z'} \\
\end{array} \right]$$

(4.36)

Wire geometry

I have now examined what happens for simpler geometries: if we consider a material very long in
the $x$ direction we can calculate the 'parallel to wire geometric integral' - $X_{xx}$, by taking the limits
$x_0 \to -\infty, x_1 \to \infty$ of (4.35). We get

$$X_{xx}(\mathbf{r}) = -\frac{\pi}{16} \left[ \begin{array}{ccc}
\frac{\sqrt{y'^2 + z'^2}}{y'} & \frac{y' - y}{y'} & \frac{z' - z}{z'} \\
\end{array} \right]$$

(4.37)

We can also calculate the 'perpendicular to wire geometric integrals' - $X_{yy}$ and $X_{zz}$. In this case
(for example for $X_{yy}$) we shall again take the limits $x_0 \to -\infty, x_1 \to \infty$ of (4.35), but only after
the exchange $x \leftrightarrow y$, to get

$$X_{yy}(\mathbf{r}) = -\frac{\pi}{16} \left[ \begin{array}{ccc}
\frac{\sqrt{y'^2 + z'^2}}{y'} & \frac{y' - y}{y'} & \frac{z' - z}{z'} \\
\end{array} \right]$$

(4.38)
Figure 4.1: (a) The calculated spin-flip rate (for the $m_F = 2 \rightarrow 1$ transition) of an atom at a distance $d$ above the center of a thin isotropic wire of width $W$, as a function of $W/d$. Blue: wire parallel to the quantization axis. Purple: wire perpendicular to quantization axis. Yellow: spin-flip rate for an infinite surface, calculated with the approximation of eq. (4.45). (b) The calculated spin-flip rate of an atom above the center of a thin isotropic wire, as a function of $\alpha$, the angle between the wire and the quantization axis, calculated for $W/d = 0.1$. A $50 \times 50$ nm$^2$ gold wire (resistivity of $\rho \approx 22$ nΩ · m) gives a spin-flip rate of $\gamma_{21} \approx 0.25$Hz.

In the case of the off-diagonal elements 'wire limits': we take the limits $x_0 \rightarrow -\infty, x_1 \rightarrow \infty$ of (4.36) (after the appropriate exchange if needed) and get $X_{xy} = X_{xz} = 0$ due to symmetry, while for $X_{yz}$ we get

$$X_{yz} = \frac{\pi}{16} \left[ \frac{1}{\sqrt{y'^2 + z'^2}} \right]_{y' = y_1 - y}^{y' = y_0 - y} \quad \left[ z' = z_1 - z \right]_{z' = z_0 - z}.$$  \hspace{1cm} (4.39)

We see that the geometric integrals give different results depending on whether the wire is elongated along the 'axis of the integral' or along a perpendicular axis. This will of course give differences also in the $Y_{ij}$ tensor. We saw earlier that in the calculation of the spin-flip rate (4.34), only the elements perpendicular to the quantization axis contribute. This means that, for a given quantization axis and a given wire, we will get different spin-flip rates, depending on whether the wire’s direction is parallel or perpendicular to the quantization axis. This can be seen in figure 4.1 (a) where we compare between the calculated spin-flip rates of the wire for both of these cases as a function of $W$ - the width of the wire. For both cases, as $W \rightarrow \infty$ the asymptotic value corresponds to the expression of an infinite surface calculated below. In order to calculate the spin-flip rates
for wires in intermediate angles, we should multiply the $Y_{ij}$ tensor with the appropriate rotation matrices. The calculated spin-flip rate of an atom above the center of a thin wire as a function of $\alpha$-the angle between the wire and the quantization axis is shown in figure 4.1 (b).

**Surface geometry**

We now consider an infinite surface geometry in the $x$-$y$ plane. For the 'parallel to surface geometric integrals', we take the limits $y_0 \to -\infty, y_1 \to \infty$ of (4.37) and get

$$X_{xx}(r) = \frac{\pi(z_1 - z_0)}{8(z_1 - z)(z_0 - z)}.$$  \hspace{1cm} (4.40)

For a thin layer $(z_1 - z_0 \ll z)$ we get

$$X_{xx} \approx \frac{\pi t}{8d^2},$$  \hspace{1cm} (4.41)

where $t$ is the thickness of the layer and $d$ is the distance to the surface. On the other hand, for the 'perpendicular to surface geometric integral' we take the limits $y_0 \to -\infty, y_1 \to \infty$ of (4.38) to get

$$X_{zz} \approx \frac{\pi t}{4d^2}.$$  \hspace{1cm} (4.42)

For summary, for a trap above a thin infinite surface in the $x$-$y$ plane we get

$$X \approx \begin{pmatrix} \frac{\pi t}{8d^2} & 0 & 0 \\ 0 & \frac{\pi t}{8d^2} & 0 \\ 0 & 0 & \frac{\pi t}{4d^2} \end{pmatrix}.$$  \hspace{1cm} (4.43)

Putting this into (4.30) we get

$$Y \approx \frac{\pi t}{8d^2\sigma_{xx}} \begin{pmatrix} 2\sigma_{yy} + \sigma_{zz} & -2\sigma_{xy} & -\sigma_{xz} \\ -2\sigma_{xy} & 2\sigma_{xx} + \sigma_{zz} & -\sigma_{yz} \\ -\sigma_{xz} & -\sigma_{yz} & \sigma_{xx} + \sigma_{yy} \end{pmatrix}.$$  \hspace{1cm} (4.44)

And for a diagonal conductivity tensor (4.18) we get

$$Y \approx \frac{\pi t}{8d^2\sigma_{xx}} \begin{pmatrix} 2\sigma_{yy} + \sigma_{zz} & 0 & 0 \\ 0 & 2\sigma_{xx} + \sigma_{zz} & 0 \\ 0 & 0 & \sigma_{xx} + \sigma_{yy} \end{pmatrix}.$$  \hspace{1cm} (4.45)
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For the isotropic case \((\sigma_{xx} = \sigma_{yy} = \sigma_{zz})\), putting this expression into (4.34) gives the same asymptotic value for \(W \to \infty\) as the calculation using equations (4.37) and (4.38). This can be seen in figure 4.1 (a).

### 4.4.4 Rotation of the anisotropic crystal

In the previous section we saw that for an isotropic material of non symmetric geometry in \(x\) and \(y\) (e.g. a wire with length much larger than width) a rotation in the \(x\)-\(y\) plane can alter the \(Y\) elements such that the spin flip rate is altered. Here we show that for a non isotropic material, even a symmetric geometry (e.g. a square) will alter the spin flip rate when rotated.

Examining (4.44) together with (4.34) for the anisotropic case, we see that, assuming our quantization axis is in the \(x\) direction, in order to have a low spin-flip rate we want \(Y_{yy}\) and \(Y_{zz}\) to be as low as possible. Let us think for example that we have a given 'layered conductance material' (good conductivity along two directions and a bad conductivity along the third axis), if we align our material in a way that the bad conductivity is along the quantization axis (i.e. \(\sigma_{xx} \ll \sigma_{yy} \sim \sigma_{zz}\)), then the spin-flip rate of our system will be relatively low. If, on the other hand, we rotate our conductor (or equivalently rotate the quantization axis) by 90 degrees, so that now the bad conductivity is along the \(y\) axis (i.e. \(\sigma_{yy} \ll \sigma_{xx} \sim \sigma_{zz}\)), then the spin-flip rate will become significantly higher. If the lifetime of a trap is dominated by the rate of thermally induced spin-flips, then the first configuration will give longer lifetimes compared to the second. Thus, we see that for a given anisotropic large surface in the \(x\)-\(y\) plane, the spin-flip rate depends on the angle between the quantization axis (defined by the local magnetic field) and the symmetry axes of the conductivity tensor. Let us examine this dependence in details: if we rotate the AEC by an angle \(\theta\) in the \(x\)-\(y\) plane, the conductivity tensor from (4.18) should be multiplied by the appropriate rotation matrices (notice that in the general case, this is not equivalent to multiplying the \(Y_{ij}\) tensor by the rotation matrices, because the later rotates also the geometry of the material. In the case of an infinite surface, both multiplications are equivalent). The new representation of
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The calculated spin-flip rate of an atom above the center of a 'layered AEC' surface (with an anisotropy ratio of $r = 3750$), as a function of $\theta$ the angle between the bad conductivity axis and the quantization axis.

The conductivity tensor will be

$$
\sigma = \begin{pmatrix}
\sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta & \cos\theta\sin\theta(\sigma_{xx} - \sigma_{yy}) & 0 \\
\cos\theta\sin\theta(\sigma_{xx} - \sigma_{yy}) & \sigma_{yy}\cos^2\theta + \sigma_{xx}\sin^2\theta & 0 \\
0 & 0 & \sigma_{zz}
\end{pmatrix}.
$$

The dependence of the resulting spin flip rate on $\theta$ is shown in figure 4.2.

Figure 4.2: The calculated spin-flip rate of an atom above the center of a 'layered AEC' surface (with an anisotropy ratio of $r = 3750$), as a function of $\theta$ the angle between the bad conductivity axis and the quantization axis.

To conclude, we have theoretically analyzed the affects of background and Johnson noise on the spin of the atom. In the latter case, we have shown that both the geometry and the structure of the conductor may impact the spin flip rate of the atom. This theory applies also with minor modifications to heating and decoherence due to noise. In the following chapter we present measurements of spin changing transitions due to background noise, and in the chapter after that we present a feasibility study of an experiment in which the theory relating to Johnson noise is tested.
Chapter 5

Observation of Asymmetric Spin-Flip Transitions in a Magnetic Trap

5.1 Spin-flip lifetime and rate equations

In the process of loading the atoms to a magnetic trap, we optically pump the atoms to the \(|F = 2, m_F = 2\rangle\) trapped state. According to the selection rules of the \(m_F\) quantum number, only transitions with \(\Delta m_F = \pm 1\) are allowed. A trapped atom with \(m_F = 2\) can flip its spin to \(m_F = 1\), but it cannot perform a direct transition to any of the \(m_F = 0, -1, -2\) states. The \(m_F = 1\) state is also a trappable state, but \(m_F = 0\) is not. Therefore in order to escape from the trap an atom needs to perform 2 transitions: from \(m_F = 2\) to \(m_F = 1\), and then from \(m_F = 1\) to \(m_F = 0\). We shall assume that atoms that have reached the \(m_F = 0\) state immediately escape from the trap (this assumption will be discussed later in section 5.6). This process raises the question: what is the total lifetime of the atoms? In reference [25], the total lifetime of the trapped states due to magnetic noise was approximated to be \(\tau_{\text{mag}} \approx \tau_{1 \rightarrow 0} + \tau_{2 \rightarrow 1}\), where \(\tau_{1 \rightarrow 0}\) and \(\tau_{2 \rightarrow 1}\) are equal to the inverse of the transition rates \(\gamma_{1 \rightarrow 0}\) and \(\gamma_{2 \rightarrow 1}\), respectively (\(\gamma_{i \rightarrow j}\) being the single atom transition rate from level \(i\) to \(j\)). A more complete approach should take into account that an atom initially at the \(|2, 2\rangle\) level, that has spin-flipped to the \(|2, 1\rangle\) level, might, at a certain probability make the opposite transition back to the \(|2, 2\rangle\) level, thereby remain trapped. So, we expect the actual lifetime to be
somewhat larger than this approximated lifetime. In order to get a quantitative estimation for this lifetime, we shall start by examining the rate equations that describe the system.

### 5.1.1 Rate equations

The rate equations that describe the dynamic occupation of the two trappable states are

\[
\begin{align*}
\frac{d}{dt} N_1(t) & = \gamma_{2\rightarrow1} N_2 - (\gamma_{1\rightarrow2} + \gamma_{1\rightarrow0}) N_1, \\
\frac{d}{dt} N_2(t) & = -\gamma_{2\rightarrow1} N_2 + \gamma_{1\rightarrow2} N_1.
\end{align*}
\]

where \( N_i \) is the number of atoms in the \( m_F = i \) level. We shall use the following definitions

\[
\begin{align*}
\alpha & = \frac{\gamma_{1\rightarrow0}}{\gamma_{2\rightarrow1}}, \\
\beta & = \frac{\gamma_{1\rightarrow2}}{\gamma_{2\rightarrow1}}, \\
\gamma & = \gamma_{2\rightarrow1}.
\end{align*}
\]

We shall assume that \( \alpha, \beta \) and \( \gamma \) are constant in time. Using these definitions, (5.1) and (5.2) can be written as

\[
\begin{align*}
\frac{d}{dt} N_2(t) & = -\gamma_{2\rightarrow1} (N_2(t) - \beta N_1(t)), \\
\frac{d}{dt} N_1(t) & = \gamma_{2\rightarrow1} N_2(t) - \gamma_{1\rightarrow2} (\beta + \alpha) N_1(t).
\end{align*}
\]

The following initial conditions will be used

\[
\begin{align*}
N_1(t = 0) & = R_0 N_{tot}(t = 0), \\
N_2(t = 0) & = (1 - R_0) N_{tot}(t = 0),
\end{align*}
\]

where \( N_{tot}(t) = N_1(t) + N_2(t) \) and \( R_0 \) is the initial ratio of atoms in the level 1. Equations (5.8) and (5.9) are easily solved analytically. The general solution is

\[
N_1(t) = N_{tot}(t = 0) \left[ \frac{1}{2\sqrt{(1+\alpha+\beta)^2 - 4\alpha}} e^{-\frac{1}{2}(1+\alpha+\beta + \sqrt{(1+\alpha+\beta)^2 - 4\alpha}) \gamma t} \times \right.
\]
\[
\left. \times (R_0(1 + \alpha + \beta + \sqrt{(\alpha - 1)^2 + 2(1+\alpha)\beta + \beta^2}) - \\
-2 + e^{\sqrt{\alpha^2 + 2\alpha(\beta - 1) + (1+\beta)^2} \gamma t} (2 - R_0(1 + \alpha + \beta) + \\
+ R_0 \sqrt{(\alpha - 1)^2 + 2(1+\alpha)\beta + \beta^2})) \right].
\]
Figure 5.1: Solution of the rate equations: (a) Number of atoms in the $m_F = 1$ level (blue), $m_F = 2$ level (purple), and total number of atoms (yellow) as a function of time. Calculated for $\gamma = 1$ Hz, $\beta = 1$, $\alpha = 3/2$ and $R_0 = 0$. (b) The function $R(t)$ as a function of time for different initial conditions. The independence of the asymptotic value on the initial conditions is clearly seen. The asymptotic value of 1/3 is with agreement to (5.13).

We define

$$R(t) = \frac{N_1(t)}{N_{\text{tot}}(t)}.$$  \hfill (5.12)

Calculating the asymptotic value of this expression we get

$$R_\infty(\alpha, \beta) \equiv \lim_{t \to \infty} R(t) = \frac{1}{2\alpha} \left( 1 + \alpha + \beta - \sqrt{(1 + \alpha + \beta)^2 - 4\alpha} \right).$$  \hfill (5.13)

This result is independent of $\gamma$ and the initial conditions. With this definition we can write the result of (5.12) as

$$R(t) = \frac{R_\infty - Ce^{-\gamma t}}{1 - \alpha R_\infty Ce^{-\gamma t}},$$  \hfill (5.14)

where $C = (R_0 - R_\infty)/(\alpha R_0 R_\infty - 1)$ and $\epsilon = 1/R_\infty - \alpha R_\infty$.

The functions $N_1(t)$, $N_2(t)$, $N_{\text{tot}}(t)$ and $R(t)$ are plotted for demonstration for specific values of the different parameters in figure 5.1.
Examining equation (5.11) in detail, we see that this equation contains a sum of two decaying exponentials with characteristic times of

\[ \tau_1 = \frac{2}{(1+\alpha+\beta+(1+\alpha+\beta)^2-4\alpha)\gamma} \]

and

\[ \tau_2 = \frac{2}{(1+\alpha+\beta-(1+\alpha+\beta)^2-4\alpha)\gamma} \]

This behavior of two decaying exponentials comes from the fact that atoms are lost only from level 1. For the system to lose atoms at a constant rate (i.e. with one decaying exponential), it should first populate level 1. Only after level 1 has reached the steady state occupation ratio, \( R_\infty \), which takes a time of \( \sim \tau_1 \), the system will decay at a constant rate. We can see this fact very nicely if at \( t = 0 \) the system is already at the steady state occupation ratio. Putting the expression of \( R_\infty \) (5.13) instead of \( R_0 \) in (5.11) we get

\[ N_{\text{tot}}(t,R_0 = R_\infty) = \exp(-t/\tau_2) \]

i.e. we are left only with the \( \tau_2 \) exponential. In the general case, we see that \( \tau_2 > \tau_1 \), meaning that the exponential of \( \tau_1 \) decays relatively fast. Therefore, after a short time we are left with a long term trap lifetime of \( \tau_2 \). It should not be surprising that \( \tau_2 = (\alpha \gamma R_\infty)^{-1} \). This can be seen by summing the two equations of (5.7) to get

\[ \dot{N}_{\text{tot}} = -\alpha \gamma N_1(t). \]  

When the system has reached the asymptotic steady state, \( N_1 = R_\infty N_{\text{tot}} \) and therefore we get

\[ \dot{N}_{\text{tot}} = -\alpha \gamma R_\infty N_{\text{tot}}(t), \] 

with an exponential solution with the characteristic time \( \tau_2 \). To conclude, if we know the transition rates (i.e. we know \( \alpha, \beta \) and \( \gamma \)) the lifetime of the trap is given by

\[ \tau = (\alpha \gamma R_\infty)^{-1} = (\gamma_{1 \rightarrow 0} R_\infty)^{-1}. \]

This result is quite interesting because even if \( \gamma \) is big, we can still have a long lifetime if \( \alpha R_\infty \) is small.

### 5.2 The values of \( \alpha \) and \( \beta \) for thermal noise

In chapter 4 we have derived expressions for the rate of thermal noise induced spin-flips (4.34). The magnitude of the magnetic field at the bottom the magnetic trap (which is usually referred as the “Ioffe-Pritchard (IP) field”) is typically of the order of one Gauss, thus, the spin-flip transition frequency is of the order of 1MHz while the temperature of the surface (\( \sim 300 \)K) corresponds to a
frequency several orders of magnitude larger. Therefore, the DC limit taken in (4.33) is acceptable.

This suggests that concerning thermal noise, the spectral density of the noise (4.6) can be regarded as flat (white noise, $S_{ij}(\omega) = \text{const}$). In this case we can calculate explicitly the values of $\alpha$ and $\beta$ using (4.34) for $i,j=y,z$:

$$\alpha = \frac{\gamma_1 \to 0}{\gamma_2 \to 1} = \frac{\left\langle 2,1 \right| F_i \left| 2,0 \right\rangle \left\langle 2,0 \right| F_j \left| 2,1 \right\rangle}{\left\langle 2,2 \right| F_i \left| 2,1 \right\rangle \left\langle 2,1 \right| F_j \left| 2,2 \right\rangle} = \frac{3}{2},$$

$$\beta = \frac{\gamma_1 \to 2}{\gamma_2 \to 1} = \frac{\left\langle 2,1 \right| F_i \left| 2,2 \right\rangle \left\langle 2,2 \right| F_j \left| 2,1 \right\rangle}{\left\langle 2,2 \right| F_i \left| 2,1 \right\rangle \left\langle 2,1 \right| F_j \left| 2,2 \right\rangle} = 1,$$

(5.18)

(the elements of the $F$ matrices can be calculated easily using $F_x = (F_+ + F_-)/2$, $F_y = (F_+ - F_-)/2i$, where $F_{\pm} |F,m_F\rangle = \hbar \sqrt{F(F+1) - m_F(m_F \pm 1)} |F,m_F \pm 1\rangle$). Putting $\alpha = 3/2$ and $\beta = 1$ into (5.13) gives us the asymptotic value for white (thermal) noise of 1/3. Returning to the result of (5.17), we get that the lifetime of atoms under the influence of white noise is $\tau = 3 \gamma_1 \to 0^{-1}$.

The meaning of $\beta = 1$ is that the atoms have equal probabilities for the transition $|2,2\rangle \to |2,1\rangle$ and vice versa. We shall regard the spin-flip rates in this case as being symmetric. A case where $\beta \neq 1$ would represent a symmetry breaking of the transition rates. This would mean that a process which is not time reversal is involved.

### 5.3 The values of $\alpha$ and $\beta$ - general case

In the general case, the spectral density of the noise is not flat. Nevertheless, if we assume that all the transition energies are equal (i.e. $E_{2\to 1} = E_{1\to 2} = E_{1\to 0}$), then as in the case of white noise, when calculating $\alpha$ and $\beta$, the spectral density terms of (4.8) cancel out and consequently we get the same values as in (5.18) and the same asymptotic value of 1/3.

### 5.3.1 Transition rates - the effect of temperature

In practice, the transition energies are only approximately equal. Atoms at different positions may experience different transition rates according to the local transition energy. In figure 5.2 we plot the one dimensional potentials of the two trapped states, ignoring gravity and the second order Zeeman shift. Let us assume that the atoms in the two states are at the same temperature (this
assumption will be justified later in section 5.3.3), and consequently their mean potential energy is $\frac{1}{2} k_B T$ above their potential trap bottom. For the sake of this simplified explanation, we take an atom positioned at the mean distance from the trap center, $d_i \equiv \sqrt{\langle x^2 \rangle_i}$. An atom in the $m_F = 2$ state undergoing spin flip to $m_F = 1$, requires stimulated emission with a photon energy $E_{2 \rightarrow 1} = V_2(d_2) - V_1(d_2)$, where $V_i$ is the potential experienced by an atom in level $i$. When the atom reaches the $m_F = 1$ level, it thermally equilibrates and its new mean distance is $d_1 > d_2$. In order to flip its spin to $m_F = 2$ or $m_F = 0$, the atom needs to absorb or emit a photon with energy $E_{1 \rightarrow 2} = E_{1 \rightarrow 0} > E_{2 \rightarrow 1}$. The two transitions sample different frequencies of the noise spectrum, giving rise to the possibility of asymmetry in the transition rates.

In order to calculate the mean transition rate of the entire atomic cloud we integrate the local spin-flip rates at different positions according to the spatial distribution of the cloud [26]:

$$\gamma_{i \rightarrow j} = \int_{-\infty}^{\infty} d\omega \int d^3r P_i(r) \Gamma_{i \rightarrow j}(\omega) \delta(V_i(r) - V_j(r) + \hbar \omega),$$

(5.19)

where $P_i(r) \propto \exp(-V_i(r)/k_B T)$ is the Maxwell-Boltzmann distribution and $\Gamma_{i \rightarrow j}(\omega)$ is the single atom transition rate, given in (4.8). We shall use a quadratic approximation for the magnitude of the magnetic field:

$$|B(x)| = B_0 + \frac{1}{2\mu} m \sum_{q=x,y,z} \omega_q^2 r_q^2$$

(5.20)
where \( \omega_q \) is the frequency experienced by the \( m_F = 1 \) state in the \( q \)-th direction, and \( \mu = \mu_B g_F \).

If gravity and non-linear Zeeman (NLZ) terms are neglected, the potentials are given by

\[
V_i(r) = m_i \left[ \mu B_0 + \frac{1}{2} m \sum_{q=x,y,z} \omega^2 q r^2_q \right]. \tag{5.21}
\]

In order to solve the integral of (5.19) we make the transformation

\[
\sqrt{\frac{m}{2k_B T}}(\omega_x x, \omega_y y, \omega_z z) = (u, v, w), \tag{5.22}
\]

\[
\rho^2 = u^2 + v^2 + w^2, \tag{5.23}
\]

then \( P_i(\rho) \propto \exp(-m_i \rho^2) \). We get

\[
\gamma_{i \rightarrow j} = 4m_i \sqrt{\frac{m_i}{\pi}} \int_0^\infty d\rho \rho^2 \exp[-m_i \rho^2] \Gamma_{i \rightarrow j}(\omega = \mu B_0/h + \rho^2 k_B T/h). \tag{5.24}
\]

We note that no matter what is the spectral density, in the linear Zeeman regime, we have the relation \( \alpha = \frac{3}{2} \beta \). This is due to the fact that the transition energies depend only on the initial state and not on the final one, and therefore (5.18) still holds.

### 1/f noise

Now suppose that the spectral density is a Lorentzian, \( S(\omega) \propto 1/(\omega^2 + \delta^2) \), then we have

\[
\gamma_{i \rightarrow j} = \Gamma_{i \rightarrow j}(\omega = \mu B_0/h)4m_i \sqrt{\frac{m_i}{\pi}} \int_0^\infty d\rho \rho^2 \exp[-m_i \rho^2] \frac{\mu^2 B_0^2 + \delta^2}{(\mu B_0 + \rho^2 k_B T)^2 + \delta^2}. \tag{5.25}
\]

The integral can be performed analytically in the limit where \( \mu B_0 \gg h\delta \), such that \( \delta \) can be replaced with 0 (typical for 1/f noise).

\[
\gamma_{i \rightarrow j} = \Gamma_{i \rightarrow j}(\omega = \mu B_0/h)4m_i \sqrt{\frac{m_i}{\pi}} \int_0^\infty d\rho \rho^2 \exp[-m_i \rho^2] \frac{1}{(1 + \rho^2/\eta)^2}, \tag{5.26}
\]

where \( \eta = \mu B_0/k_B T \) is the energy splitting at the bottom of the trap in units of thermal energy.

It follows that the ratio between the transition rates, \( \beta = \gamma_{1 \rightarrow 2}/\gamma_{2 \rightarrow 1} \), is given by

\[
\beta(\eta) = \frac{e^{\eta} \sqrt{\pi}(1 + 2\eta)\text{erfc}(\sqrt{\eta}) - 2\sqrt{\eta}}{2e^{2\eta} \sqrt{2\pi}(1 + 4\eta)\text{erfc}(\sqrt{2\eta}) - 8\sqrt{\eta}}. \tag{5.27}
\]

This function, plotted in figure 5.3, tends to 1 when \( \eta \to \infty \) and to \( 1/2\sqrt{2} \) when \( \eta \to 0 \).
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Figure 5.3: $\beta$ as a function of $\eta = \mu B_0/k_B T$ for $1/f$ noise. As temperature rises (or $B_0$ becomes smaller) the value of $\beta$ becomes different from 1, meaning that the asymmetry of the transition rates becomes larger.

5.3.2 Transition rates - the effect of gravity

Figure 5.2 shows the two potentials experienced by the atoms as being concentric. This is true for the $x$ and $y$ directions. In the $z$ direction, the centers of the trap for $m_F = 1$ and $m_F = 2$ levels are separated due to gravity. This is because gravity affects both of the states in the same way, while the magnetic force applies differently. If the atoms are very cold, the atomic cloud can become separated spatially according to the different states. This effect will cause the asymmetry between the two levels to become more severe. Gravity may be neglected if $mg^2/\omega_z^2 \ll k_B T$ i.e. the gravitational energy is much smaller than the thermal energy.

The full treatment starts with the potential

$$V_i(r) = m_i \left[ \mu B_0 + \frac{1}{2} m \sum_{q=x,y,z} \omega_q^2 r_q^2 \right] + mgz. \quad (5.28)$$

This can be written as

$$V_i(r) = m_i \mu B_0 + \frac{1}{2} mm_i(\omega_z^2 x^2 + \omega_y^2 y^2) + \frac{1}{2} mm_i \omega_z^2 (z + \frac{g}{m_i \omega_z^2})^2 - \frac{mg^2}{2m_i \omega_z^2}. \quad (5.29)$$

After the transformation given in (5.23), we get

$$P_i(r) \propto \exp[-m_i(u^2 + v^2 + (w - w_i)^2)], \quad (5.30)$$
where \( w_i = -\frac{\sqrt{m/2k_BT}}{m_iw_i} \). Calculating the integral (5.19) in spherical coordinates \((\rho, \theta, \varphi)\), (with \( w = \rho \cos \theta \)), \( P_i(r) \) is written as

\[
P_i(r) \propto \exp[-m_i\rho^2 - 2m_iw_i\rho \cos \theta] \exp(-m_iw_i^2),
\]

(5.31)

and the integral becomes

\[
\gamma_{i \rightarrow j} = 4m_i\sqrt{m_i/\pi} \pi \int_0^\infty d\rho \rho F(2m_iw_i\rho) \exp[-m_i(\rho^2 + w_i^2)] \sinh(2m_iw_i\rho) \Gamma_{i \rightarrow j}(\omega = \mu B_0/h + \rho^2k_BT/h),
\]

(5.32)

with \( F \) being the result of the angular integral:

\[
F(x) = \frac{1}{2} \int_0^\pi d\theta \sin \theta \exp[x \cos \theta] = \frac{\sinh x}{x}
\]

(5.33)

We obtain the following result for \( \gamma_{i \rightarrow j} \), which generalizes equation (5.24) for the case with gravity

\[
\gamma_{i \rightarrow j} = \frac{2}{w_i} \sqrt{m_i/\pi} \int_0^\infty d\rho \rho \exp[-m_i(\rho^2 + w_i^2)] \sinh(2m_iw_i\rho) \Gamma_{i \rightarrow j}(\omega = \mu B_0/h + \rho^2k_BT/h),
\]

(5.34)

We see both from (5.34) and from (5.24) that the spin-flip rate \( \gamma_{i \rightarrow j} \) depends only on the initial state \( i \) and not on the final state \( j \). This means that the relation \( \alpha = \frac{3}{2} \beta \) (eq. (5.18)) still holds.

5.3.3 Transition rates - corrections due to the non-linear Zeeman effect

If the magnetic field at the trap bottom \( (B_0) \) is relatively strong, corrections due to the non-linearity of the Zeeman effect should be taken into account. In this case (5.34) should be written in a more general form:

\[
\gamma_{i \rightarrow j} = \frac{2}{w_i} \sqrt{m_i/\pi} \int_0^\infty d\rho \rho \exp[-m_i(\rho^2 + w_i^2)] \sinh(2m_iw_i\rho) \Gamma_{i \rightarrow j}(E_{ij}/h + \rho^2k_BT/h),
\]

(5.35)

where we have replaced the transition energy at the magnetic field minimum, \( \mu B_0 \), by a more general term, \( E_{ij}^0 \) and the effect of second order Zeeman shift on the trapping frequencies has been neglected. For a transition resonance frequency of \( \approx 18\text{MHz} \), as in our case, \( E_{ij}^0 \) should be calculated using the Breit-Rabi formula [3]. Notice that now, when non-linear effects come into play, the transition rate, \( \gamma_{i \rightarrow j} \), depends not only on the initial state, \( i \), but also on the final state, \( j \). This means that, in this case, the relation \( \alpha = \frac{3}{2} \beta \) is no longer valid.
Justification of assumptions

In the calculations described above, we made use of the following assumptions:

- **The magnitude of the magnetic field is approximately quadratic:** This assumption is valid if the atoms are cold enough, so that effectively they are trapped inside the harmonic range of the trap (the anharmonicity of the atomic cloud is low, see chapter 7). In the experiment, we raise the IP-field ($B_{ip}$) of the magnetic trap to relatively high values ($\sim 26\text{G}$). This means that the harmonic range of the trap (7.8) is very large. Combining this with the fact that the atoms are very cold (below 1 $\mu\text{K}$) gives us a very low anharmonicity (7.11).

- **No 0 $\rightarrow$ 1 transitions:** Only atoms with $m_F = 1$ or 2 are trapped. We assume that atoms that reach one of the other levels are quickly expelled from the trap (due to repulsive magnetic forces or due to gravity) and cannot make a transition back to a trapped state. This assumption is valid if the characteristic escape time of an atom is much shorter than the characteristic spin-flip time ($1/\gamma$). We shall define the characteristic escape time as the time it takes for an atom, initially at rest, to fall to the position $z = -g/\omega_z^2$ ($g$ being the free fall acceleration), which is the “point of no return” (where gravity becomes stronger than the magnetic force). This time is equal to $\sqrt{2}/\omega_z$. In the trap we are using for this experiment, $\omega_z \approx 2\pi \times 70\text{ Hz}$ (for atoms in the $m_F = 1$ level). This means that for our system this assumption is valid if $\gamma \ll 300\text{ Hz}$ (taking into account thermal velocity yields a similar result).

- **Thermal equilibrium of all the atoms:** We assume that all the atoms (with $m_F = 1$ and $m_F = 2$) are always in thermal equilibrium with the same temperature $T$. This assumption is valid if the elastic collision rate [27] ($\gamma_{el} \approx \sqrt{\frac{2k_BT}{\pi m} n_0 \sigma_{el}}$) is much higher than the spin-flip rate $\gamma$, where $n_0$ is the peak density and $\sigma_{el}$ is the elastic collision cross section. The peak density can be calculated using

\[ n_0 = N\omega_x\omega_y\omega_z \left(\frac{m}{2\pi k_B T}\right)^{3/2}. \]  \hspace{1cm} (5.36)

Thus, we get

\[ \gamma_{el} \approx \omega_x\omega_y\omega_z \frac{N\sigma_{el} m}{2\pi^2 k_B T}. \]  \hspace{1cm} (5.37)
For $N \sim 10^{5}$, $(\omega_x, \omega_y, \omega_z) \approx 2\pi \times (10, 70, 70)$ Hz and a temperature of $1 \mu K$ we get $\gamma_{el} \sim 0.1$ Hz.

- **The kinetic energy remains almost unchanged during a spin-flip:** The relative change of the kinetic energy can be calculated by comparing the recoil energy of an RF photon to the mean kinetic energy of the atoms.

$$\frac{\langle |\Delta K| \rangle}{\langle K \rangle} = \frac{k_B T_r}{D k_B T/2} = \frac{h^2 \nu^2}{D k_B T m c^2},$$

(5.38)

where $k_B T_r = h^2 \nu^2 / 2 m c^2$ is the recoil energy, $h$ is Planck’s constant $\nu$ is the photon’s frequency, $c$ is the speed of light and we have made use of the equipartition relation $\langle K \rangle = D k_B T/2$. Calculating this explicitly for $D=3$, a frequency 20MHz and a temperature of 1 $\mu K$, we get: $\frac{\langle |\Delta K| \rangle}{\langle K \rangle} \sim 10^{-15}$. We see that the relative change of the kinetic energy is completely negligible.

### 5.4 The experimental sequence

Our experimental sequence starts with a fixed preparation of about $7 \times 10^{4}$ atoms in a magnetic trap, following the preparation stage described in section 2.2. The atoms are cooled down to a temperature of $1 \mu K$. The whole preparation stage takes about 30 seconds. After the atoms have reached the desired temperature we change the IP-field of the trap adiabatically (during half a second) to a very harmonic trap ($B_0 \approx B_{ip} \approx 26 G$). At this value of trap bottom ($B_0$), which is equivalent to a Zeeman splitting of 18MHz, the background noise is quite small and flat. At this stage the atoms are almost completely polarized to the $m_F = 2$ state. We then introduce RF noise into the system, using the same function generator and the same antenna (the copper U-wire), which are used for the RF evaporative cooling stage. The introduced noise causes the atoms to make spin-flip transitions. After a waiting time, $t$, in the trap, we release the trap by suddenly shutting off the bias magnetic fields while increasing the current in the Z-wire. These actions create a strong magnetic field gradient that pushes the atoms down. Atoms in the $m_F = 2$ level will accelerate twice as fast relative to atoms in the $m_F = 1$, therefore the atoms will become separated spatially into two clouds, each containing a different Zeeman state. This type of release, which separates between the different spin states, resembles very much the milestone experiment of
Figure 5.4: The measured ratio, $R(t) = N_1/(N_1 + N_2)$, where $N_1(N_2)$ is the number of atoms in the $m_F = 1(2)$ state, as a function of the time in which the external RF noise is applied. $\Delta f$ is the frequency difference between the RF peak frequency and the Larmor frequency at the magnetic field minimum (18 MHz). The horizontal dashed line at $R(t) = 1/3$ is the expected asymptotic value in the case of white noise, while the band incorporates the expected asymptotic values based solely on time reversal symmetry and the Zeeman splitting (including second order). One clearly sees that the simple hypothesis does not explain the observed data. The error bars, taken from data variance, are of ±0.01 and are not visible.

Stern and Gerlach back in 1922 and therefore we refer to it as a “SG-release” [28]. Once the atoms are spatially separated we image them. A computer software then analyzes the image, calculating the number of atoms in each state, and their ratios. By running a sequence of experimental cycles, each with a different waiting time, $t$, in the trap (under the influence of the induced noise), we are able to reconstruct the time evolution of $R(t)$, the occupation ratio, and compare it to the solution of the rate equations (5.14).

5.5 Results

Figure 5.4 presents the obtained results for $R(t)$, the ratio of atoms in the $m_F = 1$ state to the total number of trapped atoms versus time in the static magnetic trap. $R(t)$ is measured for various detunings of the central frequency of the noise relative to the fixed transition resonance frequency.
Figure 5.5: Experimental data compared with theory: asymptotic values $R_\infty$ of curves as in figure 5.4 as a function of $\Delta f$. The band is the theoretical prediction, Eq. (5.13), without any free parameters, utilizing the fit of the real noise we introduced into the system (red line in inset). The blue, green and red curves represent 0.5, 1, 1.5 $\mu K$, respectively. Note that contrary to cases where time reversal symmetry is applicable, such as white noise, $R_\infty$ varies between zero and one, and there is a strong asymmetry around $\Delta f = 0$.

As can be seen the behavior of different noise detunings is clearly different. For red-detuned noise ($\Delta f < 0$, meaning that the central frequency of the noise is lower than the transition frequency), the asymptotic value $R_\infty \equiv R(t \to \infty)$ is very high, while for blue-detuned noise it is very low. The spectral density of the induced noise is shown in figure 5.5 (inset). The noise fits to a narrow Lorentzian multiplied by a wider Gaussian with two sidebands.

The solid curves in figure 5.4 are fits to the form

$$R(t) = R_\infty + (R_0 - R_\infty)e^{-\tilde{\gamma}t},$$

which represents an exponential convergence from the initial value $R_0 \equiv R(t = 0)$ into an asymptotic value $R_\infty \equiv R(t \to \infty)$. The dashed line at 33% is the expected asymptotic value for white noise. The pink band represents the possible asymptotic values for a general colored noise based solely on time reversal symmetry and the Zeeman splitting (including second order). This band of $0 < R_\infty < 1/2$ is obtained by putting $\beta = 1$ and $0 < \alpha < \infty$ into (5.13). One clearly sees that this simple hypothesis does not explain the observed data.
Figure 5.6: Left figure: additional verification of the theory and demonstration of control by noise. In the main figure, we apply noise with a center frequency 0.2 MHz below the Larmor frequency (\(\Delta f = -0.2\) MHz). As predicted by the theory presented in figure 5.5, this transfers most of the atoms to the \(m_F = 1\) state. After 200 ms we rapidly change the center of the noise to \(\Delta f = +0.4\) MHz (time 0 in the graph). At this frequency, the theory predicts that no atoms will reside in the \(m_F = 1\) state, and indeed as can be seen in the figure, the noise clears the \(m_F = 1\) level. The speed in which the \(m_F = 1\) level is cleared may be controlled by varying \(\Delta f\). In the inset we show the evolution of the system when white noise is applied. The asymptotic value of this graph, at 0.344 ± 0.004, is very close to the theoretical prediction of white noise or symmetric transition, \(R_\infty = 1/3\). The error bars, taken from data variance, are of ±0.01. Right figure: population control. By applying noise at red or blue detuning, we can control the relative occupation of the \(m_F\) states, from \(\approx 70\%\) of the atoms in the \(m_F = 1\) state (left frame, top cloud) for red detuned noise, up to \(\approx 100\%\) of the atoms in the \(m_F = 2\) state (right frame, bottom cloud) for blue detuning.

The dependence of the asymptotic values obtained in figure 5.4 (with more experimental data sets which are not shown) on the detuning of the noise, \(\Delta f\), together with the theoretical prediction, obtained by (5.35), is shown in figure 5.5. As can be seen, the experimental data agrees very well with the theoretical prediction. We note that the assumptions of thermal equilibrium and no \(0 \to 1\) transitions, of the theoretical calculation (see section 5.3.3) are valid for \(|\Delta f| > 150\) kHz and \(|\Delta f| > 20\) kHz respectively.

We see that we have found an effective tool to manipulate the relative occupation of the levels. Applying red-detuned noise will pump up to \(\approx 75\%\) of the atoms to the \(m_F = 1\) level, whereas blue-detuned noise will clear this level completely, leaving 100% of the atoms in the \(m_F = 2\) level. This ability to manipulate the atomic state is presented in figure 5.6, where we have applied
the following procedure: After the regular preparation of the trap, we start by introducing red-detuned noise (with a detuning of -0.2MHz). At this detuning the system has an asymptotic steady state of \( R_\infty \approx 70\% \), (as can be seen in figure 5.5). 200ms later, after the system has reached the asymptotic steady state, we quickly change the central frequency of the noise to become +0.4MHz blue-detuned. Now the atoms become very far from the new asymptotic steady state \( R_\infty \approx 5\% \), and indeed we see a clear decrease of the relative occupation from over 60% down to \( \approx 5\% \).

Another verification of our theoretical explanation, is to check what is the asymptotic relative occupation when white noise is applied. Recalling the result of section 5.2, we expect an asymptotic value of 1/3. The inset of figure 5.6 presents the experimental results obtained when white noise is introduced. The predicted asymptotic value of \( \approx 1/3 \) is clearly obtained.

### 5.6 Calculation of the mean transition energies

The mean transition energies are calculated by multiplying the integrand of (5.19) by \( \hbar \omega \) and dividing it by the appropriate transition rate. This gives us

\[
\langle E_{i\rightarrow j} \rangle = \frac{1}{\gamma_{i\rightarrow j}} \int_{-\infty}^{\infty} d\omega \int d^3r P_i(r) \Gamma_{i\rightarrow j}(\omega) \delta(V_i(r) - V_j(r) + \hbar \omega) \hbar \omega.
\] (5.40)

#### 5.6.1 Mean transition energies - white noise, simple case

In the white noise limit, \( \Gamma_{i\rightarrow j} \) in equation (5.19) is constant. The \( \delta \) function is therefore integrated out to 1, as does the thermal distribution probability, hence \( \gamma_{i\rightarrow j} = \Gamma_{i\rightarrow j} \). Therefore, in the integral of (5.40) \( \Gamma_{i\rightarrow j} \) cancels out with \( \gamma_{i\rightarrow j} \). We are left with

\[
\langle E_{i\rightarrow j} \rangle = \int d^3r P_i(r)(V_i(r) - V_j(r)) = \langle V_i(r) - V_j(r) \rangle_i,
\] (5.41)

where \( \langle ... \rangle_i \) denotes thermal averaging over the distribution of the \( i \)-th level. If interatomic interactions, second order Zeeman and gravity are neglected, we may use the potential \( V_i(x) = m_i(\mu_B B_0 + \frac{1}{2}m\omega^2x^2) \) (for a 1D case). Equation (5.41) then becomes

\[
\langle E_{i\rightarrow j} \rangle = (m_j - m_i)(\mu B_0 + \frac{1}{2}m\omega^2\langle x^2 \rangle_i),
\] (5.42)
We assume that all the atoms are always in thermal equilibrium with temperature $T$. Therefore from the equipartition theorem we know that

$$\frac{1}{2}m\omega^2 \langle x^2 \rangle_1 = \frac{1}{2}k_BT,$$

$$\frac{1}{2}m\omega^2 \langle x^2 \rangle_2 = \frac{1}{2}k_BT.$$  \hspace{1cm} (5.43)

Introducing (5.43) into (5.42) we get the $1 \rightarrow 2$ mean transition energy

$$\langle E_{2\rightarrow1} \rangle = -\mu B_0 - \frac{1}{4}k_BT.$$  \hspace{1cm} (5.44)

For the opposite $1 \rightarrow 2$ transition, we get

$$\langle E_{1\rightarrow2} \rangle = \mu B_0 + \frac{1}{2}k_BT,$$  \hspace{1cm} (5.45)

and for the $1 \rightarrow 0$ transition, we get

$$\langle E_{1\rightarrow0} \rangle = -\mu B_0 - \frac{1}{2}k_BT = -\langle E_{1\rightarrow2} \rangle.$$  \hspace{1cm} (5.46)

Generalizing this to $D$ dimensions gives the following results

$$\langle E_{2\rightarrow1} \rangle = -\mu B_0 - \frac{1}{4}Dk_BT,$$  \hspace{1cm} (5.47)

$$\langle E_{1\rightarrow2} \rangle = -\langle E_{1\rightarrow0} \rangle = \mu B_0 + \frac{1}{2}Dk_BT.$$  \hspace{1cm} (5.48)

5.6.2 Mean transition energies - colored noise, general case

In order to calculate the transition energies for colored noise, taking into account both gravity and NLZ terms, we notice that the transition energy of the $i \rightarrow j$ transition is given by

$$\hbar \omega = E_{ij}^0 + \frac{1}{2}m \sum_q \omega_q^2 r_q^2$$

(we neglect the effect of NLZ on the trap frequencies). Equation (5.40) is equal to equations (5.19) and (5.35), except for $\hbar \omega$ which multiplies the integrand and the division of the final result by $\gamma_{i\rightarrow j}$. Hence, if we simply multiply the integrand of equation (5.35) by $\hbar \omega$ (equal to $E_{ij}^0 + \rho^2k_BT$) and divide the result by $\gamma_{i\rightarrow j}$, we get the transition energy:

$$|\langle E_{i\rightarrow j} \rangle| = E_{ij}^0 + \frac{2k_BT}{\gamma_{i\rightarrow j}w_i} \sqrt{\frac{m_i}{\pi}} \int_0^\infty d\rho \rho^3 \exp[-m_i(\rho^2 + w_i^2)] \sinh(2m_i w_i \rho) \Gamma_{i\rightarrow j}(E_{ij}^0/\hbar + \rho^2k_BT/\hbar).$$  \hspace{1cm} (5.49)
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Figure 5.7: Mean transition energies, \( \langle |E_{i \rightarrow j}| \rangle - E_{ij}^0 \) /\( k_B T \), calculated according to (5.50) (a) and (5.49) (b) for the noise we introduce to the system and a temperature of 1 µK. The expected values for white noise (equations (5.51-5.53)) are shown for comparison (dashed lines). As expected, the mean transition energies go to these values for large positive or negative detunings (where the noise is effectively white). The maximal values for the 1 \( \rightarrow \) 2 and 1 \( \rightarrow \) 0 transitions are obtained for a larger blue detuning than for the 2 \( \rightarrow \) 1 transition because the atoms at level 1 reach farther away from the trap center, making the transition energies higher. The horizontal separation in (b) between \( \gamma_1 \rightarrow 2 \) and \( \gamma_1 \rightarrow 0 \) is due to NLZ and is equal exactly to \( E_{12}^0 - E_{01}^0 \).

If NLZ terms and gravity are neglected this becomes

\[
\langle |E_{i \rightarrow j}| \rangle = \mu B_0 + \frac{4 k_B T m_i}{\gamma_{i \rightarrow j}} \int_0^{\infty} dp \rho^4 \exp[-m_i \rho^2] \Gamma_{i \rightarrow j} (\mu B_0 / h + \rho^2 k_B T / h). \tag{5.50}
\]

In the case of white noise, \( \Gamma_{i \rightarrow j} \) in (5.49) cancels out with \( \gamma_{i \rightarrow j} \) and the integral can be solved analytically, giving

\[
\langle |E_{1 \rightarrow 2}| \rangle = E_{12}^0 + \frac{3}{2} k_B T + \frac{m g^2}{2 \omega_z^2}, \tag{5.51}
\]

\[
\langle |E_{2 \rightarrow 1}| \rangle = E_{12}^0 + \frac{3}{4} k_B T + \frac{m g^2}{8 \omega_z^2}, \tag{5.52}
\]

\[
\langle |E_{1 \rightarrow 0}| \rangle = E_{10}^0 + \frac{3}{2} k_B T + \frac{m g^2}{2 \omega_z^2}, \tag{5.53}
\]

which agrees with the previous results (5.46, 5.48) for \( g = 0, D = 3 \).

Figure 5.7 shows the mean transition energies, \( \langle |E_{i \rightarrow j}| \rangle - E_{ij}^0 \) /\( k_B T \), calculated according to (5.50) (a) and (5.49) (b) for the noise we introduce to the system. The expected values for white
Figure 5.8: (a) The expected lifetime as a function of the noise detuning, calculated according to (5.17), for the cases that gravity and NLZ are neglected (blue) and taken into account (purple). We see that if gravity and NLZ are taken into account, the lifetimes become orders of magnitude longer. (b and c) The transition rates $\gamma_{i\rightarrow j}$ as a function of the noise detuning if NLZ and gravity are ignored (b) and taken into account (c). We note that the shortest lifetime in (a), occurring at $\approx 0.1\text{MHz}$, is explained in (c), as the maximal value for the product of $\gamma_{2\rightarrow 1}$ and $\gamma_{1\rightarrow 0}$. All the plots were calculated for the noise we introduce to the system (at the same amplitude) and for a temperature of $1\ \mu K$.

noise (equations (5.51-5.53)) are shown for comparison (dashed lines).

5.7 Lifetime

Figure 5.8 (a) presents the expected lifetime as a function of the noise detuning, calculated according to (5.17) for the noise we introduce to the system and a temperature of $1\ \mu K$, for the cases that gravity and NLZ are neglected (blue) and taken into account (purple). We see that if gravity and NLZ are taken into account, the lifetimes become orders of magnitude longer. The explanation of this effect is that if NLZ and gravity are ignored, the transition rates $\gamma_{2\rightarrow 1}$ and $\gamma_{1\rightarrow 0}$ both become large around the same detuning (see figure 5.8 (b)), meaning that if the noise detuning is close to this resonance value we have large transition rates both for the $2 \rightarrow 1$ and $1 \rightarrow 0$ transitions and the atoms are lost very quickly. On the other hand, if NLZ and gravity are taken into account each transition rate becomes large at different detuning of the noise (figure 5.8 (c)). $\gamma_{2\rightarrow 1}$ becomes large with a detuning closer to the one that gives a large $\gamma_{1\rightarrow 2}$, the latter preserving the atoms in the system, than to the one that gives a large $\gamma_{1\rightarrow 0}$.
Chapter 6

Experimental Outlook: Feasibility Study of the Anisotropic Experiment and New Chip Mount

We wish to check the influence of using anisotropic electrical conductors (AEC) on atom chips. As shown in [29], the theory predicts that AECs should have orders of magnitude suppression of decoherence even at room temperature, compared with isotropic conductors. The idea of the following experiment (suggested in the outlook of [29]) is to verify the theory regarding the decoherence suppression by measuring the angular dependence of the matrix elements via the spin flip rate. Here we would prefer to use a 'quasi-1D conductor' (see section 4.4.1), which would make the angle dependence more significant. Unfortunately, quasi-1D conductors are very hard to get or fabricate (see table 3.2 of [29]), hence, we shall use a 'layered conductance material' instead.

The goal of the experiment is to measure the expected dependence of the thermally induced spin-flip rate on the angle between the bad conductivity axis of a 'layered AEC' and the quantization axis described in figure 4.2. We are planning to glue 4 pieces of highly oriented pyro-graphite (HOPG) on a silicon wafer. HOPG is a layered material with resistivity values of \( \{\rho_{xx}, \rho_{yy}, \rho_{zz}\} = \{4 \cdot 10^{-7}, 4 \cdot 10^{-7}, 1.5 \cdot 10^{-3}\} \, \Omega m \) which gives an anisotropy ratio of 3750. Each piece of HOPG
will be glued with its bad conductivity axis at a different orientation relative to the quantization axis. The planned angles between the good conductivity axis and the quantization axis are: 0°, 30°, 60° and 90°. We plan to measure the lifetime of a cloud of magnetically trapped atoms, each time positioned very closely to one of the different pieces.

### 6.1 Expected results

The calculated expected spin-flip rate of a single atom placed above the 4 AEC pieces at a distance of 15 µm (taking into account the thermal noise generated by all 4 pieces) is shown in figure 6.1 (a). This was calculated using the equations in chapter 4 with a material temperature of 300K. In the experiment, the atoms will suffer from various loss mechanisms, such as background collisions, technical noise spin-flips, tunneling to the surface, etc. The different orientations of the AEC pieces will only affect the thermally induced spin-flip rate. Therefore, we will assume that the lifetime of an atom in the trap due to all of the loss-mechanisms except thermal noise is \( \tau_{BG} \). The total lifetime of the atom will be approximated by:

\[
\tau^{-1} = \tau_{thermal}^{-1} + \tau_{BG}^{-1}
\]
6.2 Distance from the chip

The lifetime of the atomic cloud is strongly affected by its distance from the atom chip. The inset of figure 6.2 (a) shows $\tau_1$ (blue) and $\tau_2$ (purple), the lifetimes above the center of the $\theta = 0^\circ$ and $\theta = 90^\circ$ AECs respectively, as a function of the height from the atom chip. Figure 6.2 (a) shows the absolute difference $\tau_1 - \tau_2$ as a function of the height. As can be seen from the figure, we get a maximal difference at a height of $\approx 40 \mu m$. Figure 6.2 (b), on the other hand, shows the relative difference between the the lifetimes (i.e. $2(\tau_1 - \tau_2)/(\tau_1 + \tau_2)$). We see that the closer the atomic
cloud is from the atom chip, the larger the relative difference becomes.

In practice, getting very close to the atom chip is not possible due to surface evaporation of the atoms. Moreover, extremely low lifetimes are hard to measure accurately, hence, we should not get too close to the atom chip, where the lifetime measurements will have large fluctuations. We have decided to work at a distance of 15 \( \mu m \). At this distance surface evaporation is negligible (calculated in [29]) while the lifetimes are long enough to be measured accurately with a relative difference of about 50%.

### 6.3 Error estimations

In the real experiment, we expect there to be numerous error factors which we must take into account. These include:

- **Errors in the distance from the chip:** With our imaging system, we are able to locate the (center of the) cloud with an error of less than 2 \( \mu m \). Height fluctuations due to noise were experimentally measured, and were found to be of less than 2 \( \mu m \) (for a cloud at a height of about 15 \( \mu m \)). Therefore, I estimate the maximal error in the distance from the chip as: 2 \( \mu m \).

- **Errors in the position above the chip:** It can be seen from figures 6.1 (a) and (b) that the spin-flip rate above the different AEC pieces is almost constant as long as the atoms are far away from the edges. The lifetime starts to change significantly only when the atoms are at about 100 \( \mu m \) from the edge. Therefore, as long as we are sure that the atoms are far enough (more than 100 \( \mu m \)) from the edges, their exact location is irrelevant.

- **The direction of the IP field is not exactly along the x direction:** The rotation angle between the IP field and the x direction was estimated using simulations of the magnetic trap as less than 2\(^\circ\) (see table 6.1 below).

- **Errors in the orientations of the AEC pieces:** As long as we are far away from the edges, at the relevant heights, the AEC pieces can be treated as infinite surfaces. Therefore, a
rotation of the crystal axis is equivalent to the opposite rotation of the quantum axis (the IP field). I estimate the error of the orientation of the AEC crystals as less than 1°. Therefore, the total 'angular error' will be taken as 3°.

- **The exact temperature of the chip is unknown:** The rise of the temperature is estimated in section 6.5 as about 15K for normal use. This number agrees to the atom-chip temperatures that we are measuring with our current mount. Anyway, as can be seen in figure 6.1, the exact temperature of the atom-chip changes the absolute lifetime values, but doesn’t affect much the lifetime difference between the two materials. Therefore, the effect of temperature changes of the atom-chip seem to be unimportant and will be neglected from now on (the temperature for the calculations will be taken as 300K).

- **Finite size effects:** The cloud is not made of a single atom; it is made out of many atoms each in a different location with a different distance from the chip. In order to take this effect into account, the expression for the spin flip rate of a single atom was (numerically) integrated over space with a Gaussian density distribution taking into account the expected orientation of the cloud. The widths of the Gaussian distributions (on each axis and for each position) as well as the different orientations of the cloud were estimated by simulations of the magnetic trap.

Taking all these considerations into account, we can make an estimation of the possible lifetime values that will be measured. The calculated expected lifetime as a function of the AEC’s orientation and the expected error-bars for the 4 actual AEC pieces are presented in figure 6.3. A clear difference between the 0° and 90° orientations is expected.

### 6.4 Loading of the AEC chip

It is very important to ensure that with the planned magnetic trap we are able to get the cloud of atoms to the exact positions required for the experiment. This process includes loading the atoms from a magneto-optical trap (MOT) at a distance of about 2.5mm from the atom-chip, bringing the atoms down to a distance of 15 µm and transferring them to their final position above the center of
one of the 4 AEC pieces. The 4 AEC pieces are located right above the center of the atom-chip \textit{i.e.} at \((x = 0, y = 0)\). The desired final locations for measuring lifetimes are therefore \((x = \pm 500 \mu m, y = \pm 500 \mu m)\). I have checked the feasibility of this loading process using an “H-Trap” which is well compatible with the design of the new mount (described in section 6.5). The different currents and bias magnetic fields required for the first (taking the cloud from a distance of 2.5mm to 15\(\mu m\) from the atom-chip) and second (taking the cloud from the center of the chip \((x = 0, y = 0)\) to the final positions \((x = \pm 500 \mu m, y = \pm 500 \mu m)\)) loading steps and the resulting simulated iso-potential plots can be found in table 6.1, figure 6.4 and figure 6.5.

**Figure 6.3:** The calculated expected lifetime as a function of the AEC’s orientation and the expected error-bars for the 4 actual AEC pieces, taking into account the considerations of section 6.3. A clear difference between the 0\(^{\circ}\) and 90\(^{\circ}\) orientations is expected.
Figure 6.4: Front (a) and side (b) views of iso-potential surfaces of the traps from the first loading step (taking the cloud from a distance of 2.5mm to 15 $\mu m$ from the atom-chip), given in table 6.1 (numbers at the right side correspond to the first column). All iso-potential are for a temperature of 40 $\mu K$ except of number 10 that corresponds to 10 $\mu K$.

Figure 6.5: Top view of iso-potential surfaces of the traps from the second loading step (taking the cloud from the center of the chip ($x = 0, y = 0$) to the final positions ($x = \pm 500 \mu m, y = \pm 500 \mu m$)), given in table 6.1 (RHS numbers correspond to the first column). All iso-potential are for a temperature of 10 $\mu K$. 

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**Table 6.1: Atom-chip loading scheme**

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6.5 Future design - a new mount

The AEC experiment described previously, may require high resolution imaging. It may also require high currents in order to create a strong magnetic barrier to suppress surface evaporation at small atom-surface distances. The main points we gave our attention to in the design of the mount were:

- All elements should be UHV proof.
- The chip position should be very accurate and very firmly held in place.
- The system should be able to run currents up to 100 Amperes for a duty cycle of 67%. Thermal expansion of the copper rods and the copper structure should be taken care of.
- Caution should be taken to prevent overheating of the atom-chip that can cause unwanted out-gassing.
- Magnetic trapping should be possible (allowing both Z and H configurations).
- The mount should physically fit both the old and the new vacuum chambers, where the new vacuum chamber has a imaging window close to its center (for high resolution imaging) and thus the mount is required to be smaller.
- Ease of use (fast and easy replacement of the atom-chip).
- In the vicinity of the atom-chip only non-magnetizable materials should be used, to prevent stray magnetic fields.
- The system should be able to withstand a large number of experimental cycles.

6.5.1 New mount overview

An overview of the suggested new mount is presented in figure 6.6. Notice that in reality the mount is actually placed upside-down, with the atom-chip facing down. From now on we will present the mount as in figure 2.1 (atom-chip facing up) for convenience.

This mount is made out of three major assemblies:
Figure 6.6: A sketch of the proposed new mount. Center: an overview of the entire mount. Left: the frame assembly. Top right: a blowup of the MACOR holder assembly. Bottom right: A blowup of the spring connector.

- MACOR holder assembly.
- Rigid frame assembly.
- Current leads assembly.

Simulations of traps generated with copper structures supported by the new mount are presented in section 6.4

6.5.2 The MACOR holder assembly

MACOR is a glass-ceramic material acting as efficient insulation and stable up to temperatures of 1000°C, with very little thermal expansion or outgassing. The MACOR holder assembly (shown in figure 6.6) is responsible for holding the atom-chip, the copper structures responsible of the
magnetic trap and two plug-connectors. In the design of this assembly we paid attention to the following points:

- **Fast and easy experimental setup replacement:** In the current setup, in order to replace an atom-chip, one has to break the old chip and reposition the new chip at the correct position. After placing the new chip, reconnection of the wiring is complicated and confusing. The general idea of the new MACOR holder is that each experiment has its own MACOR holder and when there is a need to replace the atom-chip (for a different experiment), the whole MACOR holder assembly is taken out as a single unit, including the copper structure and the atom-chip, and is replaced with a new one. One only needs to reconnect the plug connectors to reconnect all the wiring. This way, the mount is kept clean under vacuum and there is no need to open the vacuum twice or keep a spare mount. This enables a fast and easy replacement of the experiment while the old experiment will be kept at a “ready to use” situation.

- **Accurate positioning of the atom-chip:** The atom-chip must be positioned with great accuracy. In order to ensure the correct positioning of the atom-chip, we designed the MACOR holder with three contact “bumps” at three corners of the atom-chip. When gluing the atom-chip, one should just “slide” the atom-chip (from the open side) until it is being restricted by the three bumps. One corner is left unrestricted to allow some thermal expansion of the chip.

- **Thermal expansion of the copper structure:** When the experiment is running, the temperature of the copper structure may rise quite significantly due to the high currents running through it (up to 100 Amperes). This causes the copper structure to expand. Surprisingly even a relatively very small expansion of the copper wires can cause severe damage to the system; if the copper wire ends are fixed at their positions, then the wire cannot expand at its longitudinal direction and therefore, it must bend. The wire is confined by the firm MACOR holder from below and from the sides, therefore it would bend upwards, towards the atom-chip. Let’s try to get a quantitative estimation of this effect: The length of the
copper wires (such as the Z wire) is of the order of \( d \approx 35 \text{ mm} \), if we assume an expansion of \( \Delta d = 35 \mu m \) i.e. a relative expansion of 0.1\% (which corresponds to a temperature rise of about 60\(^\circ\)C for a linear thermal expansion coefficient of \( 17 \times 10^{-6} /\circ\)C), we get from a simple triangle model that the transverse disposition is equal to \( \Delta h \approx \sqrt{d \Delta d / 2} \approx 780 \mu m \). This sort of disposition will most probably break the chip. The solution we found in the new design is to let the ends of the wires have some small amount of freedom to move (about 40 \( \mu m \) in each direction). This will ensure that the expansion is in the longitudinal direction and not towards the atom-chip. For some of the wires (such as the Z wire), the accurate positioning of the central part of the wire is very important and we cannot allow movement in the longitudinal direction. These wires will be held fixed in position from the center using small pieces of MACOR that will fill in the gaps. In this way we prevent movement of the central part of the wire while allowing it to expand outwards.

- **Prevention of stray magnetic fields:** At the vicinity of the atom-chip we cannot use magnetizable metals that will produce stray magnetic fields in the system; for this reason all the screws on the MACOR holder are made of titanium (which is non-magnetizable).

### 6.5.3 The rigid frame assembly

As mentioned before, the atom-chip must be positioned with a great amount of accuracy. For this purpose the new mount uses a strong frame (see figure 6.6) that holds the MACOR holder assembly at place. The frame is built from the following pieces:

- **Steel rods:** connect the bottom flange to the titanium cross brace.
- **Titanium cross brace:** connects between the steel rods and the titanium rods. It gives the frame support and decreases torsion and perpendicular movement.
- **Titanium rods:** connect the titanium cross brace to the PEEK disk. We use titanium to prevent stray magnetic fields at the vicinity of the atom-chip, as mentioned before.
- **PEEK Disk:** Polyether ether ketone (PEEK) is a colorless organic polymer thermoplastic with excellent mechanical and chemical resistance properties that are retained to high tem-
peratures. It is very robust and UHV compatible. The PEEK disk at the top of the frame holds the MACOR holder in an exactly shaped socket that keeps the MACOR in its exact location. Even when the MACOR holder is reinstalled the socket ensures that it is held in place. The PEEK disk also holds the sockets of the plug connectors.

6.5.4 Current leads assembly

The current leads assembly contains the long copper rods that carry the high currents from the vacuum feedthroughs to the copper structures behind the chip (described in section 6.5.2). These high currents are required for the MOT and the magnetic loading of the chip micro traps.

Thermal expansion of the copper rods

Because of the UHV environment, there is almost no heat dissipation to the environment (except through the rods themselves). This can cause the copper rods to heat up and expand significantly. If the rods will expand in the direction of the atom-chip they can break it. For this reason we wanted to ensure that the expansion of the copper rods will be in the opposite direction. We do this by holding the upper end of the copper rod in place with the rigid PEEK disk while connecting the lower end to the feedthrough with a flexible 'spring connector' at the bottom shown in figure 6.6. In the design of this 'spring connector' we took care to prevent the existence of long sections with a small cross-section that would have prevented efficient heat dissipation and cause the copper structure to heat-up markedly. Even with this design, where we have only two cuts with a small cross section, the effect of these cuts is clearly seen in the temperature profile shown in figure 6.7 (see next section).

Heating of the copper structure

As mentioned before, due to the UHV environment, the copper structure can reach rather high temperatures. The temperature will be the highest at the point that is farthest away from the feedthroughs that come in contact with air, allowing some heat dissipation. The farthest away point is the center of the magnetic trap copper structure underneath the atom-chip. It is very
important to verify that the temperature of the copper structure stays low enough to prevent over-expansion of the copper and even outgassing of the chip. For this purpose I made an estimation of the steady state rise in temperature at the center of the copper structure, using a simple 1D model that assumes that all the heat is evacuated only through the two ends of the feedthroughs (through which the currents come in and go out). The steady state rise of the temperature at the center of the copper structure was evaluated using

$$\Delta T = \frac{DI^2}{k} \sum_i \left[ \frac{L_i}{A_i} \left( \frac{R_i}{2} + \sum_{j>i} R_j \right) \right], \quad (6.2)$$

where $D$ is the duty-cycle, $I$ is the current, $k$ is the thermal conductivity coefficient of copper ($\approx 400$ W/m-K) and $L_i$, $A_i$ and $R_i$ are the length, the cross-section and the resistance of the $i$-th element respectively (elements ordered from outmost to inmost). I made two calculations: one for a ‘normal scenario’ with typical values that we are currently using in our magnetic trap: $D=0.5$ and $I=50$ Ampere, and an ‘extreme scenario’ where I took the system to the edge with $D=2/3$ and a current of 100 Ampere. The calculated temperature profile of the extreme scenario is shown in figure 6.7. The temperature profile of the normal scenario is identical to that of the extreme scenario after a multiplication by $3/16$. The temperature rise at the center of the copper Z is of about 73K and 14K for the extreme and normal scenarios respectively.
Chapter 7

Theoretical Outlook: Heating in an Anharmonic Trap

In chapter 5 we considered heating due to engineered (colored) noise, and as a function of the noise parameters. In this chapter, we consider heating due to background noise, and as a function of trap parameters.

7.1 Heating in a perfectly harmonic trap

Following Japha and Band [19], we consider a general system of $N$ mutually interacting identical bosonic particles of mass $m$ in an external time-dependent harmonic potential (both trap frequencies and trap position are changing in time). The Hamiltonian for the system is given by

$$H = \sum_\alpha \left[ \frac{p_\alpha^2}{2m} + \frac{1}{2} m \sum_{i=x,y,z} \omega_i^2(t)(r_{\alpha i} - r_{0i}(t))^2 \right] + \sum_{\alpha<\beta} U(r_\alpha - r_\beta), \quad (7.1)$$

where $r_{0i}(t)$ is the $i$-th component of the time-dependent position vector of the center of the harmonic trap and $r_{\alpha i}$ is the $i$-th component of the $\alpha$-th boson. After a coordinate transformation,
\( R = \sum_\alpha r_\alpha / N, \ q_\alpha = r_\alpha - R, \) the above Hamiltonian can be written as

\[
H = \left[ \frac{P^2}{2M} + \frac{1}{2} M \sum_{i=x,y,z} \omega_i^2(t)(R_i - r_{0i}(t))^2 \right] + \sum_{\alpha=1}^{N-1} \left[ \frac{p_\alpha^2}{2m} + \frac{1}{2} m \sum_{i=x,y,z} \omega_i^2(t)q_{\alpha i}^2 + \sum_{\alpha<\beta} U(q_\beta - q_\alpha) \right],
\]

(7.2)

where we have defined \( M = Nm \) and used the relation \( \sum_{\alpha=1}^N q_{\alpha i} = 0. \) This form of the Hamiltonian clearly reveals that the motion of the center of mass of the system \( R \) is decoupled from the motion of other degrees of freedom of the system \( q_\alpha. \) Hence, the general wavefunction \( \psi(r_1, r_2, ..., r_N, t) \) may be written in product form, \( \psi(r_1, r_2, ..., r_N, t) = \psi_{CM}(R, t)\psi_{rel}(q_1, q_2, ..., q_{N-1}), \) where the relative part of the wavefunction does not depend explicitly on time, the effect of the motion (time dependence) of the harmonic potential is only on the center-of-mass part of the wavefunction, and this dependence is given via the quantity \( r_{0i}(t). \) Only the center of mass motion is influenced by shaking. The meaning of the last sentence is that shaking of a perfectly harmonic trap should not cause any heating effect (when we measure the temperature of the cloud we actually measure the expansion relative to the center of mass as described in section 3.5.2). In practice, our magnetic trap is harmonic only near the center within some range, which I will refer as the harmonic range of the trap (not to be confused with the harmonic length \( a_{ho}. \) If such a magnetic trap is shaken, the atomic cloud may enter a region where the true potential is anharmonic. In this case the center of mass motion is no longer decoupled from the relative motion, therefore the atomic cloud may heat.

### 7.2 Defining the harmonic range

#### 7.2.1 The harmonic range of a trap

I will now treat the case of a Z-trap [10] made of infinitely thin wires (the results obtained in this section are valid also for an H-trap). This trap is made from a wire, bent to a right angle “Z” shape and carrying a current \( I \) (the \( x \) axis is chosen at the direction of the central part of the wire and the \( y \) axis parallel to the “legs” of the “Z”). Two homogeneous bias fields, \( B_x \) in the \( x \) direction and \( B_y \) in the \( y \) direction, are applied by the external coils creating a total magnetic field with a
magnitude of $B_0$ at the trap center. This model is a good approximation when the distance from
the trap center to the wire ($z_0$) is much larger than the width of the wire. It will enable us to get
easy analytical expressions for the harmonic range.

**Analytic treatment**

The trap configuration described above creates at $x = 0$ a potential given by

$$V(x = 0, y, z) = \mu \sqrt{B_0^2 + \left(\frac{\mu_0 I}{2\pi}\right)^2 \left(\frac{y}{y^2 + z^2}\right)^2 + \left(B_y - \frac{\mu_0 I}{2\pi} \frac{z}{y^2 + z^2}\right)^2},$$  \hspace{1cm} (7.3)

where $\mu_0$ is the vacuum permeability. The derivation of (7.3) is quite straightforward, starting with
(4.1) and using an approximation of an infinitely long wire given in (4.9) for the central part of the
"Z". The center of the trap is at $y_0 = 0$ and

$$z_0 = \frac{\mu_0 I}{2\pi B_y}.$$  \hspace{1cm} (7.4)

As can be seen from (7.3), the potential is symmetric with respect to $y = 0$ and asymmetric with
respect to $z = z_0$.

At $z = z_0$ and assuming that we are close to the trap center (i.e. $|y| \ll z_0$) we get in the $y$
direction

$$V(x = 0, y, z = z_0) = \mu \sqrt{B_0^2 + \left(\frac{\mu_0 I y}{2\pi (y^2 + z_0^2)}\right)^2} \approx \mu B_0 \sqrt{1 + \left(\frac{\mu_0 I y}{2\pi B_0 z_0^2}\right)^2} =$$

$$= V_0 + \omega_y^2 a_y^2 \left(\sqrt{1 + (y/a_y)^2} - 1\right),$$  \hspace{1cm} (7.5)

where we have defined

$$V_0 = \mu B_0,$$  \hspace{1cm} (7.6)

$$\omega_y = \frac{2\pi B_y^2}{\mu_0 I} \sqrt{\frac{\mu}{m B_0}},$$  \hspace{1cm} (7.7)

$$a_y = \frac{\mu_0 I B_0}{2\pi B_y^2} = \frac{B_0}{B_y z_0}.$$  \hspace{1cm} (7.8)

This potential describes two regimes:

- For $|y| \ll a_y$ the potential is harmonic with a frequency $\omega_y$.  

For $|y| \gg a_y$ the potential is linear.

This means that $a_y$ may be used to describe the harmonic range of the trap in the $y$ direction. By fitting the real potential to the form given in (7.5) we get the harmonic range $a_y$.

At $y = 0$ and assuming that we are close to the trap center (i.e. $|z - z_0| \ll z_0$) we get in the $z$ direction

$$V(x = 0, y = 0, z) = \mu \sqrt{B_0^2 + \left(\frac{\mu_0 I}{2\pi z} - B_y\right)^2} = V_0 \sqrt{1 + \left(\frac{\mu_0 I}{2\pi B_0} \frac{1}{z_0} - \frac{B_y}{B_0} \frac{z_0}{z} \right)^2} = V_0 \sqrt{1 + \left(\frac{z - z_0}{a_z} \frac{z_0}{z} \right)^2} \approx V_0 \sqrt{1 + \left(\frac{z - z_0}{a_z} \frac{z_0}{z} \right)^2} \left(1 - \frac{z - z_0}{z_0} \right)^2},$$

(7.9)

where $a_z = a_y$ (7.8). This equation describes 3 regimes:

1. when $|z - z_0| \ll a_z$ and $|z - z_0| \ll z_0$ the potential is harmonic with frequency $\omega_z$.
2. when $|z - z_0| \gg a_z$ but $|z - z_0| \ll z_0$ the potential is linear $V(z) = V_0 |z - z_0| / a_z$.
3. when $|z - z_0| \sim z_0$ the potential becomes asymmetric such that it is lower when $z > z_0$ and higher when $z < z_0$.

This means that $a_z$ describes well the harmonic range of the trap in the $z$ direction. By fitting the real potential to the form given in (7.9) we get the harmonic range $a_z$.

**Numerical treatment**

The analytic treatment in the previous section is only an approximation. When better accuracy is needed, one can use a numerical simulation in order to obtain the harmonic range. Figure 7.1 (a) presents a numerical simulation result for the potential of a Z-trap in the $y$ direction. As can be seen in the figure, the exact calculated potential (blue) fits much better to the function given in (7.5) (green) than to a simple quadratic potential (purple). The same happens with the asymmetric potential in the $z$ direction (figure 7.1 (b)). The obtained harmonic ranges are $a_y = 23.4 \, \mu m$ and $a_z = 20.0 \, \mu m$. These values (calculated for a Z-trap with a center wire length of
Figure 7.1: Simulated potential cuts, in the $y$ (a) and $z$ (b) directions, for a Z-Trap. The exact calculated potential (blue) fits much better to the functions given in (7.5) and (7.9) (green) than to a simple quadratic potential (purple). The resulting anharmonic ranges $a_y$ and $a_z$ (calculated from the fits) are given. These values are equal to the value obtained by (7.8).

4mm, $I=55A$, $B_y=60G$ and $B_z=-12.1G$ which gives $B_0 \approx 1G$) are very close to the value obtained by (7.8), $a_y \approx 25.8 \mu m$ (after taking into account the actual value of $z_0$, which slightly differs from (7.4)). The good agreement between the values obtained by the numerical simulation and equation (7.8) can be seen in figure 7.2.

### 7.2.2 The anharmonicity of an atomic cloud

As discussed above, the behavior of the atomic cloud depends on whether or not the atoms have reached the anharmonic area. This means that the anharmonicity of an atomic cloud should depend not only on the properties of the trap, but also on the cloud itself. We shall use the following definition for $A_j$, the (dimensionless) anharmonicity of an atomic cloud in the $j$-th direction

$$A_j = \sqrt{\frac{\langle r_j^2 \rangle}{a_j}}. \quad (7.10)$$

Assuming that $\langle r_j^2 \rangle \approx k_B T/m\omega_j^2$ we get,

$$A_j \approx \sqrt{\frac{k_B T/m}{\omega_j a_j}}. \quad (7.11)$$
We see that for a given trap (i.e. $a_j$ and $\omega_j$ are given), the anharmonicity of the atomic cloud depends on its temperature; a higher temperature means a higher anharmonicity.

According to section 7.1, the heating rate of an atomic cloud should depend on its anharmonicity. We expect to get a faster heating rate for a larger anharmonicity of the atomic cloud. This means that for a given trap, the colder the atoms are the lower their heating rate is expected to be.

### 7.3 A proposed experiment

I conclude this chapter with a brief description of a proposed experiment that would check the expected connection between heating rates and anharmonicity. The experiment will include a sequence of heating rate measurements, each with the same preparation but with different final parameters. We plan to check the dependence of the heating rate on different parameters: the initial temperature of the atoms, the trapping frequency, the density of the atoms (taking into account the possibility of 3 body collisions that might be involved in the heating mechanism), etc. Preliminary results of the planned experiment are shown in figure 7.3, where we present the measured heating rates as a function of the initial temperatures for various radial trapping
Figure 7.3: Preliminary experimental results from our lab of heating rates as a function of the initial temperatures for various radial trapping frequencies. Frequencies. Unfortunately, this figure does not present a clear picture of the expected behavior. One of the reasons for this might be that this type of measurements requires an extremely stable system (each data point of the plot is about a half a day of measurements). Our system has undergone many changes during the time period this data has been obtained and measurements that have been repeated a few times produced different results without any clear reason. A good advice for a future experimentalist planning to perform this experiment is to concentrate the effort on very low radial trapping frequencies. At these frequencies, as we have learned during our work on the experiment described in chapter 5 the spectral density of the noise is low and flat, which makes it a simpler and more controllable environment to work with.
Chapter 8

Summary

In this work we have performed several experiments with ultacold atoms. Some of the results have been submitted for publication in


In chapter 3 we have described the characterization of a BEC, containing $\sim 10^4$ atoms, which was obtained during my work in the laboratory. We have confirmed the condensation of the atomic cloud and measured its properties, including the transition temperature, chemical potential, condensate lifetime and heating rate. The observation of a BEC is the starting point for many advanced experiments with ultracold atoms, for example a double well BEC interferometry experiment planned in our lab.

In the experiment described in chapter 5, we have observed asymmetric transition rates between Zeeman levels of magnetically trapped atoms, and found that this asymmetry strongly depends on the spectral shape of the applied noise. This effect follows from the interplay between the internal states of the atoms and their external degrees of freedom, where different trapped levels experience different potentials. We have shown that this asymmetry can be controlled by the use of engineered noise. This experiment can be described in a more general manner; we have examined the affect of colored noise on a two level system coupled to external (translational, in
our case) degrees of freedom and to a weak homogeneous field inducing transitions between the internal levels. The internal states of the system (atoms, in our case) are usually controlled by monochromatic fields, while incoherent fluctuations (noise) are usually studied under the “white noise” assumption. Little was previously known about what happens between the monochromatic and white noise limits (colored noise) with respect to control and hindering effects. The findings of this work may be utilized to control the relative population of the different Zeeman levels. Just as important, this insight may serve to better understand how noise couples to atoms in magnetic traps and may pave the way for effective schemes for combating background and technical noise, which are of great concern in atom chip experiments both for fundamental studies and technological applications. This work may also form the basis for schemes in which noise is used to combat decoherence as suggested in Ref. [30].

In the experimental outlook of chapter 6, we have performed a feasibility study for an experiment with anisotropic electrical conductors (AEC). We have shown that a clear dependence of the atomic lifetime on the AEC’s orientation should be obtained with the suggested experimental setup. This experiment would verify the theory of spin-flips in the vicinity of AECs (see chapter 4). In this chapter, we have also presented a design of a new atom chip mount that will be used for this experiment and others.

In chapter 7 we have presented a description of a proposed experiment to check an expected relation between the heating rate of an atomic cloud and its anharmonicity. This experiment may be used to examine and better understand the heating mechanisms of cold atoms as a function of the trapping parameters.

In addition to the experimental work, including BEC characterization, observation of asymmetric spin changing transition rates, new mount design and a feasibility study for two experiments, we have also done some theoretical work. This includes: the explicit calculation of the geometric integrals in chapter 4, which to the best of our knowledge have never been written this explicitly before; the theoretical explanations for the asymmetric spin changing transition rates of chapter 5, including lifetime and mean transition energies; and the definition of the anharmonicity of an atomic cloud and the harmonic range of a magnetic trap described in chapter 7.
CHAPTER 8. SUMMARY

The results of this work may prove useful for controlling atomic states by the introduction of noise and will hopefully contribute to the understanding of the affect of noise on atoms enabling better control over hindering effects in future quantum devices.
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