Two-photon spatial coherence in a two-photon temporal coherence experiment

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Two-photon interference is a fundamental phenomenon in quantum mechanics and stands at the base of numerous experimental observations. Here another manifestation of this phenomenon is described, taking place at the beam splitter in photon counting experiments. A way of measuring this new manifestation is discussed. Specifically it is shown how the r^2+t^2 term which is behind previous observations of two-photon interference, may be observed also in photon counting experiments. This serves to emphasize the fundamental similarity between all two-photon experiments. Finally, two-photon states impinging on a Y junction are shown to exhibit two-particle quantum amplification.

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Two-photon interference is one of the most important representations of quantum optics. It has been reviewed in many text books as well as numerous scientific papers. At the base of the effect stands the measured joint probability of the detection of two particles (typically photons) by two detectors. This is usually referred to as intensity correlations and is very different from the amplitude correlations (or first order coherence) which is measured in a typical interference experiment such as the well known double slit experiment.

Two-photon interference has thus far presented itself in several ways including the HBT (Hanbury Brown and Twiss) effect [1–8] and the HOM (Hong-Ou-Mandel) effect [9, 10]. Quantitatively, this phenomenon is described by the second order coherence function [11]. This function is defined as

$$g^{(2)} = \langle I_1 I_2\rangle / \langle I_1\rangle\langle I_2\rangle \tag{1}$$

where I_1 and I_2 are the currents in the two photodetectors, and the angled brackets denote the mean. An extensive description of the phenomenon appears in the book of Scully and Zubairy [12] and in numerous papers (e.g. Mandel's overview [13]). The results of a $g^{(2)}$ experiment typically depend on the type of source (e.g. thermal - two independent atoms, correlated photons from a down converter, etc.) and on the phase the particles accumulate on the way to the detectors by virtue of optical elements such as beam splitters or simply distance. For example, in the case of the HBT effect the phases are accumulated due to different paths while in the case of the HOM effect they are accumulated due to an interaction with an optical element. These are technical details which should not mask the common fundamental origin.

The beauty of two-photon interference as well as the source of many of the resulting misconceptions lies in the fact that it cannot be viewed as the interference of two waves, as is done in the double slit experiment. This was nicely exhibited by the experiment of Pittman et al. in which the two photon wave packets hit a beam splitter at different times, and still interference was observed [14]. Two-photon interference is the interference between two possible occurrences (or histories) which are indistinguishable as they lead to the same observed result.

Photon counting experiments are experiments in which the emission temporal statistics of a source are measured [15]. These experiments are related to the above experiments in the fact that both are sensitive to the source characteristics, and in the fact that what is measured is the $g^{(2)}$ intensity-intensity correlation as before (here, as a function of time between two photon detections). The difference between spatial coherence and temporal coherence is nicely described in Ref. [11]. In the following I use the term "photon counting experiment" (PCE) or "photon counting configuration" (PCC) simply to note a geometry in which both photons impinge on the beam splitter (BS) from the same port. The three types of experiments noted so far are presented in Fig. 1.

In this paper I wish to emphasize a fundamental similarity between PCEs and the experiments described above, by analyzing the states produced by the BS and ways of measuring them. In addition, I wish to point out some unique features of the PCC and HOM states.

PCEs may actually be done by one high temporal resolution detector. This arrangement indeed tests the characteristics of the source which give $g^{(2)}(\tau = 0) > 1$ for a chaotic source, $= 1$ for a coherent source and < 1 for a non-classical source. However, in practical experiments, a 50/50 BS is added and two detectors are positioned symmetrically relative to this BS so that their individual inability to detect small time gaps between photons, does not inhibit the joint detection of such gaps. Hence, one detector starts the clock while the other stops it. This is typically treated as a mere technicality, but is it?

By way of analogy, the question may be asked: if a BS introduces such strong detection correlations in the HOM two-photon interference experiment, may it also create such correlations here, when the two photons impinge from the same port?

The usual theoretical treatment suggests that a PCE with one detector is exactly the same as two detectors with a BS. Let us see why. We first define the relation between the incoming and outgoing modes of the BS. If

$\hat a^\dagger$ and $\hat a$ are the creation and annihilation operators of the incoming light mode via which the photons impinge on the BS, and if $\hat c_k$ and $\hat c_k^\dagger$ are the operators of the two outgoing modes ($k = 1, 2$), we may take the relations to be $\hat a^\dagger = (\hat c_1^\dagger + \hat c_2^\dagger)/\sqrt{2}$ and $\hat a = (\hat c_1 + \hat c_2)/\sqrt{2}$. This means that the average photon number (or expectation value) in detector 1 (in the presence of a BS) is:

$$\bar n_1 = \langle n|\hat c_1^\dagger \hat c_1|n\rangle = \langle 0|(\hat a)^n \hat c_1^\dagger \hat c_1 (\hat a^\dagger)^n|0\rangle/n! = \frac{1}{2}n \qquad (2)$$

and similarly for $\bar n_2$. The two-detector correlation is $\langle n_1 n_2\rangle = \langle n|\hat c_1^\dagger \hat c_2^\dagger \hat c_2 \hat c_1|n\rangle$. Expanding as we did previously this gives $\langle n_1 n_2\rangle = \frac{1}{4}n(n-1)$. One thus finds

$$g^{(2)}(\tau) = \frac{\langle n_1 n_2\rangle}{\bar n_1 \bar n_2} = \frac{n}{4}(n-1)/\frac{1}{4}n^2 = 1 - \frac{1}{n} \qquad (3)$$

which is exactly what one gets without a BS, namely,

$$g^{(2)}(\tau) = \frac{\langle \hat a^\dagger \hat a^\dagger \hat a \hat a\rangle}{\langle \hat a^\dagger \hat a\rangle^2} \qquad (4)$$

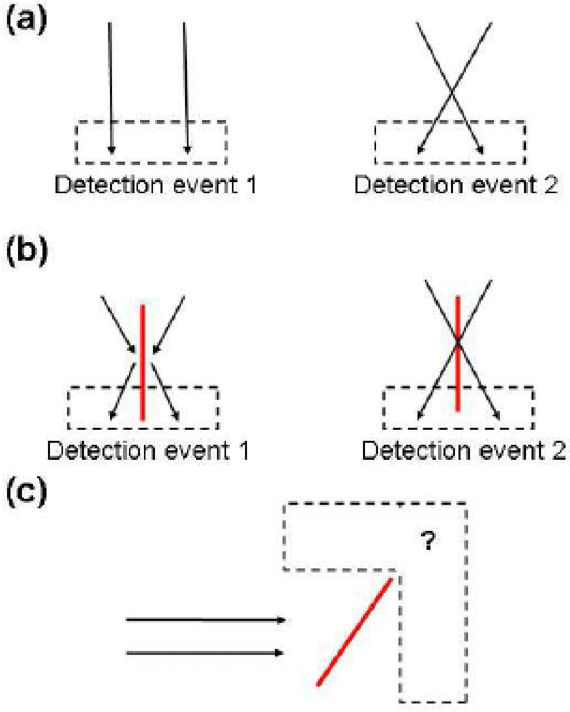

FIG. 1: Three types of two-photon experiments: (a) a HBT experiment (e.g. with two atoms at its source). (b) a HOM experiment. (c) typical photon counting experiment. In (a-b) two indistinguishable events are shown. In the first, the phase difference between the two events is due to optical path length, and in the second to the optical element and the phase difference between reflection and transmission. In (b-c) the beam splitter is shown in red. Arrows indicate photon trajectories from source to detector. In (c) there are no two indistinguishable events with a phase difference, and thus "common practice" assumes there is no two-photon interference.

Introducing a phase into the above relations between $\hat a$ and $\hat c$ changes nothing.

We thus see that indeed the splitting or the phase makes no difference. More so, the literature emphasizes that this situation, in which both particles impinge on the BS from the same side, does not create probabilities different from those for classical particles [16]. It is therefore understandable that an analysis of interference effects caused by this BS has to the best of my knowledge been neglected, and that PCEs have been to a large extent described as lying outside the family of two-photon spatial interference experiments.

To complete our introduction, let us now introduce for the sake of unitarity a $\pi/2$ phase into the reflected mode of the BS by defining $\hat a^\dagger = (\hat c_1^\dagger + i\hat c_2^\dagger)/\sqrt{2}$ and $\hat a = (\hat c_1 - i\hat c_2)/\sqrt{2}$, and verify that this indeed gives the HOM effect, where the phase does matter. For the other incoming port we have $\hat b^\dagger = (i\hat c_1^\dagger + \hat c_2^\dagger)/\sqrt{2}$. We present the different modes in Fig. 2a.

For the HOM scenario in which two correlated photons hit the BS, each photon from a different side, we have:

$$\hat a^\dagger \hat b^\dagger|0\rangle = \frac{1}{2}(\hat c_1^\dagger + i\hat c_2^\dagger)(i\hat c_1^\dagger + \hat c_2^\dagger)|0\rangle = \frac{i}{\sqrt{2}}(|2,0\rangle + |0,2\rangle) \qquad (5)$$

as for bosons the two outgoing operators commute. $|2,0\rangle$ and $|0,2\rangle$ equal $\frac{1}{\sqrt{2}}\hat c_1^\dagger \hat c_1^\dagger|0\rangle$ and $\frac{1}{\sqrt{2}}\hat c_2^\dagger \hat c_2^\dagger|0\rangle$ respectively.

For arbitrary reflection coefficients, the same result may be described in the following way (e.g. [16]):

$$\sqrt{2}rt|2,0\rangle + \sqrt{2}rt|0,2\rangle + (r^2 + t^2)|1,1\rangle \qquad (6)$$

where r and t are simply the reflection and transmission amplitudes, and $|1,1\rangle = \hat c_1^\dagger \hat c_2^\dagger|0\rangle$. For the case of a 50/50 BS one has $r = te^{i\frac{\pi}{2}}$. As usual, the squared moduli of the coefficients gives the associated probabilities. We see that the fundamental reason for the HOM two-photon interference resulting in no-coincidence counts lies entirely

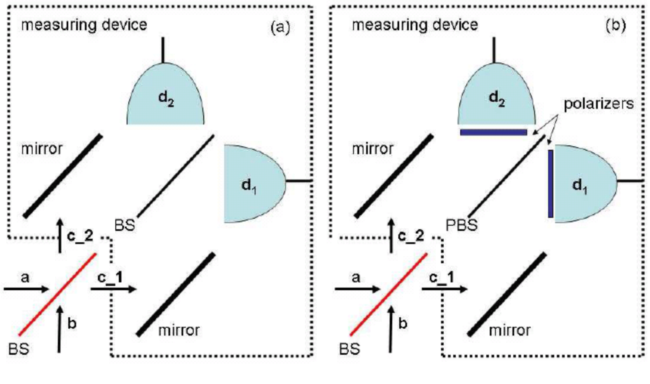

FIG. 2: A typical PCE consisting of a source (mode a) and a beam splitter (in red): (a) measured by an external beam splitter and two photo-diodes d_1 and d_2, and (b) measured by an external polarizing beam splitter with polarizers set to $+45^o$ in front of the detectors.

in the fact that the amplitude of the $|1,1\rangle$ state is r^2+t^2 which, for a 50/50 BS, is just zero.

We now go back to the operation of the BS in a PCC. The main message of this letter is to point out that exactly the same r^2+t^2 amplitude may be found in a PCC.

One may easily verify that in the case of a PCC, one has (e.g. [16]):

$$|\psi\rangle = \frac{1}{\sqrt{2}}\hat{a}^\dagger\hat{a}^\dagger|0\rangle = r^2|2,0\rangle + \sqrt{2}rt|1,1\rangle + t^2|0,2\rangle. \quad (7)$$

We see that the total amplitude that we find the state $|0,2\rangle + |2,0\rangle$ is r^2+t^2, indicating that different from the classical case, there is some possible measurement of photon number for which the probability is zero. But how can this be measured?

To further our understanding let us analyze the measurement scheme in Fig. 2a. From Eq. (7) one finds

$$|\psi\rangle = \frac{1}{\sqrt{2}}(t^2\hat{c}_1^\dagger\hat{c}_1^\dagger + r^2\hat{c}_2^\dagger\hat{c}_2^\dagger)|0\rangle + \sqrt{2}rt(\hat{c}_1^\dagger\hat{c}_2^\dagger)|0\rangle, \quad (8)$$

and upon insertion of the definitions of $\hat{c}_1$ and $\hat{c}_2$ in terms of $\hat{d}_1$ and $\hat{d}_2$, we find

$$\begin{aligned} |\psi\rangle = \frac{1}{\sqrt{2}}[(t^2(r\hat{d}_1^\dagger + t\hat{d}_2^\dagger)(r\hat{d}_1^\dagger + t\hat{d}_2^\dagger) + \\ r^2(r\hat{d}_2^\dagger + t\hat{d}_1^\dagger)(r\hat{d}_2^\dagger + t\hat{d}_1^\dagger)) + 2rt(r\hat{d}_1^\dagger + t\hat{d}_2^\dagger)(r\hat{d}_2^\dagger + t\hat{d}_1^\dagger)] = \\ (r^4+t^4)|0,2\rangle^d + 2t^2r^2|2,0\rangle^d + \sqrt{2}rt(t^2+r^2)|1,1\rangle^d + \\ 2t^2r^2|2,0\rangle^d + 2t^2r^2|0,2\rangle^d + \sqrt{2}rt(r^2+t^2)|1,1\rangle^d = -|2,0\rangle^d. \quad (9) \end{aligned}$$

We thus find that just like in the HOM effect, the t^2+r^2 factor originating from the first BS has eliminated the probability for coincidence counts in the third term. Furthermore, the above result is an outcome of a rather dramatic destructive interference between the events in which the photons are bunched after the first BS and the event where they are anti-bunched (first and fifth terms), where the first term again arises from the t^2+r^2 factor of the first BS. This eliminates the $|0,2\rangle^d$ state. However, this is not a very appealing effect due to the fact that all photons exit through the $|2,0\rangle^d$ state, a result which is identical to that obtained from two distinguishable photons transversing a Mach-Zehnder interferometer.

Let us complete the discussion by presenting a scheme which cannot be explained as a sum over single photon evolution. As is shown in Fig. 2b, we replace the second BS with a polarization BS (PBS). We assume that there is no phase difference induced by the PBS between the reflected and transmitted modes. Furthermore, we utilize a source which gives rise to two photons with perpendicular polarizations (e.g. type II down converter). Last, we place $+45^o$ polarizers in front of the detectors. As the operation of the first BS is polarization independent we find for $|\psi\rangle = \hat{a}_\parallel^\dagger\hat{a}_\perp^\dagger|0\rangle$ as in Eq. (7)

$$r^2\hat{c}_{2,\parallel}^\dagger\hat{c}_{2,\perp}^\dagger + rt(\hat{c}_{1,\parallel}^\dagger\hat{c}_{2,\perp}^\dagger + \hat{c}_{2,\parallel}^\dagger\hat{c}_{1,\perp}^\dagger) + t^2\hat{c}_{1,\parallel}^\dagger\hat{c}_{1,\perp}^\dagger. \quad (10)$$

Following the known features of a PBS, we now make the interchanges $\hat{c}_{1,\parallel}^\dagger \to \hat{d}_{1,\parallel}^\dagger$, $\hat{c}_{1,\perp}^\dagger \to \hat{d}_{2,\perp}^\dagger$, $\hat{c}_{2,\parallel}^\dagger \to \hat{d}_{2,\parallel}^\dagger$, and $\hat{c}_{2,\perp}^\dagger \to \hat{d}_{1,\perp}^\dagger$. This gives,

$$\begin{aligned} r^2\hat{d}_{2,\parallel}^\dagger\hat{d}_{1,\perp}^\dagger + rt(\hat{d}_{1,\parallel}^\dagger\hat{d}_{1,\perp}^\dagger + \hat{d}_{2,\parallel}^\dagger\hat{d}_{2,\perp}^\dagger) + t^2\hat{d}_{1,\parallel}^\dagger\hat{d}_{2,\perp}^\dagger = \\ r^2(\hat{d}_{2,+45}^\dagger + \hat{d}_{2,-45}^\dagger)(\hat{d}_{1,+45}^\dagger - \hat{d}_{1,-45}^\dagger) + \\ rt[(\hat{d}_{1,+45}^\dagger + \hat{d}_{1,-45}^\dagger)(\hat{d}_{1,+45}^\dagger - \hat{d}_{1,-45}^\dagger) + \\ (\hat{d}_{2,+45}^\dagger + \hat{d}_{2,-45}^\dagger)(\hat{d}_{2,+45}^\dagger - \hat{d}_{2,-45}^\dagger)] \\ +t^2(\hat{d}_{1,+45}^\dagger + \hat{d}_{1,-45}^\dagger)(\hat{d}_{2,+45}^\dagger - \hat{d}_{2,-45}^\dagger). \quad (11) \end{aligned}$$

where we have ignored the $1/\sqrt{2}$ factors.

Only the terms multiplied by r^2 and t^2 may give rise to a coincidence count, and as the detectors are preceded by polarizers set at $+45^o$, only the terms $r^2\hat{d}_{2,+45}^\dagger\hat{d}_{1,+45}^\dagger$ and $t^2\hat{d}_{1,+45}^\dagger\hat{d}_{2,+45}^\dagger$ may give rise to a coincidence count. As $[\hat{d}_2^\dagger, \hat{d}_1^\dagger] = 0$ we may join these two terms to give $(r^2+t^2)\hat{d}_{1,+45}^\dagger\hat{d}_{2,+45}^\dagger$. As $r^2+t^2=0$ these two terms cancel one another. Thus, the coincidence count rate will be zero ($g^{(2)}=0$). This result is a unique indication of two-photon interference and it cannot be imitated by two single photons as these would give rise to a coincidence count with a probability of 2/16 ($g^{(2)}=\frac{1}{2}$). Let us note what is trivial but perhaps insightful, and that is that the projection onto the 45^o basis by the polarizers acts as a quantum eraser erasing which-path information [17–20]. The polarizers have erased the information of whether what transpired was a $|0,2\rangle$ or a $|2,0\rangle$ event, these events being very different from the HOM events.

Finally, we analyze the difference between the HOM and PCC states in the context of quantum reflection or transmission when these states impinge on a Y junction. Quantum reflection or transmission is a term used when classical probabilities cannot explain a reflection, e.g. when a wave packet hits a potential well, or transmission, e.g. when a wave packet tunnels through a barrier. Numerous effects may be termed quantum reflection, from atoms reflected from a surface [21], through coherent back scattering of light [22] to electron transport through a ring [23]. Examples of quantum transmission are abundant as well.

There is a unique difference between the HOM state of Eq. (6) and the PCC state described by Eq. (7): while the probabilities of the two surviving bunched states in the former add to one, the probability of the single state in the latter not connected to the t^2 and r^2 terms amounts to one half. If we could therefore keep only the anti-bunched state in Eq. (7), we would be in fact forcing half of the HOM flux to be reflected back.

Let us take away all the additional elements we introduced previously and retain only the single BS of the PCC. If each outgoing port of our BS could be coupled into a Y junction to combine the two into one mode in which all three states of Eq. (7) are indistinguishable, we would add the three amplitudes to calculate the flux in the combined channel and find that the flux has dropped to $(\sqrt{2}rt)^2 = 1/2$. We would thus expect 50% of the photons to be reflected back towards the source, relative to the HOM state. While previous realizations of quantum reflection or transmission utilize single particle physics, this difference would be due to two-particle physics.

Let us review this with more detail. We have analyzed Y junctions in [24, 25]. As an example, we utilize a real 3×3 transfer matrix which we have analyzed in [26] and which transfers the vector of three outgoing amplitudes to a vector of three incoming amplitudes:

$$\begin{pmatrix} 1-2T & \sqrt{2T(1-T)} & \sqrt{2T(1-T)} \\ \sqrt{2T(1-T)} & -(1-T) & T \\ \sqrt{2T(1-T)} & T & -(1-T) \end{pmatrix},$$

where T is the "cross talk" (transfer amplitude) between the two incoming channels. We name the outgoing (combined) port with index 1 and the two incoming ports (coming from our BS to the Y junction) as 2 and 3. We thus see that $\hat{a}^{\dagger}_{2,in} = \sqrt{2T(1-T)}\hat{a}^{\dagger}_{1,out} - (1-T)\hat{a}^{\dagger}_{2,out} + T\hat{a}^{\dagger}_{3,out}$ and similarly $\hat{a}^{\dagger}_{3,in} = \sqrt{2T(1-T)}\hat{a}^{\dagger}_{1,out} + T\hat{a}^{\dagger}_{2,out} - (1-T)\hat{a}^{\dagger}_{3,out}$. As our incoming state is [Eq. (8)] $\frac{1}{\sqrt{2}}r^2(\hat{a}^{\dagger}_{2,in})^2 + \frac{1}{\sqrt{2}}t^2(\hat{a}^{\dagger}_{3,in})^2 + \sqrt{2}rt\hat{a}^{\dagger}_{2,in}\hat{a}^{\dagger}_{3,in}$, we find the outgoing state to be

$$\begin{aligned} |\psi\rangle = {} & \frac{1}{\sqrt{2}}r^2(\sqrt{2T(1-T)}\hat{a}^{\dagger}_{1,out} - (1-T)\hat{a}^{\dagger}_{2,out} + T\hat{a}^{\dagger}_{3,out})^2 + \\ & \frac{1}{\sqrt{2}}t^2(\sqrt{2T(1-T)}\hat{a}^{\dagger}_{1,out} + T\hat{a}^{\dagger}_{2,out} - (1-T)\hat{a}^{\dagger}_{3,out})^2 + \\ & \sqrt{2}rt(\sqrt{2T(1-T)}\hat{a}^{\dagger}_{1,out} - (1-T)\hat{a}^{\dagger}_{2,out} + T\hat{a}^{\dagger}_{3,out}) \times \\ & (\sqrt{2T(1-T)}\hat{a}^{\dagger}_{1,out} + T\hat{a}^{\dagger}_{2,out} - (1-T)\hat{a}^{\dagger}_{3,out}) \end{aligned} \tag{12}$$

Thus, the probability for having $\hat{a}^{\dagger}_{1,out}\hat{a}^{\dagger}_{1,out}$ is

$$|\frac{1}{\sqrt{2}}(r^2+t^2) + \sqrt{2}rt|^2|[2T(1-T)]|^2 = \frac{1}{2}[2T(1-T)]^2 \tag{13}$$

where we have used for the equality $t = \frac{1}{\sqrt{2}}$ and $r = \frac{i}{\sqrt{2}}$.

For the HOM state [Eq. (6)], one has

$$|\psi\rangle = rt(\hat{c}^{\dagger}_1\hat{c}^{\dagger}_1 + \hat{c}^{\dagger}_2\hat{c}^{\dagger}_2)|0\rangle + (t^2+r^2)(\hat{c}^{\dagger}_1\hat{c}^{\dagger}_2)|0\rangle, \tag{14}$$

which, following the same path that led to Eq. (13), gives

$$|2rt + (t^2+r^2)|^2|[2T(1-T)]|^2 = [2T(1-T)]^2, \tag{15}$$

where we have again used the same t and r.

One may hypothesize that this indicates that bunched states scatter forward better than anti-bunched states, but this is not so as this difference is independent of the value of the reflection and transmission coefficients.

As for indistinguishable particles $\frac{1}{\sqrt{2}}\hat{a}^{\dagger}_{1,out}\hat{a}^{\dagger}_{1,out} = |2,0,0\rangle$, the final transmission (forward scattering) probability of the PCC state would be $[2T(1-T)]^2$ and for the HOM state $2[2T(1-T)]^2$. As expected, the former is smaller by a factor of two relative to the latter.

For an input state of two independent photons one finds:

$$[|r+t|^2|\sqrt{2T(1-T)}|^2]^2 = [2T(1-T)]^2. \tag{16}$$

As for two independent photons $\hat{a}^{\dagger}_{1,out}\hat{a}^{\dagger}_{1,out} = |2,0,0\rangle$, the final probability for forward scattering is $[2T(1-T)]^2$. We thus find the forward going flux of the PCC state to be the same as the flux of two independent photons. The flux of the HOM state, being twice as large, may then be named quantum transmission enhancement or amplification.

To conclude, we have shown that the $t^2 + r^2$ interference term responsible for the HOM effect may be observed also in the photon counting experiment configuration when utilizing the correct measurement basis. These findings are another step in the conceptual unification of all two-photon experiments. In addition, we have shown that when different two-photon states impinge on a Y junction, one finds different forward going fluxes.

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