

Moduli stabilization, SUSY breaking and Cosmology

Ram Brustein



אוניברסיטת בן-גוריון

PRL 87 (2001), hep-th/0106174

PRD 64 (2001), hep-th/0002087

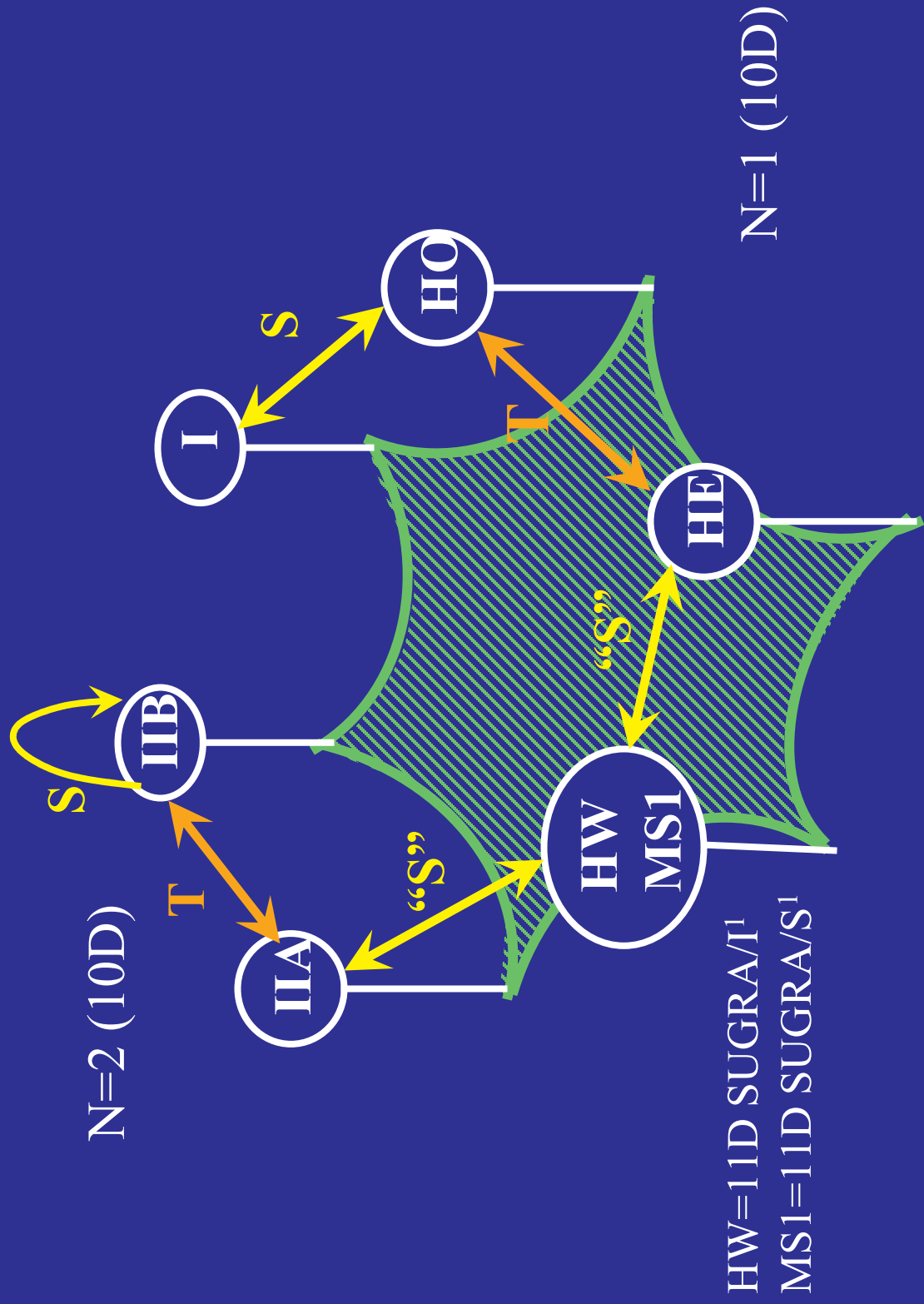
hep-th/0205042

hep-th/0212344

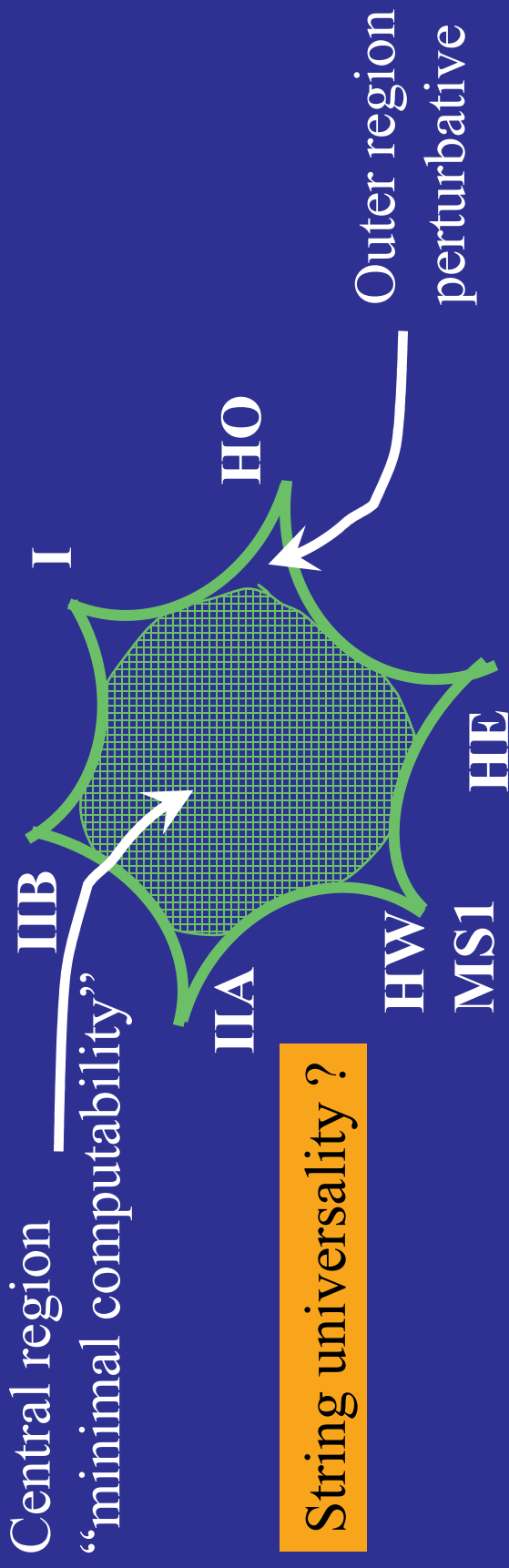
with S. de Alwis, E. Novak

- * Moduli space of effective theories of strings
- * Outer region of moduli space: problems!
- * “central” region:
 - * stabilization
 - * interesting cosmology

String Theories and 11D SUGRA



String Moduli Space



String universality ?

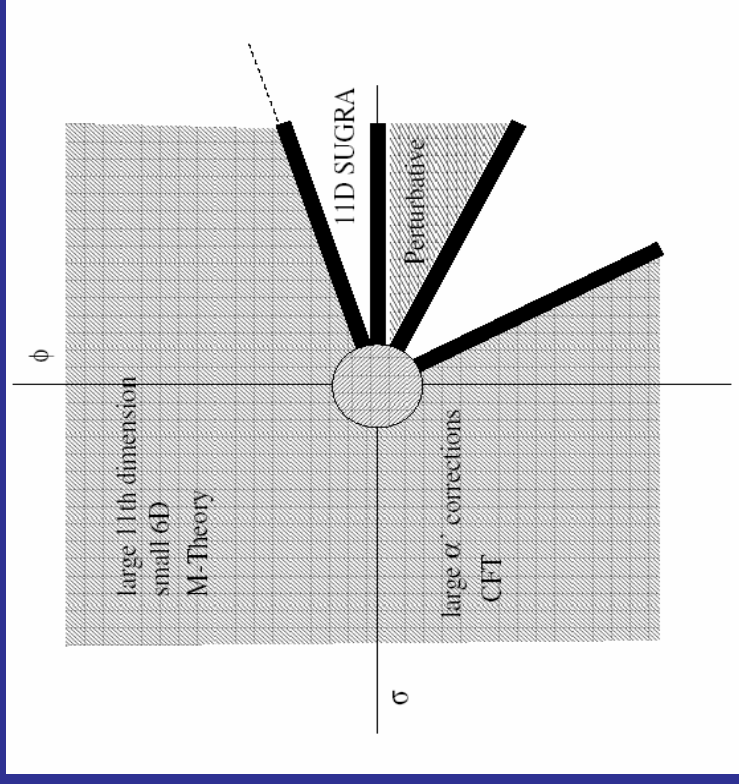
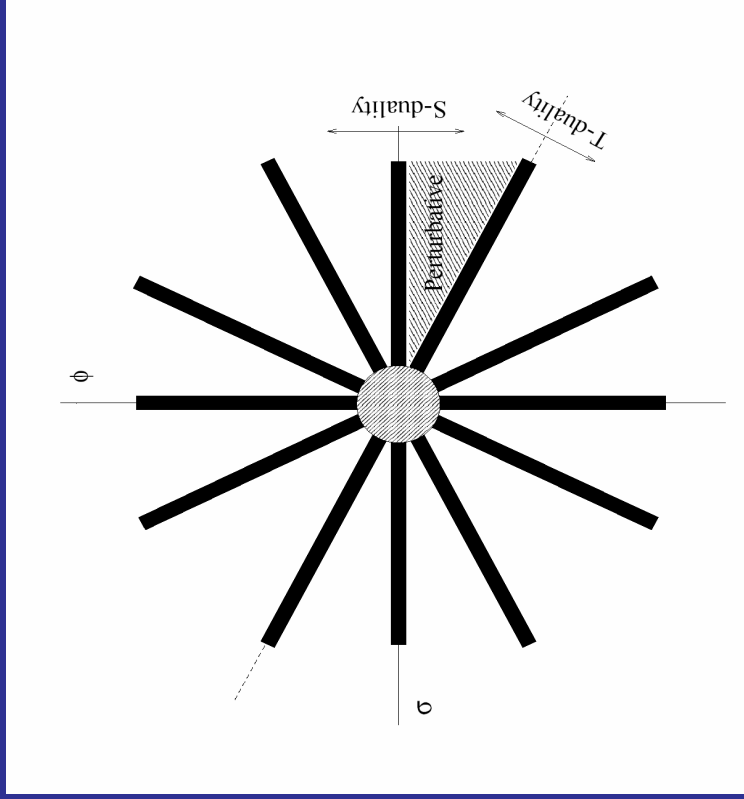
Requirements

- $D=4$
 - $N=1$ SUSY $\rightarrow N=0$
 - $CC < (m_{3/2})^4$
 - SM (will not discuss)
 - Volume/Coupling moduli
- T S

Perturbative theories =
phenomenological disaster

- SUSY + massless moduli
- Gravity \neq Einstein's
- Cosmology

Cosmological moduli space



$$ds_{10}^2 = G_{MN} dx^M dx^N = \frac{1}{2\sqrt{3}} e^{\sigma + \frac{1}{2}\phi} (e^{-\frac{2}{\sqrt{3}}\sigma} g_{\mu\nu} dx^\mu dx^\nu + g_{mn}^c dy^m dy^n)$$

$$ds_{11}^2 = e^{\frac{1}{2\sqrt{3}}\sigma - \frac{1}{6}\phi} (e^{-\frac{2}{\sqrt{3}}\sigma} g_{\mu\nu} dx^\mu dx^\nu + g_{mn}^c dy^m dy^n) + e^{\frac{4}{3}\phi} dz^2$$

“Lifting Moduli”

- Perturbative
 - Compactifications
 - Brane Worlds
- Non-Perturbative
 - SNP = Brane instantons
 - Field-Theoretic, e.g., gaugino-condensation
- Generic Problems
 - **P**ractical **C**osmological **C**onstant **P**roblem
 - Runaway potentials (not solved by duality)

BPS Brane-instanton SNP's

From hep-th/0002087

Euclidean wrapped branes

Potential $V \sim e$ -action

Complete under duality

Type I		HO	
brane	action	brane	action
$D5$	$\frac{1}{g_I} \left(\frac{R}{l_I}\right)^6$	$F5$	$\frac{1}{g_{HO}^2} \left(\frac{R}{l_{HO}}\right)^6$
$D1$	$\frac{1}{g_I} \left(\frac{R}{l_I}\right)^2$	$F1$	$\left(\frac{R}{l_{HO}}\right)^2$
HE		HW	
brane	action	brane	action
$F5$	$\frac{1}{g_{HE}^2} \left(\frac{R}{l_{HE}}\right)^6$	$M5$ transverse	$\left(\frac{R}{l_{II}}\right)^6$
$F1$	$\left(\frac{R}{l_{HE}}\right)^2$	$M2$ longitudinal	$\left(\frac{R}{l_{II}}\right)^2 \frac{\rho}{l_{II}}$
IIA		MS1	
brane	action	brane	action
$D0$	$\frac{1}{g_{IIA}} \frac{R}{l_{IIA}}$	KK graviton	$\frac{R}{\rho}$
$F1$	$\left(\frac{R}{l_{IIA}}\right)^2$	$M2$ longitudinal	$\left(\frac{R}{l_{II}}\right)^2 \frac{\rho}{l_{II}}$
IIA		IIB	
brane	action	brane	action
$D0$	$\frac{1}{g_{IIA}} \frac{R_{IIA}}{l_{II}}$	$D-1$	$\frac{1}{g_{IIB}}$
$D0$	$\frac{1}{g_{IIA}} \frac{R}{l_{II}}$	$D1$	$\frac{R R_{IIB}}{g_{IIB} l_{II}^2}$
$F1$	$\frac{R R_{IIA}}{l_{II}^2}$	KK MM	$\frac{R}{R_{IIB}}$
$F1$	$\frac{R^2}{l_{II}^2}$	$F1$	$\frac{R^2}{l_{II}^2}$

$$D5 : V \sim e^{-\left[e^{\frac{\sqrt{3}}{2}\sigma} + \frac{1}{2}\phi \right]}$$

$$D1 : V \sim e^{-\left[e^{\frac{1}{2\sqrt{3}}\sigma} - \frac{1}{2}\phi \right]}$$

Outer Region

Moduli – chiral superfields of N=1 SUGRA,

N=1 SUGRA

$$V = e^{-K} (F_i K^{i\bar{j}} F_{\bar{j}} - 3|W|^2)$$

$$K = K(S, S^*), \quad W = W(S)$$

$$F_S = \partial_S W + K_S W$$

Pert. Kahler $K = -\ln(S + S^*)$

Steep potentials

$$\frac{|(S + S^*) \partial_S^{(n+1)} W|}{|\partial_S^n W|} \gg 1, \quad n = 0, 1, 2, 3.$$

$$W(S) = \sum_i e^{-\beta_i S}, \quad \text{with } \text{Re } \beta_i \gg 1.$$

e.g: $\text{Re } S = e^{-2\phi} \sqrt{\frac{1+\cos\theta}{2}}$

$\Lambda < 0$

$$V(S, S^*) = (S + S^*) F(S, S^*) F^*(S, S^*) - \frac{3}{S + S^*} W(S) W^*(S^*)$$

Outer Region Stabilization?

$$\partial_S V = (S + S^*) \partial_S^2 W(S) F^* - \frac{2}{S + S^*} F W^*,$$

$$\partial_{S^*} V = (S + S^*) \partial_{S^*}^2 W^*(S^*) F^* - \frac{2}{S + S^*} F^* W.$$

$$F(S, S^*) = \partial_S W(S) - [1/(S + S^*)] W(S)$$

Extremum:

$$(S + S^*)^2 \partial_S^2 W(S) F^* = 2 F W^*$$

(ii)

$$F \neq 0 \quad \text{and} \quad (S + S^*)^2 \partial_{S^*}^2 W(S) F^* = 2 F W^*$$

Two types:

(i) $F = 0$.

Min?, Max?, Saddle?

$$H = \frac{\partial^2 V}{\partial S_R \partial S_R} \frac{\partial^2 V}{\partial S_I \partial S_I} - \left(\frac{\partial^2 V}{\partial S_R \partial S_I} \right)^2$$
$$= -4(\partial_{SS} V \partial_{S^* S^*} V - \partial_{SS^*} V \partial_{S^* S} V).$$

Case (i)

$$H = 4 \left((S+S^*) |\partial_S^2 W|^2 - \frac{2}{(S+S^*)^3} |W|^2 \right)^2 - \frac{4}{(S+S^*)^2} |\partial_S^2 W|^2 |W|^2.$$

$$\partial_{S_R S_R}^2 V \approx 2(S+S^*) |\partial_S^2 W|^2 > 0.$$

Case (i) is a minimum

Case (ii)

$$H \approx -4(S+S^*)^2 |\partial_S^3 W|^2 |F|^2.$$

Case (ii) is a saddle point

In general, max or saddle, but never min !

Outer Region Cosmology: Slow-Roll?

- Without a potential: 4D, 5D, 10D, 11D : “fast-roll”

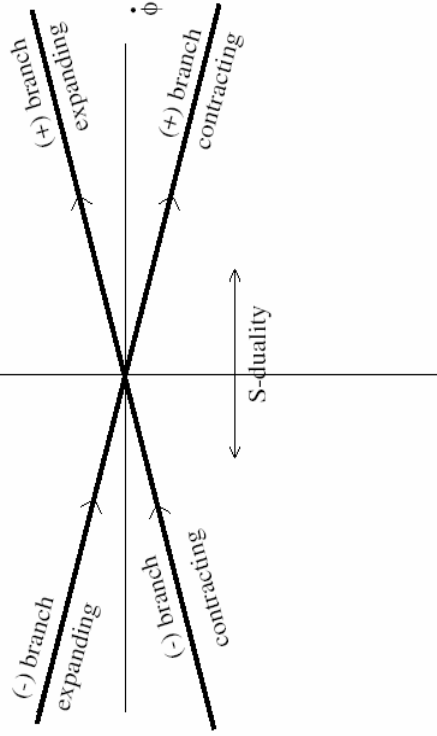
$$\begin{aligned}
 a(t) &= a(1)|t|^{\frac{1}{3}} \\
 \phi(t) &= \phi(1) + p_\phi \ln|t| \\
 \sigma(t) &= \sigma(1) + p_\sigma \ln|t|
 \end{aligned}$$

5D – same solutions!

$$\begin{aligned}
 ds_5^2 &= C_5^2 |t|^{-\frac{1}{3} p_\sigma} (-dt^2 + a^2(1) |t|^{\frac{2}{3}} dx^i dx^i) + C_5^{-4} |t|^{\frac{1}{3} p_\sigma + p_\phi} dz^2 \\
 \sigma_5 &= \frac{\sqrt{3}}{2} \sigma(1) - \frac{1}{2} \phi(1) + \left(\frac{\sqrt{3}}{2} p_\sigma - \frac{1}{2} p_\phi \right) \ln|t| \\
 C_5 &= e^{-\frac{1}{4\sqrt{3}} \sigma(1) - \frac{1}{4} \phi(1)} .
 \end{aligned}$$

$$\begin{aligned}
 \phi(1) &\rightarrow -\phi(1) \\
 p_\phi &\rightarrow -p_\phi ,
 \end{aligned}$$

S-duality



T-duality

$$\begin{aligned}
 p_\phi &\rightarrow -\frac{1}{2} p_\phi - \frac{\sqrt{3}}{2} p_\sigma \\
 p_\sigma &\rightarrow -\frac{\sqrt{3}}{2} p_\phi + \frac{1}{2} p_\sigma ,
 \end{aligned}$$

- **With a potential**

Ansatz

$$a(t) = a(1)|t|^{p_a}$$

$$\psi_1(t) = \psi_1(1) + p_1 \ln |t|$$

$$\psi_2(t) = \psi_2(1) + p_2 \ln |t|$$

Solution

$$\gamma p_1 = -2$$

$$p_a^2 = \frac{1}{6} \left(\frac{1}{2} p_1^2 + \frac{1}{2} p_2^2 + A e^{\gamma \psi_1(1)} \right)$$

$$p_a = \frac{1}{2} \left(\frac{1}{2} p_1^2 + \frac{1}{2} p_2^2 \right)$$

$$-p_1 + 3p_a p_1 + A \gamma e^{\gamma \psi_1(1)} = 0$$

$$-p_2 + 3p_a p_2 = 0 \quad .$$

Use to find properties of solutions with real potential

$$V(\phi, \sigma) = A e^{\alpha \sigma + \beta \phi}$$

$$\psi_1 = \frac{\alpha \sigma + \beta \phi}{\sqrt{\alpha^2 + \beta^2}}$$

$$\psi_2 = \frac{-\beta \sigma + \alpha \phi}{\sqrt{\alpha^2 + \beta^2}}$$

$$\gamma = \sqrt{\alpha^2 + \beta^2}$$

$$p_a = \frac{1}{\gamma^2}$$

realistic steep potential

$$V'/V \sim 1/g_s, 1/g_s^2, V'/V \sim R/l_s$$

No slow-roll for real steep potential

Central Region

Our proposal:

- Parametrization with $D=4$, $N=1$ SUGRA
- Stabilization by SNP effects @ string scale
- Continuously adjustable parameter
- SUSY breaking @ lower scale by FT effects
- PCCP o.k. after SUSY breaking



VADIM: CAN YOU HAVE A CONTINUOUSLY ADJUSTABLE PARAMETER THAT IS NOT A MODULUS? ARE 2 AND 3 CONSISTENT
OFER: KACHRU ET AL CENTRAL REGION.

Stable SUSY breaking minimum

**Two Moduli, S (susy breaking direction),
T (orthogonal), $m_{3/2}/M_P = \epsilon \sim 10^{-16}$**

- (a) a minimum of the potential at (S_0, T_0) : $\partial_S V|_{\min} = 0, \partial_T V|_{\min} = 0$
- (b) with broken SUSY: $F_S|_{\min} \sim 0(\epsilon)$
- (c) and a small cosmological constant: $V|_{\min} < 0(\epsilon^2)$
- (d) in the central region: $ReS_0, ReT_0 \sim 1$.

In addition, for the SUSY preserving direction T we have

- (e) $F_T|_{\min} = 0$.

$$V = e^{-K} \left(F_i K^{i\bar{j}} F_{\bar{j}} - 3|W|^2 \right) \quad (1)$$

$$F_S = \partial_S W + K_S W ; F_T = \partial_T W + K_T W. \quad (2)$$

$$K^{S\bar{T}}|_{min} = 0 \quad (3)$$

$$(b),(c),(e) \text{ \& } (2) \rightarrow W|_{min}, \partial_S W|_{min}, \partial_T W|_{min} \sim O(\varepsilon) \quad (4)$$

(a),(b),(e) \& (2,3,4) \rightarrow

$$\partial_T F_S|_{min}, \partial_S \partial_T W|_{min}, \partial_S F_T|_{min} \sim O(\varepsilon) \quad (5)$$

With more work

$$W|_{\min}, \partial_S W|_{\min}, \partial_T W|_{\min}, \partial_S^2 W|_{\min},$$

$$\partial_{ST}^2 W|_{\min}, \partial_S^3 W|_{\min} \sim O(\varepsilon).$$

- Higher derivatives in S (> 3) and T (> 1), & mixed derivatives of order > 2 generically $O(1)$.
- In SUSY limit, in T direction, V is steep, all derivatives > 2 generically $O(1)$ @ min. In S direction, potential is very flat around min.
- Masses of SUSY breaking S moduli $o(\varepsilon)$ in general masses of T moduli $O(1)$.

Simple example

$$W_{SNP} = a_4(S - \tilde{S}_0)^4$$

$$\Delta W_{FT} = \sum b_i e^{-\beta_i S}$$

$$\Delta W_{FT} = \sum c_i e^{-\beta_i(S-S_0)}, \text{ so } c_i \sim O(\varepsilon)$$

$$W = a_4(S - S_0)^4 + a_3(S - S_0)^3 +$$

$$a_2(S - S_0)^2 + a_1(S - S_0) + a_0$$

$$a_4 \sim O(1), a_3 = -\frac{1}{6} \sum c_i \beta_i^3, a_2 = \frac{1}{2} \sum c_i \beta_i^2, a_1 = -\sum c_i \beta_i$$

$$a_0 = (\pm \sqrt{3K_{S\bar{S}}} - K_S) a_0.$$

- Reasonable working models,
- Additional SUSY preserving $\Lambda < 0$ minima!

Scales & Shape of Moduli Potential

- The width of the central region

In effective 4D theory:

kinetic terms multiplied by $M_s^8 V_6 (M_{11})^9 V_7$ in M).

Curvature term multiplied by same factors

“Calibrate” using 4D Newton’s const. $8\pi G_N = m_p^{-2}$

$$\Gamma = \frac{1}{2} \int d^4x \left\{ m_p^2 \sqrt{-g} R + m_p^2 \partial \psi \partial \psi \right\}$$

→ Typical distances are $O(m_p)$

$$W \approx M_S^3 \Rightarrow V \approx M_S^6 / m_p^2 \equiv \Lambda^4$$

NO VOLUME FACTORS!!!

Banks

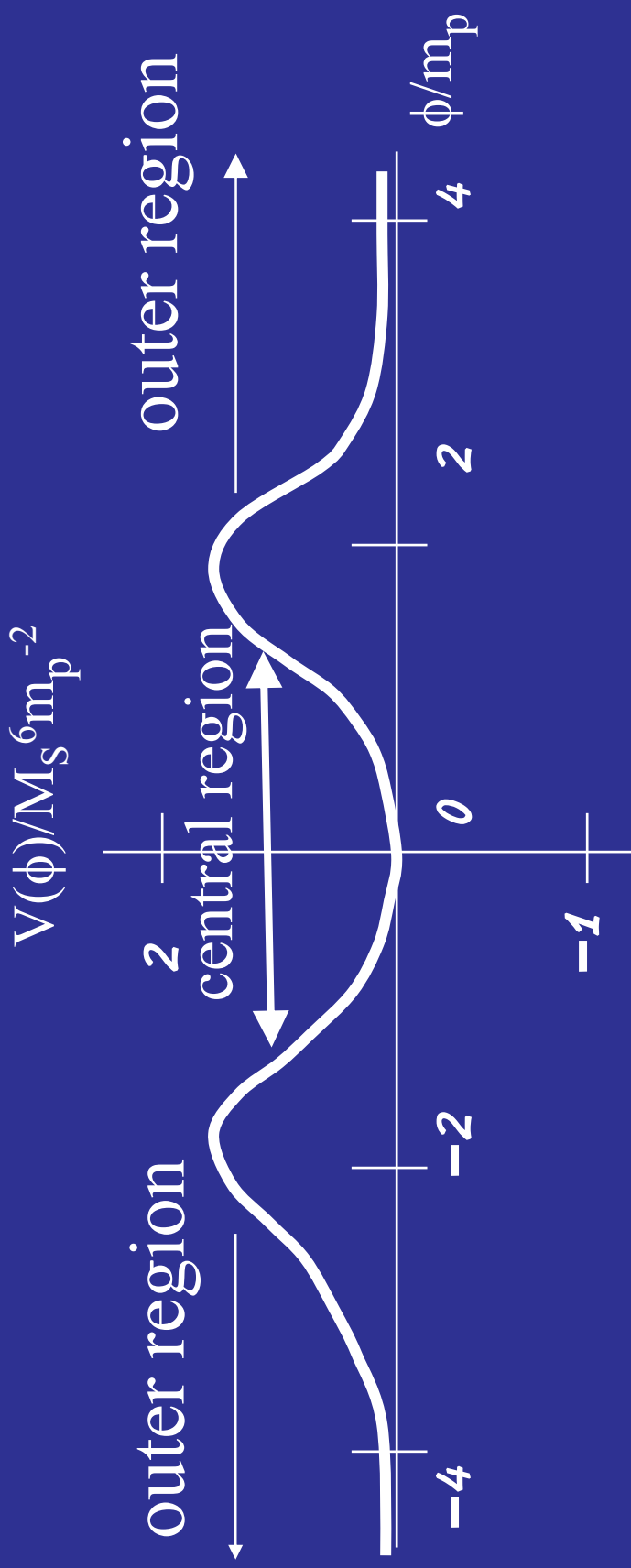
Numerical examples:

$$\Lambda_I = 8.6 \times 10^{16} \text{ GeV} \left(\frac{\alpha_{YM}}{1/25} \right)^{3/4} g^{3/4}$$

$$\Lambda_{HW} = 7.6 \times 10^{16} \text{ GeV} \left(\frac{\alpha_{YM}}{1/25} \right)^{-1/4} (4M_{GUT} V_6^{1/6})^{-3/2}$$

$$\begin{aligned} \Gamma &= \int d^4x \left\{ \frac{1}{2} m_p^2 \partial\psi \partial\psi - \Lambda^4 V(\psi) \right\} \\ &= \int d^4x \left\{ \frac{1}{2} \partial\phi \partial\phi - \Lambda^4 V(\phi / m_p) \right\} \end{aligned}$$

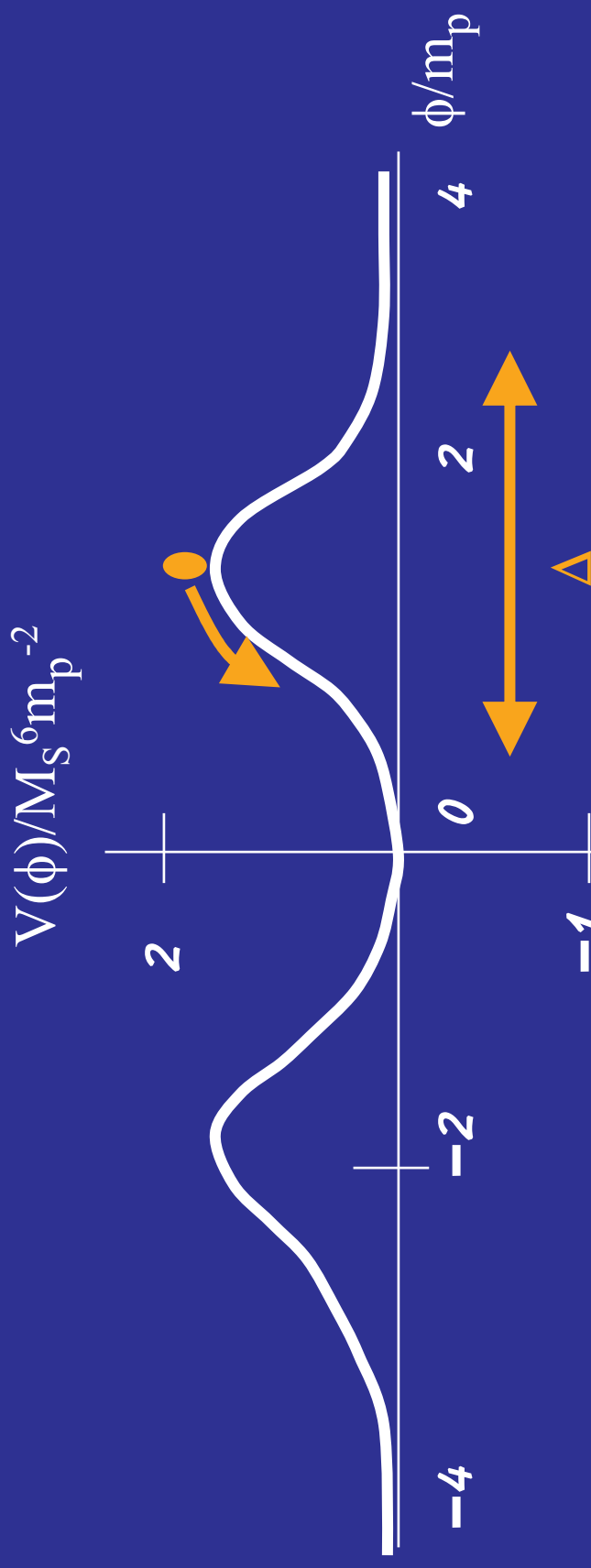
- The shape of the potential



zero CC min. & potential
vanishes @ infinity →
intermediate max.

Inflation: constraints & predictions

- Topological inflation



δ – wall thickness in space

$$(\Delta/\delta)^2 \sim \Lambda^4$$

$$H^2 \sim 1/3 \Lambda^4/m_p^2$$

$$\text{Inflation} \Leftrightarrow \delta H > 1 \Leftrightarrow \Delta > m_p$$

CMB anisotropies and the string scale

Slow-roll parameters

$$\varepsilon \sim 2 \frac{m_p^2}{\mu^2} (\phi - \phi_{\max})^2, \eta \sim V''(\phi_{\max})$$

The “small” parameter

$$\mu^2 = \frac{2m_p^2}{|V''(\phi_{\max})|}$$

Number of e-folds

$$N(\phi, \phi_{\text{end}}) = \frac{1}{\sqrt{2}m_p} \int_{\phi}^{\phi_{\text{end}}} \frac{1}{\sqrt{\varepsilon(\phi)}} d\phi$$

Sufficient inflation

$$\phi_{\text{init}} - \phi_{\text{max}} \cong \mu \text{Exp}(-[120(m_p / \mu)^2])$$

Qu. fluct. not too large

$$\phi_{\text{init}} - \phi_{\text{max}} \prec \frac{H}{2\pi} \Rightarrow V''(\phi_{\text{max}}) \prec 1/6$$

$$\Lambda^2 \cong 6.5 \times 10^{16} \text{ GeV} (|V''(\phi_{\text{max}})| / 2)^{1/4} \text{Exp}(-[25|V''(\phi_{\text{max}})|])$$

For consistency need $|V''| \sim 1/25$

For our model

$$\varepsilon_{\text{CMB}} \propto (V'/V)^2 \sim 0$$

$$\eta_{\text{CMB}} \approx V''(\phi_{\text{max}}) < 0$$

$$n_s = 1 - 4\varepsilon_{\text{CMB}} + 2\eta_{\text{CMB}}$$

$$r = 13.7\varepsilon_{\text{CMB}}$$

$$n_s = .92 - .08(25|V''(\phi_{\text{max}})| - 1)$$

$$r \approx 0$$



$$.76 < n_s < .97$$

$$r \approx 0$$

✓ WMAP

$$|1/3 < 25|V''| < 3 \rightarrow$$

If consistent:

$$M_s \approx 1.7 \times 10^{17} \text{ GeV} \left(\frac{P_\xi^{1/2}}{10^{-4}} \right)^{1/3} (1 - n_s)^{-1/6} e^{-\frac{25}{3}(1 - n_s)}$$

Summary and Conclusions

- **Stabilization and SUSY breaking**
 - Outer regions = trouble
 - Central region: need new ideas and techniques
 - Prediction: “light” moduli
- **Consistent cosmology:**
 - Outer regions = trouble
 - Central region:
 - scaling arguments
 - Curvature of potential needs to be “smallish”
 - Predictions for CMB