Entropy Bounds, Holography & 2nd Law (in Cosmology)

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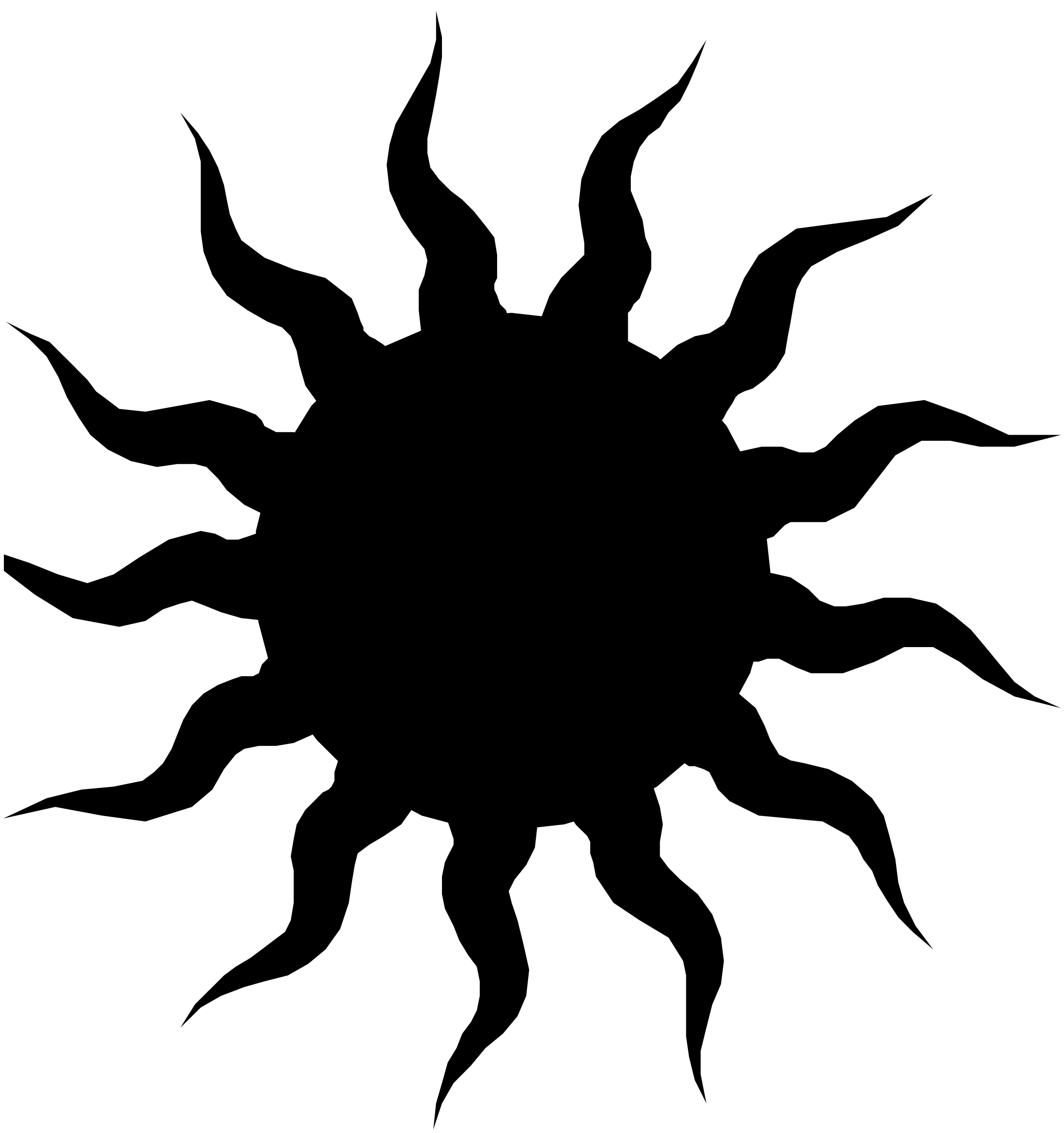
אוניברסיטת בן-גוריון

•gr-qc/9904061=PRL 84 (00)
•hep-th/9907032=PLB 471 (00)
with S. Foffa & R. Sturani
•hep-th/9912055=PRL 8? (00)
with G. Veneziano

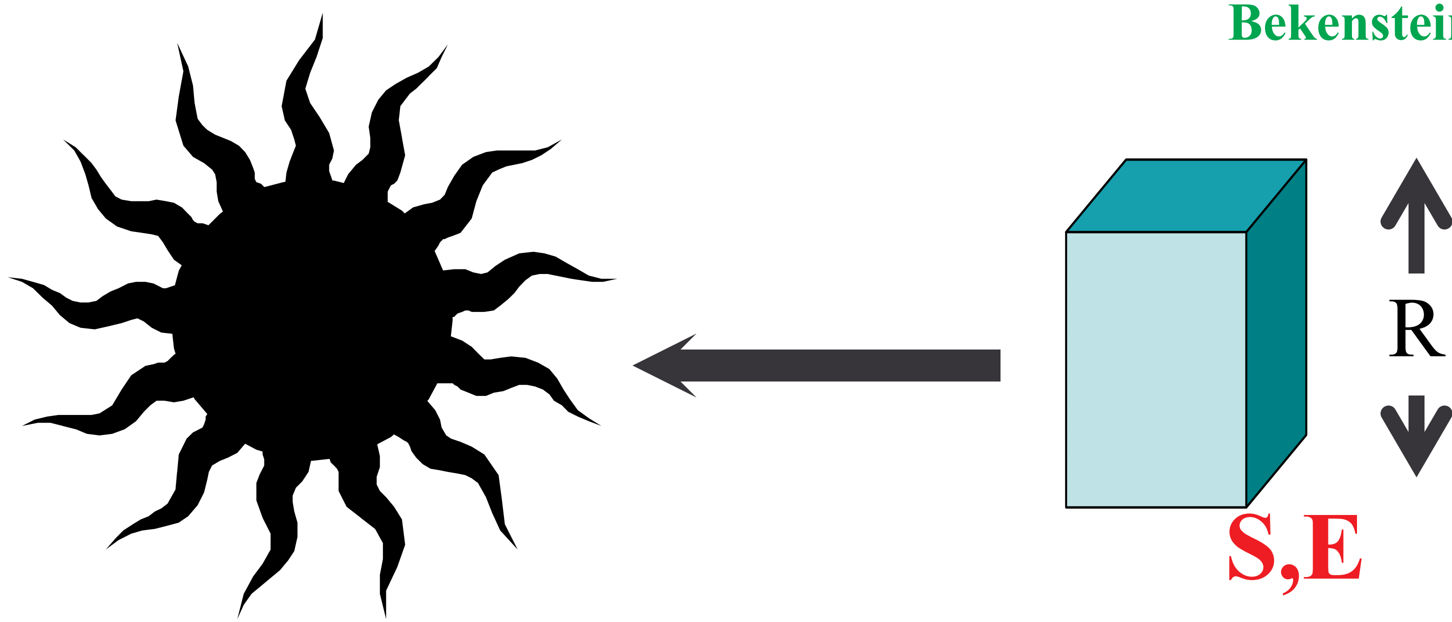
- Entropy bounds, holography, Causal Entropy Bound (CEB)
- Quantum & Geometric entropies
- GSL



Entropy Bounds

BEB : $S \leq ER$

Bekenstein '81



Too much entropy/ too little energy ==> ~~**GSL**~~

(For systems of limited gravity $R > R_g = 2\,E\,G_N$)

Entropy Bounds

Apply BEB to the universe ?!

U is ***not*** a system of limited gravity …

Bekenstein '89

$$R \approx H^{-1}, E \approx \rho R^3$$

$$M_P^2 H^2 \approx \rho, \rho \approx N T^4, s \approx N T^3$$

$$s \le \rho R \Rightarrow N T^3 \le N T^4 \frac{M_P}{\sqrt{N T^4}} \quad \Rightarrow \quad \begin{cases} T \le \dfrac{M_P}{\sqrt{N}} \\ H = \dfrac{\sqrt{N} T^2}{M_P} \le \dfrac{M_P}{\sqrt{N}} \end{cases}$$

Upper bound on curvature

Holography

BEB is not compatible with QFT!

$$S \le ER, \quad E < \frac{R}{G_N} \quad \Rightarrow S \le \frac{R^2}{G_N} = \frac{R^2}{l_P^2}$$

QFT: $$S \approx \frac{V}{l_{UV}^3} \approx \frac{R^3}{l_P^3} >> \frac{R^2}{l_P^2}$$

'tHooft '93
Susskind '95

Holographic principle:

Any physical system can be completely specified by data stored on its boundary, without exceeding a density of one bit per Planck area.

*(adapted from **Bousso hep-th/9911002**)*

Holography

Holographic entropy bound

$$S \leq S_{HOL} = \frac{A}{l_P^2}$$

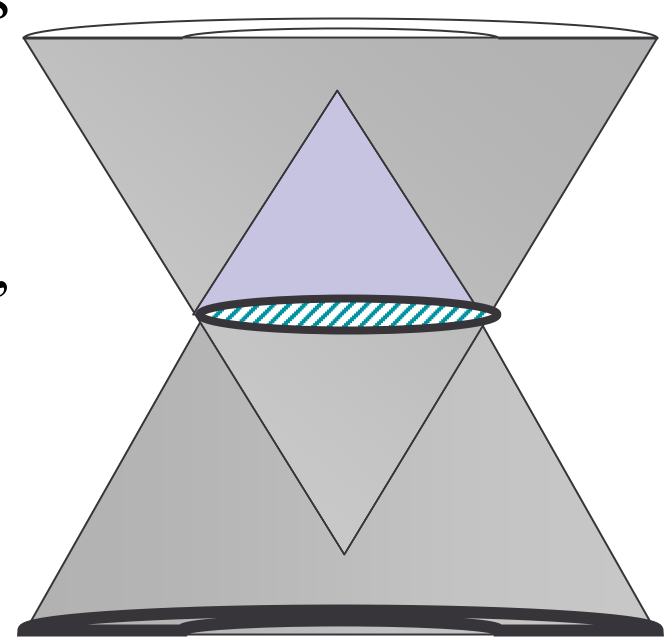
But what is S ?

Bousso: use light-sheets=2+1D collections of light-rays orthogonal to surface

FS: past ingoing light-sheet- wrong!

B:

1. light-sheet of decreasing area = “inside” with converging geodesics $\theta<0$
2. Stop when $\theta>0$ caustic ~ singularity need “space-like projection”

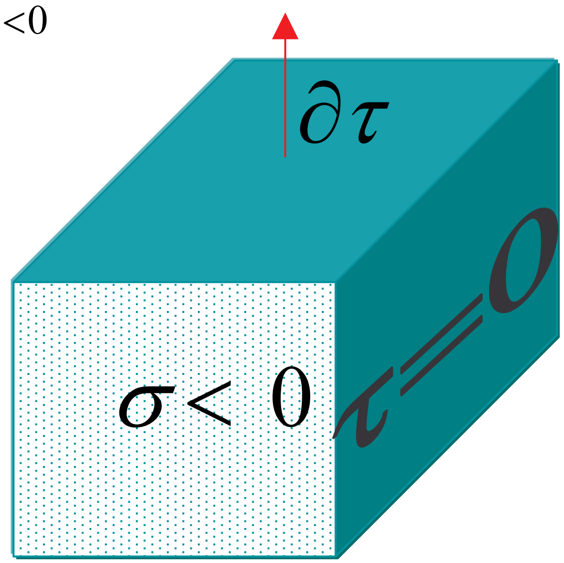


Entropy Bounds

CEB : $S_{\sigma<0} \leq S_{CEB}$ **R.B. & Veneziano '00**

$$S_{CEB} = \frac{1}{l_P^2} \int_{\sigma<0} d^4x \sqrt{-g}\,\delta(\tau) \sqrt{\mathrm{Max}_{\pm}\left[\left(R_{\mu\nu} \pm R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R\right) \quad \partial^\mu \tau \partial^\nu \tau\right]}$$

$$= \frac{1}{l_P \sqrt{\hbar}} \int_{\sigma<0} d^4x \sqrt{-g}\,\delta(\tau) \sqrt{\mathrm{Max}_{\pm}\left[\left(T_{\mu\nu} \pm T_{\mu\nu} \mp \tfrac{1}{2} g_{\mu\nu}\, T\right) \quad \partial^\mu \tau \partial^\nu \tau\right]}$$



$$\frac{S}{V} \leq \sqrt{\frac{E}{V}} \Rightarrow S \leq \sqrt{EV}$$

Entropy Bounds

Local form of CEB :

$$s_\mu \lambda^\mu \le \frac{1}{l_P\sqrt{\hbar}}\sqrt{\mathrm{Max}_\pm\left[\left(T_{\mu\nu} \pm T_{\mu\nu} \mp \tfrac{1}{2} g_{\mu\nu}\, T\right) \quad \lambda^\mu \lambda^\nu\right]}$$

$$s \le \frac{1}{l_P\sqrt{\hbar}}\sqrt{\mathrm{Max}\left[\tfrac{\rho}{3} - p, \rho + p\right]}$$

in cosmology:

$$T^\mu{}_\nu = diag(\rho, p, p, p)$$

CEB $\Longrightarrow$ A bound on curvature

(for RD FRW)

$$NT^3 \le M_P\sqrt{NT^4} \Longrightarrow H, T \le \frac{M_P}{\sqrt{N}}$$

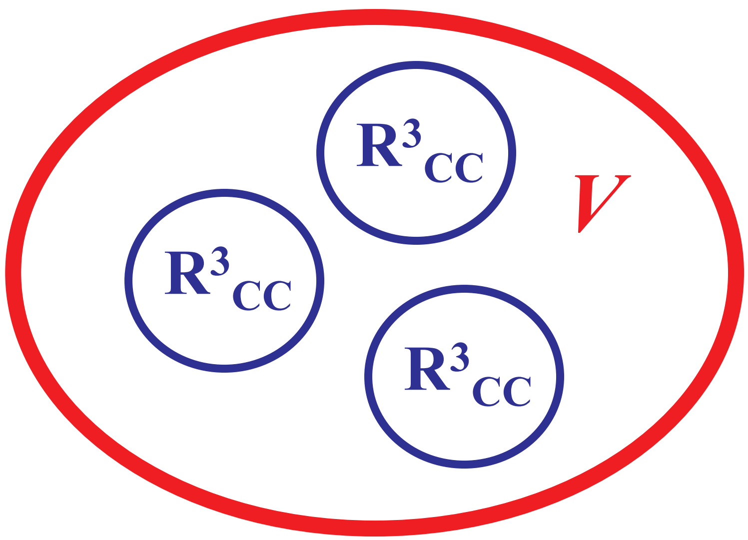
Derivation of CEB :

(*i*) Entropy is maximized by the largest stable BH (s) that can fit in a region

(*i i*) The largest stable BH is determined by causality: BH horizon < R_{CC}

$$n_H = \frac{V}{R^3_{cc}}, \quad S^H = \frac{R^2_{cc}}{\ell_P^2}$$

$$S \le n_H S^H = \frac{V}{R^3_{cc}} \frac{R^2_{cc}}{\ell_P^2} = \frac{V}{R_{cc}\ell_P^2}$$



Find R_{CC} : use cosmological perturbations

Comparison between entropy bounds

$$S_{CEB} \approx \frac{1}{\ell_P}\frac{1}{\sqrt{\hbar}}\sqrt{EV} = \sqrt{\frac{ER}{\hbar}\frac{R^2}{\ell_P^2}} = \sqrt{S_{BEB}\cdot S_{HOL}}$$

limited gravity $\quad R > \frac{E}{\ell_P^2} \quad S_{BEB} \leq S_{CEB} \leq S_{HOL}$

not limited gravity $\quad R < \frac{E}{\ell_P^2} \quad \cancel{S_{HOL}} \leq S_{CEB} \leq S_{BEB}$

Bousso is o.k.

Quantum entropy:
Entropy of quantum fluctuations

Modes "freeze"/ "thaw"
"exit" / "reenter"

$$\Psi_k'' + 2H\Psi_k' + k^2\Psi_k = 0$$

$$s_{\mathbf{quantum}} = \int_{k_{\min}}^{k_{\max}} d^3k \underbrace{\ln n_k}_{f(k)} \Rightarrow \Delta S_{quantum} \approx -\mu \Delta n_H$$

$$\mu \propto N$$

$$k_{\max} \approx M_P$$
Constant !

Quantum entropy is real !

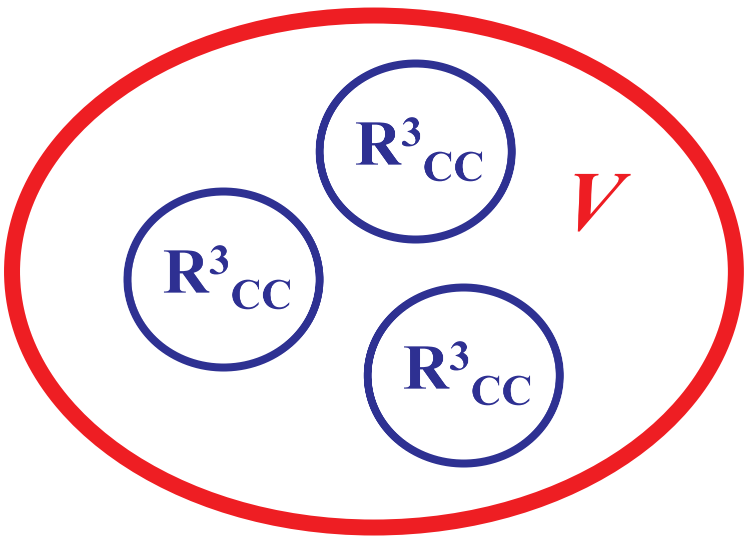
So what about 2nd law ?

Proposed resolution:

R.B., PRL 84 '00

Proof in progress

Causal boundary has geometric entropy



$$S_G^H = \frac{R_{CC}^2}{G_N}$$

Entropy bounds:
Geometric entropy dominates

G S L

Generalized second law

R.B., PRL 84 '00

$$\boxed{\begin{aligned} dS &= dS_{Classical} + dS_{Quantum} \\ &= dn_H S^H + n_H dS^H - \mu N dn_H \geq 0 \end{aligned}}$$

In cosmology: $R_{CC} = H^{-1}$, $n_H = \frac{a^3}{(H^{-1})^3}$, $S^H = \frac{H^{-2}}{l_P^2}$

$$(3H + 3\tfrac{\dot{H}}{H})n_H(S^H - \mu N) - 2\tfrac{\dot{H}}{H} n_H S^H \geq 0$$

$H > 0$, $|\dot{H}| << H^2$ $\Rightarrow H \leq \frac{M_P}{\sqrt{N}}$

Conclusions

א Holography modified by causality

ב Singularity thms. modified by entropy bounds

ג Hint: shortest length scale $\dfrac{M_P}{\sqrt{N}}$