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Self-similar solutions for imploding z-pinch shells in magnetized plasmas

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Imploding z-pinch shells carrying power law in time total currents $\sim t^S$ are investigated. Time-space separable self-similar solutions with cylindrical symmetry are explicitly developed. The problem is treated asymptotically in high thermal-conductivity within the model of magnetized resistive plasmas, while the ionization and radiation processes are ignored.

1. Introduction

A time-space separable self-similar quasi-equilibrium solutions are developed for a cylindrical z-pinch shell imploding under the pressure of an azimuthal magnetic field that is produced by the axial current.

The self-similar equilibrium solutions are of great interest due to their possible role as attractors at at long times. Such solutions were previously found for full cylindrical z-pinches carrying power law in time total currents $\sim t^S$ with the pinch

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radius $\sim t^{(1-3S)/2}$, imploding at $S > 1/3$ and exploding otherwise (Bud'ko et al. 1994). The solutions are conventionally related to either exact (Coppins et al. 1988 at $S = \pm 1/3$) or long-time asymptotic (Bud'ko et al. 1994 at $S > -1/5$, see also Shtemler and Mond 2005 hereinafter called SM 2005) mechanical equilibrium.

Those studies were carried out in the context of a plasma model that is more realistic than ideal magneto-hydrodynamics (MHD). It is assumed that converting the magnetic energy into the kinetic energy of z-pinches would add some dissipation and thereby the model of the resistive and thermally conductive plasma was adopted, while ionization and radiation effects which breaks the symmetry that underlies the self-similar solutions were neglected. Such solutions have been developed for two limiting cases of magnetized and unmagnetized plasmas (Coppins et al. 1988, Coppins et al. 1992, Bud'ko et al. 1994). In the case of magnetized plasma, i.e. high values of $\Omega_{ci}\tau_{ii}\gtrsim 1$ (Ω_{ci} is the ion cyclotron frequency, τ_{ii} is the ion-ion collision time), explicit Bessel-type solutions were obtained in the limit of infinite thermal conductivity (isothermal approximation), which leaves the radius of the full z-pinch undetermined. In the limit of high but finite thermal conductivity, the ohmic heating and entropy terms are neglected in the leading order energy equation, however they are taken into account by the solvability condition for the next order energy equation, and thus determines the pinch radius and influences the leading order approximate solution (SM 2005). A small parameter of the asymptotic procedure that is proportional to the square root of the electron-to-ion mass ratio naturally emerges in the magnetized plasma model. Explicit equilibrium solutions at high thermal-conductivity strongly simplify the modeling compared with the limit of unmagnetized plasmas where the thermal-conductivity is small (Coppins et al. 1992).

Although gas-puff z-pinches are described in the limit of the unmagnetized plasma rather than in the limit of the magnetized plasma (see also section 6), the concept of the imploding shell observed in the gas-puff z-pinches (Gregorian et al 2003a) forms the basis of the present study of quasi-equilibrium states in magnetized plasmas.

The recent experimental investigations of gas-puff z-pinches (Gregorian et al 2003a, Gregorian et al 2005a,b, Kroupp et al 2007, see discussion in section 6) infer to the model of the annular z-pinch shell configurations, which differs from that for full cylindrical z-pinches. The whole imploding phase may be separated into three stages: an initial stage that is characterized by a near-linear time variation of the total current and almost constant outer radius of the shell, an intermediate stage, and the final stagnation stage that corresponds to the saturation of the total current and the plasma closing to the pinch axis (Davara et al 1998). At the final stagnation stage of the implosion the total mass of the imploding-plasma significantly increases, the kinetic plasma energy is converted into plasma internal energy and radiation is accompanied by shock waves and magneto-hydrodynamic turbulence (Gregorian et al 2005a,b and Kroupp et al 2007). During the intermediate stage, the imploding shell is characterized by a density that is significantly higher than in the dilute plasma inside the shell as well as by the highest radial velocities of plasma, since the magnetic field energy is mainly spent on the radial flow. Although the inner ionization front propagates inward to the center of the pinch and ionizes the new portion of working gas, the plasma-mass rise is small, $\sim 10\%$, within the intermediate time interval. During this time the sweeping effect is small and the plasma column that contains most of the plasma mass has a nearly annular geometry. That annular region of the z-pinch plasma was named in (Gregorian et al 2003) to as the 'imploding shell' (hereinafter named imploding shell).

In the present modeling the previous study of a quasi-equilibrium self-similar solutions for full z-pinches (SH 2005) is expanded on z-pinch shells. It is sought that ionization and radiation processes as well as the shock-wave and sweeping effects are ignored. Instead, based on the concept of the imploding shell, the effective inner boundary of the imploding shell is introduced at which the conventional kinematic and dynamic conditions are satisfied. That problem with annular geometry rather never been studied theoretically. It is conjugated that the present qualitative model of the quasi-equilibrium imploding shell may be better adopted to description of z-pinch plasmas than the full z-pinch configuration.

The quasi-equilibrium solutions of the MHD model which depend on the time and radius only are also exact solutions of the Hall MHD model since the Hall term in Ohm's law is identically zero. In this respect its may serve as basic states for studying the stability of Hall MHD plasmas, similar to that carried out for the full cylindrical z-pinches, (see e.g. Shtemler and Mond 2006 and references therein). The stability analysis is motivated by some experimental observations (see discussion in section 6) that demonstrate small-scale turbulent ion motion in the outer radius of the z-pinches. The characteristic length scale of that turbulence may be compared with wave lengths of perturbations excited in the Hall regime λ_H

$$\lambda_H = \frac{c}{\omega_{pi}} = \frac{c}{2eZ}\sqrt{\frac{m_i}{\pi \hat{n}_i}} \sim 0.1cm. \qquad (1.1)$$

where c is the light velocity, ω_{pi} is the ion plasma frequency, ion current density averaged over the shell radius $\hat{n}_i \sim 10^{16}cm^{-3}$ and the average ion charge $Z = 2$.

This paper is organized as follows. In the next section the basic governing relations are presented. Self-similar quasi-equilibrium solutions for z-pinch plasma shells are described in section 3. In section 4 asymptotic expansions in the limit of high thermal conductivity are carried out. Results of numerical simulations for the imploding shells are presented in section 5. Applicability of the model of magnetized plasmas to conditions of gas-puff z-pinches, and Hall instability in them are discussed in section 6. The principle results and conclusions of the study are given in section 7. A closure condition that determines the outer radius of the pinch shell is derived in Appendix A. Explicit solution of the problem is carried out in Appendix B.

2. Governing relations

2.1. The physical model

A resistive MHD model for quasi-neutral magnetized plasma that accounts for thermal conductivity is considered. The displacement current, viscosity, ionization, radiation shock wave as well as sweeping effects are neglected:

$$\frac{\partial n}{\partial t} + \nabla(n\mathbf{V}) = 0, \tag{2.1a}$$

$$m_i n \frac{D\mathbf{V}}{Dt} = -\nabla P + \frac{1}{c}\mathbf{j} \times \mathbf{B}, \tag{2.1b}$$

$$\frac{1}{\gamma-1} n^{\gamma} \frac{D}{Dt}(P n^{-\gamma}) = \eta_{\perp} j^2 - \nabla \cdot \mathbf{Q}, \;\; \mathbf{Q} = -K_{\perp}\nabla T, \tag{2.1c}$$

$$P = 2nT, \tag{2.1d}$$

$$\mathbf{j} = \frac{1}{4\pi} c \nabla \times \mathbf{B}, \tag{2.1e}$$

$$\frac{\partial \mathbf{B}}{\partial t} + c\nabla \times \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0, \qquad \mathbf{E} = \eta_{\perp}\mathbf{j} - \frac{1}{c}\mathbf{V} \times \mathbf{B}. \tag{2.1f}$$

Here $\mathbf{V}$, $T = T_i = T_e$ and $P = P_e + P_i$ are the plasma velocity, temperature and pressure, $P_k = n_k T_k$; m_k are the electron and ion masses, $k = e, i$; $n \approx n_i = n_e/Z$ is the number density, Z is the effective ion charge of quasi-neutral plasma; $\gamma = 5/3$ is the specific heat ratio; $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric fields; $\mathbf{j}$ is the current density; c is the speed of light; $D/Dt = \partial/\partial t + (\mathbf{V} \cdot \nabla)$; t is the time; $\mathbf{Q}$ is the ion heat flux; $\eta_{\perp}$ and $K_{\perp}$ are the cross-field Spitzer resistivity and ion thermal conductivity (Lifshits and Pitaevskii 1981):

$$K_{\perp} = \frac{a_i n^2}{B^2 T^{1/2}}, \qquad \eta_{\perp} = \frac{a_e}{T^{3/2}},$$

$$a_i = aZ^2e^2c^2\sqrt{m_i}, \qquad a_e = aZe^2\sqrt{m_e/2}, \qquad a = \frac{8}{3}\, l\sqrt{\pi},$$

$l \approx 14$ is Coulomb logarithm, e is the electron charge; and for magnetized plasmas

$$\Omega_{ci}\tau_{ii} \gg 1, \qquad \Omega_{ci} = \frac{ZeB}{m_i c}, \qquad \tau_{ii} = \frac{m_i^{1/2}T^{3/2}}{4\pi Z^4 e^4 l\, n}. \tag{2.2}$$

Assuming the cylindrical symmetry, the physical system is described in a cylindrical frame of reference (r, θ, z), subscripts r, θ and z denote the corresponding projections. The boundary conditions for the thermally isolated z-pinch shell for vanishing sheet current are as follows Kruskal & Schwarzchild (1954):

$$P = 0,\ \ V_r = V_{out},\ \ Q_r = 0,\ \ B_\theta = B_{out} \ \ \text{at } r = r_{out}, \tag{2.3a}$$

$$V_r = V_{in},\ \ Q_r = 0,\ \ B_\theta = 0 \qquad \text{at } r = r_{in}. \tag{2.3b}$$

Here r_k and V_k the pinch radius and boundary velocity for the outer and inner edges, $k = out, in$. Outside the pinch the toroidal magnetic field is potential:

$$B_{out}(r,t) = \frac{2I(t)}{cr} \qquad \text{at } r \geq r_{out}. \tag{2.4}$$

The system of relations (2.1)-(2.4) is complemented by integral conditions for the total current, $I(t)$, that is a known function on time, and for the constant line density, N, of the imploding shell:

$$2\pi \int_{r_{in}}^{r_{out}} j_z\, rdr = I(t),\ \ 2\pi \int_{r_{in}}^{r_{out}} n\, rdr = N \equiv const. \tag{2.5}$$

Due to Ampere law in (2.1) and the relation (2.4), the relation (2.5) for the total current is satisfied identically and is omitted from further consideration. The initial data is not relevant here since time-space separable self-similar solutions are sought.

2.2. Dimensionless problem

The characteristic normalizing scales marked by the subscript * may be expressed through four independent scales, the characteristic radius r_* and time t_* as well as the given dimensional line density N and total current I_*:

$$m_* = m_i,\ V_* = \tfrac{r_*}{t_*},\ T_* = \tfrac{I_*^2}{c^2 N},\ n_* = \tfrac{N}{r_*^2},\ j_* = \tfrac{I_*}{r_*^2},\ P_* = n_* T_*,$$
$$B_* = \tfrac{I_*}{c r_*},\ E_* = \tfrac{r_* B_*}{c t_*}, \eta_* = \tfrac{a_e}{T_*^{3/2}},\ K_* = \tfrac{a_i P_*^2}{T_*^{5/2} B_*^2},\ Q_* = \tfrac{a_i P_*^2}{T_*^{3/2} B_*^2 r_*}. \tag{2.6}$$

Substituting (2.7) into (2.1)-(2.5) results in

$$\tfrac{\partial n}{\partial t} + \tfrac{1}{r}\tfrac{\partial}{\partial r}(r\, n\, V_r) = 0\,, \tag{2.7a}$$

$$\Pi_i n\,(\tfrac{\partial V_r}{\partial t} + V_r \tfrac{\partial V_r}{\partial r}) = -\tfrac{\partial P}{\partial r} - j_z B_\theta\,, \tag{2.7b}$$

$$\tfrac{1}{\gamma-1} n^\gamma \big[\tfrac{\partial}{\partial t}(P n^{-\gamma}) + V_r \tfrac{\partial}{\partial r}(P n^{-\gamma})\big] = \Pi_R \eta_\perp j_z^2 - \Pi_T \tfrac{1}{r}\tfrac{\partial}{\partial r}(r Q_r)\,, \tag{2.7c}$$

$$P = (1+Z) n \eta_\perp^{-2/3}, \tag{2.7d}$$

$$j_z = \tfrac{1}{4\pi r}\tfrac{\partial}{\partial r}(r B_\theta)\,, \tag{2.7e}$$

$$\tfrac{\partial B_\theta}{\partial t} - \tfrac{\partial}{\partial r}(\Pi_R \eta_\perp j_z - V_r\, B_\theta) = 0. \tag{2.7f}$$

Here temperature and radial heat flux are expressed through the resistivity

$$T = \eta_\perp^{-2/3},\quad Q_r = \frac{2P^2}{3(1+Z)^2 B^2}\frac{\partial \eta_\perp}{\partial r}.$$

The dimensionless boundary conditions at the pinch edges, the integral conservation laws and condition for magnetized plasmas, are given by

$$V_r = V_{out},\ \ B_\theta = \tfrac{2I(t)}{r},\ \ Q_r = 0,\ \ P = 0,\ \ \text{at}\ r = r_{out}, \tag{2.8a}$$

$$V_r = V_{in},\ \ B_\theta = 0,\ \ Q_r = 0\ \ \text{at}\ r = r_{in}, \tag{2.8b}$$

$$2\pi \textstyle\int_{r_{in}}^{r_{out}} n\, r dr = 1, \tag{2.8c}$$

$$\Omega_{ci}\tau_{ii} \sim \Pi_M \gg 1. \tag{2.8d}$$

The dimensionless parameters in the relations (2.7)-(2.9) are expressed through the characteristic scales as follows:

$$\Pi_I = \frac{m_i r_*^2}{T_* t_*^2},\ \Pi_T = \frac{\Pi_R}{\epsilon},\ \Pi_R = \frac{t_* a Z^2 e^2 c^2 \sqrt{m_i}}{T_*^{3/2} r_*^2}\epsilon,\ \Pi_M = \frac{2T_*^2 r_*}{3acZ^3 e^3 \sqrt{\pi N m_i}}, \tag{2.9}$$

where $\epsilon \lesssim 1$ is proportional to the square root from the electron-to-ion mass ratio

$$\epsilon = \frac{a_e c^2}{a_i} \equiv \frac{1}{Z}(\frac{m_e}{2m_i})^{1/2}. \tag{2.10}$$

Alternatively, the inertial, heat-transfer and magnetization parameters Π_I, Π_T and Π_M may be expressed through the resistivity parameter and Lundquist number Lu that is defined through the characteristic Alven speed V_a

$$\Pi_I = \frac{1}{\Pi_R^2 Lu^2},\ \Pi_T = \frac{\Pi_R}{\epsilon},\ \Pi_M = \epsilon Lu\Pi,\ (\Pi = \frac{2c}{3Ze}\sqrt{\frac{m_i}{\pi N}}), \tag{2.11a}$$

$$Lu = \frac{V_a r_*}{c^2\eta_*} \equiv \frac{T_*^2 r_*}{c^2 aZ^2e^2m_i}\frac{1}{\epsilon},\ (V_a = \frac{B_*}{\sqrt{m_i n_*}} \equiv \frac{T_*}{\sqrt{m_i}}). \tag{2.11b}$$

The condition for the plasma magnetization is satisfied if $\Pi_M \gtrsim 1$. Quasi-equilibrium state is settled if the inertial term in the momentum equation is small $\Pi_I \gtrsim 1$. That condition presumes that the characteristic time t_*, during which the equilibrium self-similar solution is settled, should be less than the current pulse duration of the imploding z-pinch shell. Since the current pulse duration is bounded from the above, the latter condition essentially restricts the admissible values of z-pinch parameters consistent with the solution developed in the present study: the equilibrium self-similar isothermal solution should be settled during the life-time of the z-pinch. Note also that the condition for applicability of the isothermal approximation is $\Pi_T \gtrsim 1$. According to Eqs. (2.11) all the above three conditions, are satisfied if

$$Lu \gtrsim \epsilon^{-1}, \qquad \Pi_R \gtrsim \epsilon. \tag{2.12}$$

3. Self-similar solutions

For cylindrical z-pinch shells, the same self-similarity law is valid as for conventional full cylindrical z-pinches (Bud'ko et al 1994, see also SM 2005):

$$\begin{gathered}
t = \tilde{t},\ r = \tilde{r}\alpha(\tilde{t}),\ V_r(r,t) = \tilde{r}\dot{\alpha}(\tilde{t}),\ a_r(r,t) = \tilde{r}\ddot{\alpha}(\tilde{t}),\\
r_k(t) = \tilde{r}_k\alpha(\tilde{t}),\ V_k(t) = dr_k/dt,\ a_k(t) = dV_k/dt,\\
I(t) = \tilde{I}\,\tilde{t}^S,\ j_z(r,t) = \tilde{j}(\tilde{r})\alpha^{\kappa_1}(\tilde{t}),\ B_\theta(r,t) = \tilde{B}(\tilde{r})\alpha^{\kappa_2}(\tilde{t}),\\
T(r,t) = \tilde{T}(\tilde{r})\alpha^{\kappa_3}(\tilde{t}),\ P(r,t) = \tilde{P}(\tilde{r})\alpha^{\kappa_4}(\tilde{t}),\\
\eta_\perp(r,t) = \tilde{\eta}(\tilde{r})\alpha^{\kappa_5},\ n(r,t) = \tilde{n}(\tilde{r})\alpha^{\kappa_6}(\tilde{t}),\ Q_r(r,t) = \tilde{Q}(\tilde{r})\alpha^{\kappa_7}(\tilde{t}).
\end{gathered} \tag{3.1}$$

Here $a_r(r,t) = DV_r(r,t)/Dt$ is the plasma acceleration; $a_k(t)$ is the pinch boundary acceleration; $\tilde{r}_k \equiv const$; $k = in, out$; $\alpha(\tilde{t}) = \tilde{t}^{\kappa_0}$; the upper-dots denote the derivatives of $\alpha(\tilde{t})$ with respect to time $\tilde{t}$ and

$$\begin{aligned} &\kappa_0 = \tfrac{1-3S}{2},\ \kappa_1 = \tfrac{4S-1}{\kappa_0},\ \kappa_2 = \tfrac{5S-1}{2\kappa_0},\ \kappa_3 = \tfrac{2S}{\kappa_0}, \\ &\kappa_4 = \tfrac{5S-1}{\kappa_0},\ \kappa_5 = -\tfrac{3S}{\kappa_0},\ \kappa_6 = \tfrac{3S-1}{\kappa_0},\ \kappa_7 = \tfrac{7S-3}{2\kappa_0}. \end{aligned} \tag{3.2}$$

In the present study we focus on imploding z-pinch shells with total current exponent $S > 1/3$. The arbitrary dimensionless total current amplitude $\tilde{I}$ in relations (3.1) may be set to unity ($\tilde{I} = 1$) due to the choice of the characteristic total-current amplitude I_*. However, more convenient another choice of I_* used in Section 5.

Substituting Eqs. (3.1)-(3.2) into Eqs. (2.8) results in:

$$0 = \frac{d\tilde{P}}{d\tilde{r}} + \tilde{j}\tilde{B}, \tag{3.3a}$$

$$\frac{d\tilde{\eta}}{d\tilde{r}} = \epsilon \frac{3(1+Z)^2}{2\tilde{r}} \frac{\tilde{B}^2}{\tilde{P}^2} \int_0^{\tilde{r}} \tilde{r}(\tilde{\eta}\tilde{j}^2 - \frac{\Gamma}{\Pi_R}\tilde{P}) d\tilde{r},\ (\tilde{Q} = \frac{2}{3(1+Z)^2} \frac{\tilde{P}^2}{\tilde{B}^2} \frac{d\tilde{\eta}}{d\tilde{r}}), \tag{3.3b}$$

$$(1+Z)\tilde{n} = \tilde{P}\tilde{\eta}^{2/3}, \qquad \left(\tilde{T} = \tilde{\eta}^{-2/3}\right), \tag{3.3c}$$

$$\tilde{j} = \frac{1}{4\pi\tilde{r}} \frac{d}{d\tilde{r}}(\tilde{r}\tilde{B}), \tag{3.3d}$$

$$S\tilde{B} - \Pi_R \frac{d}{d\tilde{r}}(\tilde{\eta}\tilde{j}) = 0. \tag{3.3e}$$

Here $\Gamma = 1 + 3S(\gamma - 5/3)/(\gamma - 1)$ so that $\Gamma = 1$ everywhere below for $\gamma = 5/3$.

The kinematic boundary conditions and the mass balance relations are satisfied identically for the solution (3.1), while the rest of the boundary conditions at the pinch edges and the integral condition for the line density are:

$$\tilde{B} = 0,\ \tilde{P} = 0,\ \tilde{Q} = 0 \ \text{ at } \tilde{r} = \tilde{r}_{out}, \tag{3.4a}$$

$$\tilde{B} = 0, \qquad \tilde{Q} = 0 \ \text{ at } \tilde{r} = \tilde{r}_{in}, \tag{3.4b}$$

$$\pi\tilde{\eta}^{2/3} \int_{\tilde{r}_{in}}^{\tilde{r}_{out}} \tilde{P}\tilde{r} d\tilde{r} = (1+Z)/2. \tag{3.4c}$$

4. High-thermal-conductivity asymptotic expansions

Further simplifications are still possible in the problem (3.3)-(3.4), which contains the small parameter ϵ. The solution of Eqs. (3.3)-(3.4) is sought in the limit of high thermal conductivity in the form of a power series in ϵ (SM 2005):

$$\tilde{f}(\tilde{r}) = f^{(0)}(\rho) + \epsilon f^{(1)}(\rho) + \ldots,\ \rho_k = \rho_k^{(0)} + \epsilon\rho_k^{(1)} + \ldots, \tag{4.1a}$$

$$\rho = \tilde{r}/\lambda,\ \rho_k = \tilde{r}_k/\lambda, \qquad (k = in, out), \tag{4.1b}$$

where $\tilde{f}(\tilde{r})$ stands for any of the unknown functions; the new independent variable ρ is scaled by an arbitrary positive constant λ that is determined below.

Substituting (4.1) into (3.3)-(3.4) yields in the leading order in ϵ (the mass balance equation is satisfied identically for the self-similar solutions):

$$\frac{dP}{d\rho} + \lambda j\, B = 0, \tag{4.2a}$$

$$\frac{d\eta}{d\rho} = 0, \tag{4.2b}$$

$$(1+Z)n = P\eta^{2/3},\ \big(T = \eta^{-2/3}\big), \tag{4.2c}$$

$$j = (4\pi\lambda\rho)^{-1}\frac{d}{d\rho}(\rho B), \tag{4.2d}$$

$$\lambda S\, B - \Pi_R \eta \frac{dj}{d\rho} = 0. \tag{4.2e}$$

Here the superscript denoting the approximation order with respect to ϵ is dropped everywhere below unless this leads to misunderstanding.

The boundary conditions for the heat flux are identically satisfied in the isothermal approximation (4.2b), while the rest of the boundary conditions at the pinch edges, and the integral condition for the line density are as follows:

$$B = 0 \quad \text{at } \rho = \rho_{in}, \qquad B = 2/(\lambda\rho), \quad P = 0 \quad \text{at } \rho = \rho_{out}, \tag{4.3a}$$

$$\pi\lambda^2 \textstyle\int_{\rho_{in}}^{\rho_{out}} n\rho d\rho = (1+Z)/2. \tag{4.3b}$$

The differential equation for the magnetic field reduces to the standard form for

Bessel functions for the value of λ chosen as follows:

$$\frac{d}{d\rho}\big[\frac{1}{\rho}\frac{d}{d\rho}(\rho B)\big] - B = 0,\ \lambda = \Lambda\sqrt{\eta},\ \Lambda = \frac{1}{2}\sqrt{\frac{\Pi_R}{\pi S}}. \tag{4.4}$$

Using Eqs. (4.3)-(4.4) yields the following general solution of the system (4.2):

$$T = C_0,\ \ (\eta = C_0^{-3/2}), \tag{4.5a}$$

$$B = C_1 I_1(\rho) + C_2 K_1(\rho), \tag{4.5b}$$

$$j = \frac{1}{4\pi\lambda}[C_1 I_0(\rho) - C_2 K_0(\rho)], \tag{4.5c}$$

$$P + 2\pi\lambda^2 j^2 = \frac{1}{4\pi}C_3. \tag{4.5d}$$

Here $I_m(\rho)$ and $K_m(\rho)$ are the Bessel functions ($m = 0, 1$).

Four boundary conditions (4.3), closed by the solvability condition (A2) for the energy equation in the first-order approximation in ϵ (see Appendix A), lead to an algebraic system of five equations for the six unknown variables, the coefficients C_j ($j = 0, 1, 2, 3$) together with the outer and inner radiuses of the pinch (or, equivalently, for the outer radius of the shell ρ_{out} and the radius ratio $A = \rho_{in}/\rho_{out}$). Thus, substituting Eqs. (B7)-(B9) for the coefficients C_j ($j = 0, 1, 2, 3$) and the solution of the eigenvalue equation (B6) for ρ_{out} vs A (Appendix B) into Eqs. (4.5) provides explicit expressions for the radial profiles of magnetic field, electric current, pressure etc.

5. Results of numerical simulations for the imploding shells

5.1. Solution of the eigenvalue equation for the outer radius

The eigenvalue equation (B6) in Appendix B that relates the outer and inner radiuses of the pinch shell, rewritten in terms of the outer radius ρ_{out} and the inner-to-outer radius ratio $A = \rho_{in}/\rho_{out}$ is

$$D(\rho_{out}; A, S) = 0. \tag{5.1}$$

In Figure 1 the dependence is depicted of ρ_{out} upon $A = \rho_{in}/\rho_{out}$ that was obtained by the numerical solution of the eigenvalue equation for $S = 2.5$. Note in particular that $\rho_{out} \approx 17$ for $A = 0.8$.

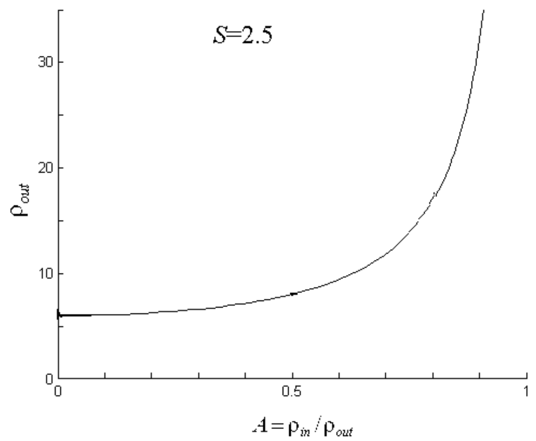

Figure 1. The outer radius of the imploding z-pinch shell ρ_{out} vs $A = 0.8$ ($S = 2.5$).

5.2. Characteristics of z-pinch shells for typical parameters of magnetized plasma

The following parameters of the imploding shell (which are assumed to be known for given experimental conditions) will be adopted as the basic ones in the present modeling: the inner-to-outer radius ratio $A = \rho_{in}/\rho_{out}$, the line density N, the total-current exponent S, the constant value of the average ion charge Z, the constant temperature T, as well as two parameters of the model - the characteristic time and radius t_* and r_*. It is convenient here to fit a given temperature T by the choice of the characteristic total current I_*. The characteristic time t_* is naturally associated with the time of the equilibrium to settle, it may be estimated by fitting the observed values of the total current or the outer radius to that in the self-similar solution, the characteristic size is chosen to be equal to the shell thickness,

$$r_* = (1 - A)r_{out}, \tag{5.2}$$

while the outer radius is determined by relations (4.1b), (4.4):

$$\frac{r_{out}}{r_*} = \frac{1}{2}\sqrt{\frac{\Pi_R}{\pi S}}\frac{T^{3/4}}{T_*^{3/4}}\rho_{out}, \tag{5.3}$$

where T_* is determined in Eqs. (2.7), and ρ_{out} is the solution of eigenvalue equation (5.1) Note that the dimensional outer radius r_{out} is independent of r_*, since the dimensionless resistivity parameter Π_R in Eq. (2.10) is proportional to $1/r_*^2$.

In Table 1 some results are presented in mixed SI-Gauss units for typical parameters of a z-pinch plasma at $t = t_*$. The dimensionless parameters of the model, Π_R, Π_I, Π_M and Lu may be calculated according to Eqs. (2.9) and (2.11). It is seen from the Table 1 that parameters of the imploding shell satisfy sufficiently well the conditions of the model applicability (2.12). Note that the conditions (2.12) depends upon only two determining parameters the Lundquist number Lu, and the resistivity parameter Π_R. In turn the Lundquist number depends on the characteristic temperature and radius $Lu \sim T_*^2 r_*$, while the resistivity parameter depends additionally on the characteristic time, $\Pi_R \sim t_*/(T_*^{3/2} r_*^2)$. Hence for any given characteristic radius r_* the Lundquist number will satisfy the former of (2.12) for sufficiently large T_*, while t_* is the principal parameter, whose value allows to satisfy the second of conditions (2.12). Note also that for t_* larger than $85ns$ adopted in the Table 1 the resistivity parameter Π_R will be larger and the conditions (2.12) will be better satisfied. The pressure at the inner edge $P_{in} = P(r_{in})$ and characteristic wave lengths of perturbations excited in the Hall regime λ_H, calculated by using Eq. (1.1), are also presented in Table 1.

In Figure 2 the radial profiles of the pressure, density, magnetic field and electric current are depicted for typical parameters of z-pinch imploding shells. Note that the magnetic field and electric current have their maximum at the outer edge of the shell, while maximum of the number density (pressure) is located at the effective

Table 1. Characteristics of z-pinch shell for typical parameters of the plasma, $S = 2.5, A = 0.8, T = 0.25keV$, $\rho_{out} \approx 17$, $t_* = 85ns$, $\epsilon = 0.017/Z$, $\Pi_T = \Pi_R/\epsilon$.

T	Z	$10^{-17}N$	I_*	r_{out}	P_{in}	Π_R	$10^5\Pi_I$	Π_M	$10^{-3}Lu$	$10^{-16}\hat{n}$	λ_H
keV	$-$	m^{-1}	kA	cm	MPa	$-$	$-$	$-$	$-$	cm^{-3}	cm
3.1	1.	0.25	11	0.17	0.7	0.13	2	50	1.6	0.75	0.26
3.1	1.	0.5	16	0.17	1.5	0.13	2	35	1.6	1.5	0.18
3.1	1.	1.0	22	0.17	2.9	0.13	2	25	1.6	3.0	0.13
4.7	2.	0.25	14	0.24	0.5	0.06	4	22	2.7	0.3	0.2
4.7	2.	0.5	19	0.24	1.1	0.06	4	15	2.7	0.62	0.15
4.7	2.	1.0	27	0.24	2.2	0.06	4	11	2.7	1.25	0.1

inner edge. The behavior of the number density at the effective inner edge differs its from the actual inner edge where the number density must be equal to zero.

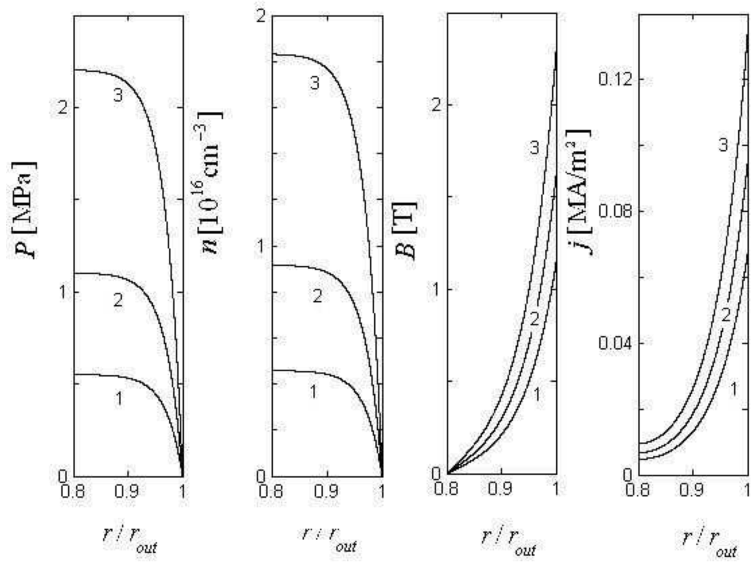

Figure 2. Dimensionless radial profiles of pressure, density, magnetic and electric fields. $S = 2.5$, $Z = 2$, $A = 0.8$, $T = 0.25keV$, $\Pi_R \approx 0.06$, $\Pi_I \approx 4 \cdot 10^{-5}$, $\Pi_M \approx 15$, $Lu \approx 2.7 \cdot 10^3$, $\epsilon \approx 8 \cdot 10^{-3}$ (see Table 1). Curves 1, 2 and 3 correspond to $N = 1 \cdot 10^{17} cm^{-3}, 2 \cdot 10^{17} cm^{-3}, 3 \cdot 10^{16} cm^{-3}$, respectively.

6. Discussion

Let us discuss applicability of the magnetized plasma model to conditions of gas-puff z-pinches, and Hall instability in them. Annular gas-puff devices are characterized by a sufficiently large initial radius and the driving pulse width of the total current is comparable to the total implosion time. Although, duration of the whole process is too short to reach a Bennett-like mechanical equilibrium (Ryutov et al 2000), the concept of the imploding shell by Gregorian et al (2003) rather assumes that the shell is in the equilibrium due to small duration of the imploding-shell stage compared to the total implosion time, and thin shell thickness compared with the the entire thickness of the plasma shell.

The gas-puff device in (Gregorian et al 2003) produces an annular shell of the working gas (CO_2). The total duration of the implosion is $\sim 620ns$, while the duration of the imploding shell stage is from $100ns$ from $435ns$ to $535ns$ (Fig. 1b in Gregorian et al 2003). At this period, the outer boundary of the plasma shell moves radially from $r_{out} \approx 2cm$ to $r_{out} \approx 1.1cm$, the thickness of the imploding shell during this period is $r_{out} - r_{in} \approx (0.1 \div 0.15)cm \pm 0.1cm$ wide, while the entire plasma shell is about $0.6cm$ wide. Then, the following characteristic scales may be adopted: the time instant of the start of the imploding shell stage $t_* = t_1 \approx 435ns$, the ratio of the inner-to-outer radius $A = r_{in}/r_{out} \approx 0.9cm$ and the mean thickness of the imploding shell $r_* = r_{out} - r_{in} \approx 0.125cm$. Furthermore, during that time the total current does not exceed the value I_{max} (Fig. 1, Davara et al 1998): $I_* < I_{max} < 175kA$. Since the imploding shell is charged with the mean ionization degree $Z \sim 3 \div 4$. (Fig. 7, Gregorian et al 2005a), the following effective value may be adopted: $Z \approx 4$. The dimensional line density of the imploding plasma in those experiments is $N_w \approx (10 \pm 3)\mu g/cm$ for CO_2 gas (reduced to 70% of imploding shell mass this yields $0.7 \cdot (10 \pm 3)\mu g/cm)$ or, equivalently,

$$N = \frac{N_w[\mu g/cm]}{N_{CO2} m_i[\mu g]} \sim 10^{17} cm^{-1},$$

where N_{CO2} is the particle number in one molecule of CO_2. The average value of the electron number density observed in experiments (Fig. 5b in Gregorian et al 2003) is $\hat{n}_e = Z\hat{n}_i \sim 4 \cdot 10^{17} cm^{-3}$, this yields $\hat{n}_i \sim 1 \cdot 10^{17} cm^{-3}$.

The above estimations for I_{max}, N, r_* and Z yield the following value for the magnetization parameter (using Eq. (2.6)):

$$\Pi_M = \frac{2T_*^2 r_*}{3acZ^3e^3\sqrt{\pi N m_i}} \equiv \frac{2I_*^4 r_*}{3ac^5Z^3e^3N^{5/2}\sqrt{\pi m_i}} < \frac{2I_{max}^4 r_*}{3ac^5Z^3e^3N^{5/2}\sqrt{\pi m_i}} \approx 0.06.$$

Thus, the plasma magnetization condition $\Pi_M \gtrsim 1$ is violated, and the limit of unmagnetized plasma is rather applicable to the experimental conditions in Gregorian et al (2003). Note also that the estimation $\Pi_M \sim 0.06$ is also valid for the case of unmagnetized plasma, since the magnetization condition has the same form as for magnetized plasma up to a constant factor of the order of unit (Coppins et al 1992).

Experiments by Gregorian et al (2005a) demonstrate the small-scale turbulent ion motion in the outer radius of the z-pinch shell with wavelength $\lambda \sim c/\omega_{pi} < 0.05cm$. Such small scale perturbations may be rather excited by the Hall instability. Indeed, adopting the above estimations $\hat{n}_i$ and Z yield for the wave length excited in the Hall regime (see Eq. (1.1)): $\lambda_H = c/\omega_{pi} \sim 0.02cm$.

7. Summary and Conclusions

Let us summarize the results obtained for z-pinch imploding shells. Time-separable self-similar quasi-equilibrium solutions are developed. The concept of the imploding shell proposed by (Gregorian et al 2003) for interpretation of the experimental data for gas-puff z-pinches in unmugnetized plasmas, lies in the basis of the present modeling of z-pinches in magnetized resistive plasmas. The intermediate-time interval of the implosion that corresponds to the imploding shell for which the radiation and ionization processes as well as shock wave and sweeping effects are ignored.

The problem is treated asymptotically in the limit of high, but finite thermal conductivity (similar to that for full cylindrical z-pinches, SM 2005). The resulting characteristics of the quasi-equilibrium z-pinch shells are determined by the following input values: the ion charge Z, the ratio of the inner-to-outer radius A, the total current exponent S, the line density N and the constant temperature T as well as the characteristic settling time of the equilibrium t_* (the characteristic length r_* chosen equal to the shell thickness). The model determines the outer radius of the shell, while the inner shell radius is known up to an arbitrary factor. The radial profiles of the magnetic and electric fields, pressure, density etc are calculated for typical parameters of plasmas. In comparison of the present qualitative modeling with the results of experiments or numerics, the model outer interface must be associated with the location of the magnetic field maximum that separates the plasma from the surrounding, while the effective inner boundary of the imploding shell should be associated with the location of the density (pressure) maximum. The estimations has been made which demonstrate the possibility of the Hall instability for typical parameters of plasmas.

Appendix A. Closure condition

Following SM (2005), the boundary conditions (4.3) are closed by considering the first-order approximation in ϵ of the energy equation (3.3b):

$$\frac{d\eta^{(1)}}{d\rho} = \frac{3(1+Z)^2}{2} Q^{(1)}(\rho)\frac{B^2(\rho)}{P^2(\rho)}, \; Q^{(1)}(\rho) = \frac{1}{\rho}\int_{\rho_{in}}^{\rho} \rho[\eta j^2(\rho) - \frac{\Gamma}{\Pi_R}P(\rho)]d\rho. \quad \text{(A 1)}$$

Here P and B are given by the zero-order approximation in ϵ, as above the superscript 0 is omitted with no misunderstanding, but superscript 1 for the first-order approximation is preserved with no confusion.

According to the boundary conditions at the outer pinch edge in (4.3a) the pressure is zero, while the magnetic field here is non zero, $P(\rho_{out}) = 0$, $B(\rho_{out}) \neq 0$, as a result, Eq. (A 1) is singular at $\rho = \rho_{out}$ for $Q^{(1)}(\rho_{out}) \neq 0$. Hence, the coefficient $Q^{(1)}(\rho)$ must vanish at $\rho = \rho_{out}$ in order to avoid that singularity (SM 2005):

$$\int_{\rho_{in}}^{\rho_{out}} \rho[\Gamma P(\rho) - \Pi_R \eta j^2(\rho)]d\rho = 0. \tag{A 2}$$

Let us reduce the closure condition (A 2) to an equivalent but more convenient form. Multiplying Eq. (4.5d) by $2S\rho$, using definition of $\lambda = \Lambda\sqrt{\eta}$ in Eq. (4.4) and integrating the result over the pinch radius from ρ_{in} to ρ_{out} yields:

$$\int_{\rho_{in}}^{\rho_{out}} \rho[2SP(\rho) - \Pi_R \eta j^2(\rho)]d\rho = \frac{S}{4\pi} C_3 \alpha_0(\rho_{in}, \rho_{out}),\ \alpha_0 = \frac{\rho_{out}^2 - \rho_{in}^2}{2}. \tag{A 3}$$

Adding Eqs. (A 2) and (A 3) results in:

$$\int_{\rho_{in}}^{\rho_{out}} \rho P(\rho)d\rho = \frac{1}{4\pi}\frac{S}{2S+\Gamma} C_3 \alpha_0(\rho_{in}, \rho_{out}). \tag{A 4}$$

Substituting Eq. (4.3c) into Eq. (A 4) and using (4.5a) reduces the closure condition (A 2) to the following form:

$$4\frac{1+Z}{\Lambda^2} C_0^{5/2} - \frac{2S}{2S+\Gamma} \alpha_0(\rho_{in}, \rho_{out}) C_3 = 0. \tag{A 5}$$

Appendix B. Explicit solution of the boundary value problem

Using Eqs. (4.5) and the definition of λ in Eqs. (4.4), let us rewrite the conditions (4.3) and the closure condition to the following form:

$$I_1(\rho_{out})C_1 + K_1(\rho_{out})C_2 = \tfrac{2}{\Lambda \rho_{out}} C_0^{3/4}, \tag{B 1a}$$

$$C_3 - [I_0(\rho_{out})C_1 - K_0(\rho_{out})C_2]^2 = 0, \tag{B 1b}$$

$$I_1(\rho_{in})C_1 + K_1(\rho_{in})C_2 = 0, \tag{B 1c}$$

$$4\tfrac{1+Z}{\Lambda^2}C_0^{5/2} - \alpha_0 C_3 + \alpha_{1,1}^2 C_1^2 + \alpha_{2,2}^2 C_2^2 - 2\alpha_{1,2}C_1C_2 = 0, \qquad \text{(B 1d)}$$

$$4\tfrac{1+Z}{\Lambda^2}C_0^{5/2} - \tfrac{2S}{2S+\Gamma}\alpha_0 C_3 = 0, \qquad \text{(B 1e)}$$

where by denoting the linear operator $2\Delta\Phi(\rho) = \Phi(\rho_{out}) - \Phi(\rho_{in})$

$$\alpha_0(\rho_{in}, \rho_{out}) = \textstyle\int_{\rho_{in}}^{\rho_{out}} \rho d\rho \equiv \Delta\{\rho^2\}/2,$$

$$\alpha_{1,2}(\rho_{in}, \rho_{out}) = \textstyle\int_{\rho_{in}}^{\rho_{out}} \rho I_0(\rho)K_0(\rho)d\rho \equiv \Delta\{\rho^2[I_0(\rho)K_0(\rho) + I_1(\rho)K_1(\rho)]\}/2,$$

$$\alpha_{1,1}(\rho_{in}, \rho_{out}) = \textstyle\int_{\rho_{in}}^{\rho_{out}} \rho I_0^2(\rho)d\rho \equiv \Delta\{\rho^2[I_0^2(\rho) - I_1^2(\rho)]\}/2,$$

$$\alpha_{2,2}(\rho_{in}, \rho_{out}) = \textstyle\int_{\rho_{in}}^{\rho_{out}} \rho K_0^2(\rho)d\rho \equiv \{\rho^2[K_1^2(\rho) - I_1^2(\rho)]\}/2. \qquad \text{(B2)}$$

The algebraic system (B1) determines five unknown variables, the coefficients C_j ($j = 0, 1, 2, 3$) and the outer radius of the pinch ρ_{out} (the inner-to-outer radius ratio A and hence $\rho_{in} = A\rho_{out}$ is assumed to be known in the equilibrium state). Then system (B1) may be reduced to the form:

$$c_1 I_1(\rho_{out}) + c_2 K_1(\rho_{out}) = 1, \qquad \text{(B 3a)}$$

$$c_3 - [c_1 I_0(\rho_{out}) - c_2 K_0(\rho_{out})]^2 = 0, \qquad \text{(B 3b)}$$

$$c_1 I_1(\rho_{in}) + c_2 K_1(\rho_{in}) = 0, \qquad \text{(B 3c)}$$

$$c_0 - c_3\alpha_0 - 2c_1c_2\alpha_{1,2} + c_1^2\alpha_{1,1}^2 + c_2^2\alpha_{2,2}^2 = 0, \qquad \text{(B 3d)}$$

$$c_0 - \tfrac{2S}{2S+\Gamma}\alpha_0 c_3 = 0, \qquad \text{(B 3e)}$$

where the following variable change has been made:

$$c_0 = (1+Z)\rho_{out}^2 C_0,\; c_1 = \frac{\rho_{out}\Lambda C_1}{2C_0^{3/4}},\; c_2 = \frac{\rho_{out}\Lambda C_2}{2C_0^{3/4}},\; c_3 = \frac{\rho_{out}^2\Lambda^2 C_3}{4C_0^{3/2}}. \qquad \text{(B 4)}$$

The subsystem (B3b)- (B3d) of the system (B3) may be reduced to a system of two linear equations with respect to c_1^2 and c_3:

$$\chi_0 c_3 - \chi_1 c_1^2 = 0, \qquad \text{(B 5a)}$$

$$\chi_2 c_3 - \chi_3 c_1^2 = 0, \qquad \text{(B 5b)}$$

where

$$\chi_0(\rho_{in}) = K_1^2(\rho_{in}),\ \chi_1(\rho_{in},\rho_{out}) = I_0(\rho_{out})K_1(\rho_{in}) + I_1(\rho_{in})K_0(\rho_{out}),$$

$$\chi_2(\rho_{in},\rho_{out}) = \frac{\Gamma}{\Gamma+2S}K_1^2(\rho_{in})\alpha_0,$$

$$\chi_3(\rho_{in},\rho_{out}) = K_1^2(\rho_{in})\alpha_{1,1} + I_1^2(\rho_{in})\alpha_{2,2} + 2I_1(\rho_{in})K_1(\rho_{in})\alpha_{1,2}.$$

The system (B5) has a non-trivial solution at zero discriminant

$$D(\rho_{in},\rho_{out}) \equiv \chi_0\chi_3 - \chi_1\chi_2 = 0. \tag{B 6}$$

Then, assuming the outer radius of the pinch ρ_{out} to be found from the eigenvalue equation (B6) for any given A $(0 < A < 1)$ and omitting one of the equations, e.g. Eq. (B3d) from the subsystem (B3b)-(B3e), the rest equations (B3) provide for expressions for the coefficient c_k $(k = 0, 1, 2, 3)$:

$$c_0 = \frac{2S}{2S+\Gamma}\frac{\chi_1}{\Delta^2}\alpha_0,\ c_1 = \frac{K_1(\rho_{in})}{\Delta},\ c_2 = \frac{I_1(\rho_{in})}{\Delta},\ c_3 = \frac{\chi_1}{\Delta^2}, \tag{B 7}$$

where $\Delta(\rho_{in},\rho_{out}) = I_1(\rho_{out})K_1(\rho_{in}) + I_1(\rho_{in})K_1(\rho_{out})$. Then Eqs. (B4) provide for expressions for the unscaled coefficient C_k $(k = 0, 1, 2, 3)$:

$$C_0 = \frac{c_0}{(1+Z)\rho_{out}^2},\ C_1 = \frac{2c_1C_0^{3/4}}{\Lambda\rho_{out}},\ C_2 = \frac{2c_2C_0^{3/4}}{\Lambda\rho_{out}}c_2C_0^{3/4},\ C_3 = \frac{4c_3C_0^{3/2}}{\Lambda^2\rho_{out}^2}. \tag{B 8}$$

Substituting Eqs. (B8) into Eqs. (4.5) provides explicit expressions for the radial distributions of temperature, magnetic field, current and pressure.

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