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Thermomagnetic instability in hot discs

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ABSTRACT

A linear stability analysis of ionized discs with a temperature gradient and an external axial magnetic field is presented. It is shown that both hydromagnetic and thermomagnetic effects can lead to the amplification of waves and make discs unstable. The conditions under which the instabilities grow are found and the characteristic growth rate is calculated. The regimes at which both the thermomagnetic and magnetorotational instabilities can operate are discussed.

Key words: accretion, accretion discs, MHD, MRI.

1 INTRODUCTION

The origin of turbulence in astrophysical discs is often attributed to hydrodynamic and hydromagnetic instabilities that can occur in differentially rotating stratified gas. The magnetorotational instability (MRI), first investigated by Velikhov (1959) and Chandrasekhar (1960) and later fully recognized by Balbus & Hawley (1991), is usually considered as one of the possible candidates to generate such turbulence and is thought to play an important role in the evolution and dynamics of astrophysical accretion discs. The growth of the instability in weakly ionized magnetized discs is of interest for models of star formation and the subsequent evolution of protostellar discs. Numerical simulations of the MRI in accretion discs (Hawley, Gammie & Balbus 1995; Brandenburg et al. 1995; Matsumoto & Tajima 1995; Torkelsson et al. 1996; Arlt & Rüdiger 2001) show that turbulence generated can enhance essentially the angular momentum transport. The MRI has been studied in detail for both stellar and accretion disc conditions (see, e.g., Fricke (1969); Safronov (1969); Acheson (1978); Balbus & Hawley (1991); Kaisig, Tajima & Lovelace (1992); Zhang, Diamond & Vishniac (1994)). In fact, the MRI can occur only in a relatively weak magnetic field, but a sufficiently strong field can suppress the instability completely (Balbus & Hawley 1991). This is related to the fact that the MRI is basically a long wavelength phenomena in the sense that it is stabilized for wavelengths shorter than $\lambda_{cr} \sim 2\pi c_A/\Omega$ where c_A is the Alfven velocity and Ω is the angular velocity. Defining $\beta = c_s^2/c_A^2$ where c_s is the sound speed, we can estimate that the critical wavelength λ_{cr} is longer than the half-thickness of the disc, $H \sim c_s/\Omega$ for $\beta > 1$. More rigorous calculations for high and low values of the radial wave number k_r may be found in Coppi & Keyes (2003), and Liverts & Mond (2009), respectively. In particular, it has been shown by Liverts & Mond (2009) that the number of unstable MRI modes is decreasing with β. For example, it was found that there exist only three unstable modes in the disc for $\beta \sim 10^3$. It is the goal of the present paper to study the effect of the temperature gradient on stability of magnetized discs in the case of high beta. It will be shown that the range of unstable wavelengths is significantly widened due to the vertical temperature gradients and can extend to values shorter than the disc thickness even in the case $\beta > 1$.

In discs, the centrifugal force almost is balanced by the gravitational force, while the vertical structure approximately is determined by hydrostatic equilibrium. The asymptotic analysis [see, e.g.,Regev (1983), Klužniak & Kita (2000), Ogilvie (1997), Shtemler, Mond, & Rüdiger (2009)] reveals that the vertical temperature gradient is comparable to that of the density in the case of a small aspect ratio. The destabilizing effect of a temperature gradient on Alfvén waves has been studies first by Gurevich (1963); Gurevich & Gel'mont (1967) and later on by Dolginov & Urpin (1979); Urpin (1981) for plasma in hydrostatic equilibrium. It has been shown that the thermomagnetic instability (TMI) can arise that transforms a fraction of the thermal energy into magnetic one. Also, Coppi (2008) has shown that the vertical temperature gradient in discs combined with the radial rotation shear give rise to vertically localized ballooning instabilities, the growth rate of which increases with

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the temperature gradient. Thus, for a super adiabatic temperature gradient that can be obtained due to a strong heating source around the equatorial plane, the growth rate of the ensuing ballooning modes is found to be comparable to that of the "cylindrical" MRIs.

In this paper, we consider the stability of hot magnetized discs taking into account the temperature gradient. We will take a look at the MRI and TMI in various ranges of the wavelength and determine the domains in a parameter space where each of those instabilities is dominant.

2 BASIC EQUATIONS

The dynamics of a magnetized disc is governed by the momentum, continuity, and induction equations:

$$\rho(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}) = -\vec{\nabla}p + \vec{J}\times\vec{B} + \rho\vec{G}, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0, \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla}\times\vec{E}, \tag{3}$$

where $\vec{G}$ is gravity.

The current density that is given by Ampére's law:

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}, \tag{4}$$

where μ_0 is magnetic constant and $\vec{B}$ satisfies a divergence-free condition

$$\vec{\nabla}\cdot\vec{B} = 0, \tag{5}$$

These set of equations should be complemented by the generalized Ohm's law that expresses the electric field $\vec{E}$ in terms of the electric current $\vec{J}$ and gradients of the thermodynamic quantities. It should be noted that due to quasi-neutrality of plasma the independent thermodynamic variables are only the temperature and pressure. Then, the Ohm's law reads (see, e.g., Braginskii (1963))

$$\vec{E} = -\vec{v}\times\vec{B} - \frac{1}{qn_e}\vec{\nabla}P_e + \hat{\eta}\cdot\vec{j} + \hat{\Lambda}\cdot\vec{\nabla}T, \tag{6}$$

where the last two terms represent the galvanomagnetic and thermomagnetic effects, correspondingly. The tensors $\hat{\eta}$ and $\hat{\Lambda}$ depend on the magnetic field $\vec{B}$ and, as a result, the transport properties of plasma are anisotropic with substantially different properties along and across the magnetic field if the field is sufficiently strong. The effect of the magnetic field on the transport properties is characterized by the magnetization parameter $a_{\rm e} = \omega_B\tau$ where $\omega_B = qB/m_{\rm e}$ is the gyrofrequency of the electrons and τ is their relaxation time (see, e.g., Spitzer (1978)). Even poorly conducting protostellar discs can be strongly magnetized if the electrons are the main charge carriers (Wardle 1999).

The tensor terms in Eq.(6) can be written as follows

$$\hat{\eta}\vec{J} = \eta\vec{J} + \eta_1(\vec{J}\times\vec{B}) + \eta_2\vec{B}(\vec{B}\cdot\vec{J}), \tag{7}$$

$$\hat{\Lambda}\vec{\nabla}T = \Lambda\vec{\nabla}T + \Lambda_1(\vec{\nabla}T\times\vec{B}) + \Lambda_2\vec{B}(\vec{B}\cdot\vec{\nabla}T), \tag{8}$$

where the second terms in both expressions represent the Hall and the Nernst effects, correspondingly. The coefficients of these expressions can be obtained from the relations given by Braginskii (1963); Lifshitz & Pitaevskii (1981). It is convenient to represent the effects of the temperature gradients by introducing two quantities that have the dimension of velocity:

$$\vec{u}_{1T} = \Lambda_1\vec{\nabla}T \tag{9}$$

$$u_{2T} = \Lambda_2(\vec{B}\cdot\vec{\nabla}T) \tag{10}$$

both of which are of the order of $u_{1T} \sim u_{2T} \sim k_B\tau/m_e|\vec{\nabla}T|$ for moderate values of the magnetization parameter $a_{\rm e}$ with k_B being the Boltzmann constant.

The basic state of the considered rotating discs is assumed to be characterized by an angular velocity that depends on the radial coordinate alone, $\Omega(r)$, and by hydrostatic equilibrium along the rotation axis z; $(r;\varphi;z)$ are the cylindrical coordinates. Thus, assuming polytropic equation of state, asymptotic analysis of the steady-state properties of the discs demonstrates that the vertical temperature gradients are much larger than the radial ones (Regev 1983; Klužniak & Kita 2000; Shtemler, Mond, & Rüdiger 2009). Consequently, as a model problem, in the current calculations the steady-state temperature is assumed to vary only vertically. In addition, the background magnetic field is assumed to be a constant parallel to rotation axis z.

As a first step toward studying the TMI it is instructive to estimate the relative importance of the thermomagnetic and galvanomagnetic terms contribution to Ohm's law. For moderate values of the magnetization parameter and wavelength of perturbations measured by c_A/Ω (typical axial wave number for the MRI spectrum), assuming the disc thickness as a characteristic length scale for the temperature gradient one finds that the thermomagnetic terms in the Ohm's law and the induction equation are much bigger than dissipative ones arising due to resistivity terms if

$$\frac{k_B T}{m_e c^2}\,\omega_p^2 \tau^2 \gg \sqrt{\beta} \tag{11}$$

where $\omega_p = \sqrt{n_e q^2/\varepsilon_0 m_e}$ is the plasma frequency with n_e being the number density of electrons and ε_0 being the electric constant. This condition can be satisfied for a very hot plasma with arbitrary values of magnetic field and small or moderate number density. Thus for example, Boettcher, Liang, & Smith (1998) have investigated Galactic black hole candidates. Applying their model to GX-339-4 the relevant data is that the disc temperature is of the order of 10^6K with number density of the order of $10^{24}m^{-3}$, and a corona with temperature of the order of 10^8K. Such range of parameters, with somewhat higher temperatures, has also been obtained by Artemova et al. (2006). According to (11) such conditions are indeed favorable for the onset of the TMI. The latter is consequently not suppressed by resistivity so that the galvanomagnetic terms can be dropped from Ohm's law.

Even though condition (11) may be fulfilled in optically thick as well as in optically thin environments (as was exemplified in the previous paragraph) we restrict the current discussion to media such that radiative heat exchange is efficient in that medium, thus any thermal transport due to the electron conduction may be neglected and the energy equation may be dropped out. This indeed simplifies the subsequent calculations. To this end one may refer to optically thick medium where the radiative transport can be described as diffusive heat transport with a thermal diffusivity that depends on the temperature and therefore a temperature gradient can be well exist. It should be emphasized though, that the TMI may also be excited in optically thin discs, however in such environments, the energy equation should be taken into account.

Consider a linear stability of the discs under axisymmetric short wavelength perturbations that propagate in z-direction. The perturbed electric field $\vec{E}$, magnetic field $\vec{b}$, current density $\vec{j}$ and hydrodynamic velocity $\vec{v}$ vary in time and space according to $\exp(ikz - i\omega t)$ so local approximation is used. Retaining the thermomagnetic terms in Ohm's law (6), invoking incompressibility that results in zero pressure perturbations (the latter also means that the perturbations are restricted to Alfvén modes) and assuming Keplerian angular velocity $\Omega(r)$ one can reduce linearized Eqs.(1)-(6) to the following system of equations:

$$\left[\begin{pmatrix} \omega^2 + 3\Omega^2 - \omega_A^2 & -2i\Omega\omega \\ 2i\Omega\omega & \omega^2 - \omega_A^2 \end{pmatrix} - \begin{pmatrix} -\omega\omega_{1T} - 2i\Omega\omega_{2T} & -\omega\omega_{2T} + 2i\Omega\omega_{1T} \\ \omega\omega_{2T} - \frac{1}{2}i\Omega\omega_{1T} & -\omega\omega_{1T} - \frac{1}{2}i\Omega\omega_{2T} \end{pmatrix}\right]\vec{b} = 0. \tag{12}$$

Here the characteristic frequencies are the Alfvén $\omega_A = kc_A$ and the thermomagnetic $\omega_{1,2T} = ku_{1,2T}$ frequencies.

Before turning to the stability analysis it is instructive to notice that for long wave lengths perturbations all the matrix elements of second matrix in square brackets of the last system of equations are small and consequently the thermomagnetic effects can be neglected. In that case the MRI is the dominant mode of instability. We will elaborate on that point in the next section. The main interest of the current work however, is the case for which the matrix elements of second matrix in square brackets of the equations (12) is comparable to the first one. This happens when: $\omega_{1,2T} \sim \Omega$, that indeed could occur in hot discs.

3 DISPERSION EQUATION FOR THERMOMAGNETIC WAVES

The dispersion relation that is obtained from Eqs.(12) reads

$$\omega^4 + a_3\omega^3 + a_2\omega^2 + a_1\omega + a_0 = 0, \tag{13}$$

where

$$a_3 = 2\omega_{1T} \tag{14}$$

$$a_2 = -\Omega^2 - 2\omega_A^2 + \omega_{1T}^2 + \omega_{2T}^2 - i\frac{3}{2}\Omega\omega_{2T} \tag{15}$$

$$a_1 = -2\omega_{1T}(\Omega^2 + \omega_A^2) \tag{16}$$

$$a_0 = -3\Omega^2\omega_A^2 + \omega_A^4 - \Omega^2(\omega_{1T}^2 + \omega_{2T}^2) + \frac{i}{2}\Omega\omega_{2T}(3\Omega^2 - 5\omega_A^2). \tag{17}$$

If $\Omega = 0$, Eq. (13) reduces to the dispersion relation for Alfvén waves modified by the temperature gradients (see Gurevich (1963); Gurevich & Gel'mont (1967); Dolginov & Urpin (1979); Urpin (1981)). If $\Omega \neq 0$ but $\nabla T = 0$, Eq.(13) reduces to the dispersion relation for the MRI obtained by Velikhov (1959), and Balbus & Hawley (1991), which describes the combined effect of inertial and Alfvén waves propagating in rotating discs.

It is instructive to examine a regime when the both MRI and TMI may operate. This implies the limit of thick disc ($\beta \gg 1$) and $\omega_{1,2T} \leqslant \Omega$. For typical wave number of the MRI spectrum (Ω/c_A) such regime reveals itself if $\zeta \equiv u_{2T}/c_A \leqslant 1$.

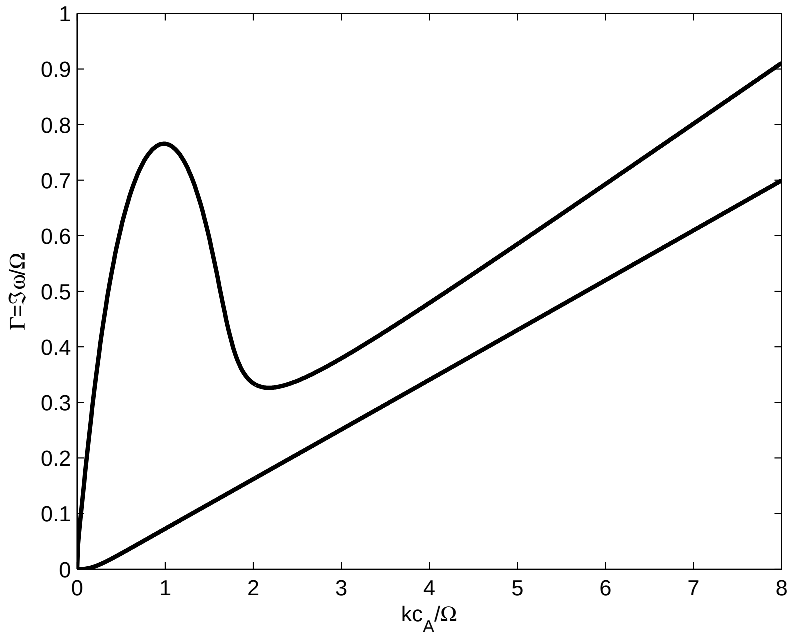

Figure 1. The growth rate, obtained by numerical solutions of eq.(13) for $u_T/c_A = 0.2$.

Furthermore one should note that in hot discs with the values of temperatures and number density quoted above the inequality (11) can be hold for such regime and thus magnetic diffusivity may be neglected. A solution of the dispersion equation for $\zeta \equiv u_{2T}/c_A = 0.2$ is depicted in Fig. 1. It is seen from this figure that there are two modes that become unstable. The upper curve corresponds to the MRI modified by the temperature gradient whereas the lower one describes a new instability that occurs due to the temperature gradient. It can be also obtained from the analytical consideration that properties of the instability are qualitatively different in the limiting cases of $\omega_A \leqslant \Omega$ (relatively long wavelengths) and $\omega_A \geqslant \Omega$ (relatively short wavelengths). In the long wavelength limit ($\omega_A \leqslant \Omega$), the instability is predominantly the MRI with small corrections caused by the temperature gradient. In particular, the temperature gradient is responsible for a finite phase velocity which is vanishing in the isothermal case. In this limit, the eigenvalue is given by

$$\omega = \pm(\frac{\Omega}{4}\frac{u_{2T}}{c_A} + i\sqrt{3}\omega_A). \tag{18}$$

The corresponding eigenvectors are

$$\vec{b} = \{\frac{2\omega_A}{\sqrt{3}\Omega}, \mp 1\}^T b, \quad \vec{v} = i\{\mp\frac{2\omega_A}{\Omega}, -\frac{4\omega_A^2}{\sqrt{3}\Omega^2}\}^T \frac{b}{\sqrt{\mu_0\rho}}, \tag{19}$$

These eigenvectors represent the pair of linearly polarized waves that propagate in the opposite directions along the z-axis. The wave propagating in the positive direction (this direction has been assumed conventionally by setting a positive value for Ω) is exponentially growing but the other one is decaying.

The another branch of modes in the limit of relatively long wavelengths is characterized by $\omega \sim \Omega$ and small growth rate that is given by

$$\Gamma = Im\omega \approx \pm\frac{5}{4}\omega_{2T}\frac{\omega_A^2}{\Omega^2} \tag{20}$$

The eigenvalue and eigenvector for that branch are

$$\omega = \pm(\Omega + i\frac{5}{4}\omega_{2T}\frac{\omega_A^2}{\Omega^2}), \quad \vec{b} = \{1, \mp 2i\}^T b, \quad \vec{v} = \{1, \frac{i}{2}\}^T \frac{b}{\sqrt{\mu_0\rho}}. \tag{21}$$

These eigenvectors describe the pair of left elliptically polarized waves that propagate in the opposite directions. In the absence of temperature gradients both waves are stable and represent inertial waves of the pure hydrodynamic nature.

In the limit of relatively short wavelengths (at $\omega \sim \omega_A \geqslant \Omega$) and for large temperature gradients such as $u_T/c_A \leqslant 1$ the thermomagnetic effects play an important role in the development of instability. The Alfvén waves can be significantly modified by the temperature gradient and become unstable beyond the stability limit of the classical MRIs. The growth rate in this regime is given by

$$\Gamma = \Im\omega \approx \frac{\omega_{2T}}{2}(1 \pm \frac{2\omega_{1T} + \Omega}{4\omega_A}) \tag{22}$$

Finally, in the short wavelength limit and under the condition $u_T/c_A \leqslant 1$ the rotation can be neglected and the problem reduces to that described by Gurevich & Gel'mont (1967).

In order to gain deeper understanding into the nature of the TMI in the short wavelength limit, the dispersion equation is rewritten in terms of $b^{\pm} \equiv b_x \pm i b_y$ in the limit $\Omega \to 0$. Then, the dispersion equation reads

$$\left[\omega^2 + \omega(\omega_{1T} \mp i\omega_{2T}) - \omega_A^2\right] b^{\pm} = 0 \tag{23}$$

This equation describes four circularly polarized waves. One pair of waves propagates along the magnetic field, and its frequency is

$$\omega = \omega_A - \frac{\omega_{1T}}{2} \pm i\frac{\omega_{2T}}{2}. \tag{24}$$

The corresponding eigenvectors are

$$b^{\pm} = b, \quad b^{\mp} = 0, \quad \frac{v^{\pm}}{c_A} = \left[-1 - \frac{\omega_{1T}}{2\omega_A} \pm i\frac{\omega_{2T}}{2\omega_A}\right]\frac{b}{B_0}, \quad v^{\mp} = 0. \tag{25}$$

The unstable wave (upper sign) is right circularly polarized while the damped wave (lower sign) is left circularly polarized.

The wave for which the phase shift between perturbations of the magnetic field and velocity is smaller than π (π is the appropriate value for pure Alfvén waves) grows exponentially. Another wave for which the phase shift between the magnetic and velocity perturbations is larger than π is damped. The other pair also comprises of two right and left polarized waves that propagate in the opposite direction to the magnetic field. One of these waves grows exponentially whereas the other is damped in the same manner as in the first pair. Summarizing, right circularly polarized waves propagating in the opposite directions along the magnetic field grow exponentially with the growth rate

$$\Gamma = \frac{\omega_{2T}}{2} \tag{26}$$

The picture of circularly polarized unstable waves allows an additional interpretation of the instability. To that end it should be noted that $j^{*\pm} = \mp k b^{*\pm}/\mu_0$ where j^* is the complex conjugate amplitude of the current density of the circularly polarized waves. The thermomagnetic contribution to the perturbed electric field is anti Hermitian as it can be seen from Eq.(25). Then, it is easy to show that the average work over the wave period is given by

$$\langle \vec{E} \cdot \vec{j} \rangle = -\frac{\omega_{2T}}{2\mu_0}|b|^2. \tag{27}$$

The time derivative of the average energy $\langle W \rangle$ for the wave propagating along the external magnetic field is given by

$$\frac{\partial \langle W \rangle}{\partial t} = \frac{\partial}{\partial t}\int \langle \rho\frac{v^2}{2} + \frac{b^2}{2\mu_0}\rangle d^3r = -\int \langle \vec{j} \cdot \vec{E} \rangle d^3r. \tag{28}$$

This equation together with Eq. (27) clearly shows that the wave energy grows exponentially with the rate 2Γ.

4 SUMMARY

We have considered the thermomagnetic instability in ionized non-isothermal Keplerian discs that are threaded by an external magnetic field. It has been shown that, in the short wavelength limit, the temperature gradient allows to develop the thermomagnetorotational instability for wavelengths satisfying the condition $\omega_A \gg \Omega$ for which the MRI does not occurs. The growth rate of this instability can be comparable to that of the MRI. Physically, the thermomagnetorotational instability transforms a fraction of the thermal energy carried out by the heat flux into the magnetic energy of Alfvén waves (modified by the temperature gradient, see Gurevich (1963); Gurevich & Gel'mont (1967); Dolginov & Urpin (1979); Urpin (1981)). The presence of shear, however, changes substantially the growth rate of instability. The considered combined magnetorotational and thermomagnetic instability can play an important role in dynamics of astrophysical discs.

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