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THE HALL INSTABILITY OF WEAKLY IONIZED, RADIALLY STRATIFIED, ROTATING DISKS

Edward Liverts and Michael Mond
Department of Mechanical Engineering, Ben-Gurion University of the Negev, P.O. Box 653, Beer-Sheva 84105, Israel; eliverts@bgu.ac.il

and

Arthur D. Chernin
Sternberg Astronomical Institute, Moscow State University, University Prospect 13, Moscow 119899, Russia
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ABSTRACT

Cool weakly ionized gaseous rotating disks are considered by many models to be the origin of the evolution of protoplanetary clouds. Instabilities against perturbations in such disks play an important role in the theory of the formation of stars and planets. Thus, a hierarchy of successive fragmentations into smaller and smaller pieces as a part of the Kant-Laplace theory of formation of the planetary system remains valid also for contemporary cosmogony. Traditionally, axisymmetric magnetohydrodynamic (MHD) and, recently, Hall-MHD instabilities have been thoroughly studied as providers of an efficient mechanism for radial transfer of angular momentum and of radial density stratification. In the current work, the Hall instability against nonaxisymmetric perturbations in compressible rotating fluid in external magnetic field is proposed as a viable mechanism for the azimuthal fragmentation of the protoplanetary disk and, thus, perhaps initiates the road to planet formation. The Hall instability is excited due to the combined effect of the radial stratification of the disk and the Hall electric field, and its growth rate is of the order of the rotation period. This family of instabilities is introduced here for the first time in an astrophysical context.

Subject headings: instabilities — MHD — planetary systems: protoplanetary disks

1. INTRODUCTION

Linear mode analysis provides a useful tool for gaining important insight into the relevant physical processes that determine the stability of rotating fluid configurations. The importance of magnetic fields in rotating disks has been demonstrated by the rediscovery of the magnetorotational instability (MRI) in which hydrodynamically stable flows with an angular velocity that is decreasing outward are highly unstable when threaded by a weak magnetic field (Balbus & Hawley 1991). That investigation has been carried out in the magnetohydrodynamic (MHD) limit and invoked a number of approximations appropriate to the study of the evolution of long-wavelength perturbations in the weak magnetic field limit. The inclusion of the Hall electric field (Hall MHD) is a relatively recent development (Wardle & Ng 1999; Wardle 1999; Balbus & Terquem 2001; Sano & Stone 2002a, 2002b; Salmeron & Wardle 2003; Desch 2004; Urpin & Rudiger 2005; Rudiger & Kitchatinov 2005). The Hall electric field plays an important role in the disk's dynamics when the coupling between the electrons and heavy particles (ions and neutrals) of the fluid is weak. In such cases, the inertial length of the ions is longer than the characteristic perturbation's length scale, and consequently, the motions of the ions and electrons are decoupled. Indeed, it has been shown that the Hall electric field has a profound effect on the structure and growth rate of unstable modes like the MRIs. Furthermore, as will be shown below, the Hall term gives rise to new branches of unstable modes. In particular, it is shown that the Hall term, in the presence of radial stratification, excites nonaxisymmetric instabilities.

As an astrophysical interest, we mention magnetically supported cool molecular clouds and their dynamics. In the disk of a typical spiral galaxy, the magnetic field strength is usually estimated to be of several to more than 10 μG, while in some regions of spiral galaxies, the magnetic field strength may be higher than several tens of microgauss (Sofue et al. 1986; Beck et al. 1996). For a protoplanetary disk, the magnetic field can be significantly higher. By these estimations and others, it is almost certain that MHD density waves should also play an important role in the dynamics and evolution of structures within a magnetized gas disk (Fan & Lou 1996). As is well known in the classical nebular hypothesis by Kant and Laplace, the condensation in a protoplanetary rotating disk plays an important part in forming stars and planets. That part of the Kant-Laplace theory remains valid also for a contemporary cosmogony. The "standard" theory of the multistage accretionary formation of planets, or the so-called core accretion mechanism (Safronov 1972; Pollack et al. 1996), remained the most popular until recently, when it was criticized by Boss (2002, 2003) and others. The main problem of the latter is the timescale, which is longer than estimates of the lifetime of many planet-forming disks (Taylor 1962; Feigelson & Montmerle 1999). In any case, all theories rely on instabilities as a mechanism to transform a relatively uniform rotating gaseous disk into a planetary system. That is, at an early stage, the protosolar nebulae are formed by fragments that separated from a molecular cloud. Planetary formation is thought to start with inelastically colliding gaseous and dust particles settling to the central plane of the rotating nebula to form a thin layer around the plane. During the early evolution of the disk it is believed that the dust particles also coagulate into comets and planetesimals. On attaining a certain critical thickness (and correspondingly low temperature) small in comparison with the size of the disk, as a result of a local gravitational collapse the nebula disintegrates into the central body and a number of separate protoplanets. Instabilities arise as the thickness of the disk is reduced (Gurevich & Chernin 1978; Safronov 1972). If a rotating gaseous disk has a large vertical thickness owing to a high internal temperature, then it is stabilized against gravitational instabilities by thermal motion (Gurevich & Chernin 1978). In Boss (2004) it is demonstrated that convective cooling is able to cool the disk midplane at the desired rate to produce clumps in marginally unstable disks. The physical phenomena treated in this paper occur during the stage of evolution of the protoplanetary cloud when the dust and gas in the disk start to condense into planetesimals and a star with current luminosity emerges at the center of the nebula.

The rest of paper is organized as follows. In § 2.1 we present basic equations and state our assumptions. In § 2.2 we present the dispersion relation to be solved. In § 2.3 we pay particular attention to the conditions for the existence of complex-conjugate roots of the dispersion relation. We present our conclusions and discussion in § 3.

2. HALL-MHD EQUATIONS AND THE DISPERSION RELATION

2.1. *Basic Equations*

We consider a thin rotating gaseous disk with angular velocities $\Omega(r)$, where G is the gravitational constant, M is the mass of the central body, and r is the distance from the center of the rotating disk. The thickness of the disk can be estimated by $c_s/\Omega(r)$, where c_s is the sound speed. The disk is made of partially ionized plasma where ions are well coupled to the neutrals while the electrons are not. However, charge neutrality, $n_e = n_i$, is assumed to be valid. The disk is immersed in a magnetic field directed along the rotation axis (defined as the z-axis in our frame of reference). Following Braginskii (1965), the equations that govern the evolution of the two fluid system, namely, the heavy particles (ions and atoms) and the electrons, are the momentum equation,

$$\rho \frac{d\boldsymbol{v}}{dt} = -\nabla p + \frac{1}{c}\boldsymbol{j} \times \boldsymbol{B} - \rho \frac{GM}{r^3}\boldsymbol{r}, \qquad (1)$$

where ρ is the density of the heavy particles (ions and atoms), and the generalized Ohm's law,

$$m_e \frac{d\boldsymbol{u}_e}{dt} = e\boldsymbol{E} + \frac{e}{c}\boldsymbol{v} \times \boldsymbol{B} - \frac{1}{n_e c}\boldsymbol{j} \times \boldsymbol{B} + T_e \frac{\nabla n_e}{n_e} - \frac{e\boldsymbol{j}}{\sigma_R}, \qquad (2)$$

where T_e is the electron temperature (measured in units of energy), σ_R is the electrical conductivity, and the relationship $\boldsymbol{j} = en_e(\boldsymbol{v} - \boldsymbol{u}_e)$ has been employed. In addition, it has been assumed in deriving equation (2) that the electrons are isothermal. The generalized Ohm's law as given in equation (2) differs from the corresponding equation of MHD theory by the term on the left-hand side, which describes the effect of electron inertia, by the third term on the right-hand side, which is the Hall effect, by the fourth term, which describes the effect of the electrons' pressure, and by the last term, which represents the drag force acting on the electrons. In addition, it is convenient to write the induction equation by substituting the electric field from equation (2) into Faraday's equation, which then becomes

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \left(\frac{\boldsymbol{j} \times \boldsymbol{B}}{en_e}\right) - \frac{m_e c}{e}\nabla \times \frac{d\boldsymbol{u}_e}{dt} + \nabla \times (\eta \nabla \times \boldsymbol{B}), \qquad (3)$$

where $\eta = c^2/4\pi\sigma_R$ is the magnetic diffusivity. The relative importance of the various terms in equation (3) may be investigated by considering appropriate length and timescales. Thus, consider, for example, the second term on the right-hand side of equation (3) (the Hall term). If we assume that $\nabla \approx 1/L$, $j \approx cB/4\pi L$ (displacement current is neglected), and $v \sim v_{\rm A}$, where $v_{\rm A} = B/(4\pi\rho)^{1/2}$ is the Alfvén velocity and L is a typical length scale of the density inhomogeneity, then that term will be of the same order as the convective electric field term if $\ell_i \approx \sqrt{y_e}L$, where

$$\ell_i = \frac{c}{\omega_{pi}} = \frac{c\sqrt{m_i}}{\sqrt{4\pi e^2 n_i}}, \quad y_e = \frac{\rho_i}{\rho},$$

where y_e is the ion mass density fraction and ρ_i is the mass density of the ions. The ionization fraction $x_e = n_e/n_n$ (here n_n is the number density of the neutrals) is equal to y_e only if the ions and the neutrals are of the same species. Otherwise, as indeed commonly happens, the ratio of y_e to x_e is approximately given by the mass ratio of the dominant ions to that of the dominant neutrals. Thus, the Hall term is important if the length scale L is of the same order of magnitude or less than the ions' inertial length ℓ_i divided by $\sqrt{y_e}$, i.e.,

$$L \le \ell_i/\sqrt{y_e}. \qquad (4)$$

Similarly, the ratio of the electron inertia term (the second to last term on the right-hand side of eq. [3]) to the convective term is given by

$$\frac{\omega}{\Omega_e}\frac{c}{4\pi e n_e L} \approx \frac{1}{\sqrt{4\pi\rho}},$$

where Ω_e is the electron Larmor frequency and it is assumed that the frequency ω is high enough to estimate $du_e/dt \approx \omega j/en_e$. Thus, the electron's inertial term is important for inhomogeneities which are characterized by $L \approx c/\omega_{pe} = \ell_e$. As far as the timescale is concerned, it is seen from the relationship $v_{\rm A}/\ell_i = \sqrt{y_e}\Omega_i$ (Ω_i is the ion Larmor frequency) that for the Hall term to be important the frequency should be $\omega > \sqrt{y_e}\Omega_i$, while the electron inertia should be retained if $\omega > \Omega_e$. Note that in the case of $\omega \gg \sqrt{y_e}\Omega_i$ the electrons drift in the wave's electric field, while the ions are immobile. Thus, if $\sqrt{y_e}\Omega_i \ll \omega \ll \Omega_e$, the second term on the right-hand side of equation (3), namely, the Hall term, is the leading term. This is the reason to term such an approximation Hall MHD and the waves in that regime Hall waves. It is seen from equation (3) that in such a case

$$\omega \approx \frac{c^2\Omega_i}{\omega_{pi}^2 L^2} = \frac{v_{\rm Hd}}{L},$$

where $v_{\rm Hd} = v_{\rm A}^2/y_e\Omega_i L$ is the phase velocity of the Hall waves in the presence of density gradients whose scale length is L. Such waves exist merely due to electron drift in the electric field of the Hall waves. It should be noted from the discussion above that the conditions for the Hall term to be significant are more easily satisfied as the fraction of ionization is decreasing. Finally, it should be noted that for low enough densities the ohmic dissipation term in equation (3) (the last term on the right-hand side) is negligible in comparison with the Hall term (Balbus & Terquem 2001). Additional support to that point of view is provided by Jin (1996), who estimated the ratio of the rotation period to the ohmic dissipation time to be of the order of $10^{-3}k^2H^2$. As will be seen in the subsequent sections, the growth rates of the Hall instability are of the order of the rotation period, and the relevant wavelengths satisfy $kH < 1$. Hence, the effect of ohmic dissipation will be neglected from now on, bearing in mind that it may nonetheless lower the growth rates of the investigated instabilities.

2.2. *The Linearized Equations and the Dispersion Relation*

We consider a differentially rotating disk for which the steady state is characterized by an angular velocity $\Omega(r)$, where r is the distance from the disk's center, and r-dependent density $\rho(r)$. The disk is threaded by an axial magnetic field $\boldsymbol{B} = B_0(r)\hat{z}$. As the steady state is assumed to be axisymmetric, and in addition, the steady state magnetic field is axial, the steady state Hall electric field is zero. As a result, the basic state of the flow is described by ideal MHD.

The equations governing the linear stability of the rotating disk may be derived from equations (1)–(3) and the continuity equation by assuming that the perturbations of the steady rotation are of the form

$$f(r, \theta) = f(r) \exp\left[i(m\theta - \omega t)\right], \tag{5}$$

where $f(r, \theta)$ stands for the perturbation of any of the physical variables that describe the system. It should be noted that, strictly speaking, equation (5) can be used only for rigid rotation, as differential rotation results in nonexponential perturbations. Nonetheless, that fact accentuates the nondependence of the Hall instability on the rotation shear, in contrast to MHD instabilities like MRIs. A full description of the temporal evolution of perturbations in a differentially rotating disk will be described in a forthcoming publication. The amplitudes of the perturbed θ-component of the velocity and density and the z-component of the magnetic field can be expressed in terms of the amplitude of the radial component of the perturbed velocity. This results in an ordinary differential equation for $u_r(r)$ that should be solved with appropriate boundary conditions, thus yielding an eigenvalue problem. However, for purposes of demonstration we first consider the simplified case of local approximation by assuming that $(1/u_r)(\partial u_r/\partial r) \ll m/r$. This means that radial gradients of perturbations are much smaller than the azimuthal gradients. After linearization, equations (1)–(3) and the continuity equation assume the form

$$i(\omega - m\Omega)u_r + 2\Omega u_\theta \cong 0, \tag{6}$$

$$(\omega - m\Omega)u_\theta + i\frac{\chi^2}{2\Omega}u_r = \frac{m}{r}c_s^2\frac{\sigma}{\rho} + \frac{m}{r}V_{\rm A}^2\frac{b_z}{B_0}, \tag{7}$$

$$-\frac{1}{y_e\rho\Omega_i}\frac{m}{r}\nabla_r\frac{B_0^2}{8\pi}\frac{\sigma}{\rho} + \frac{m}{r}u_\theta - \left[\omega - m\left(\Omega + \frac{\ell_i^2\Omega_i}{Lr}\right)\right]\frac{b_z}{B_0} = 0, \tag{8}$$

$$(\omega - m\Omega)\frac{\sigma}{\rho} - \frac{m}{r}u_\theta = 0, \tag{9}$$

where σ is the perturbation of the density, χ is the epicyclic frequency given by $\chi = (4\Omega^2 + 2r\Omega\, d\Omega/dr)^{1/2}$, and u_r, u_θ, and b_z are the perturbed radial and azimuthal components of the fluid velocity and the axial component of the magnetic field, respectively. It has also been tacitly assumed that y_e is a constant parameter that characterizes the steady state as well as the perturbed disk. This assumption greatly simplifies the calculations by avoiding the ionization dynamics while preserving the main features of the Hall instability. In order to see that, it is first noted that typical ionization rates are much larger than typical rotation frequencies. Hence, since the inverse growth rates of interest are of the order of the rotation period, it is plausible to assume that the system is in ionization equilibrium at each time during the perturbations. In order to describe that, the model suggested for a wide range of ionization processes,

$$y_e = c(T)\rho^{-1/2},$$

where $c(T) \propto T^{1/4}$ is given in terms of the recombination and ionization rate coefficients (Fromang et al. 2002), is adopted. The latter yields in the polytropic case

$$y_e = c\rho^{(\gamma-3)/4}, \tag{10}$$

where γ is the adiabatic index. Now, the variations of y_e are of importance only in the second term on the right-hand side of equation (3) whose linearized form is given by equation (8). Thus, ∇n_e in equation (3) is given by $\nabla(\rho y_e)$, which in turn is given due to equation (10) by $\nabla\rho(1 + d\ln y_e/d\ln\rho)$. As a result, influence of the variations in the ionization fraction is manifested in the modification of the relevant length scales in equation (8) by the same factor, $(1 + d\ln y_e/d\ln\rho)$, which is of order 1. According to equation (10), the latter may range between 1/2 for the isothermal case ($\gamma = 1$) through 2/3 for the adiabatic case ($\gamma = 5/3$) and 3/4 for $\gamma = 2$. Furthermore, as will be seen in § 2.3, one of the parameters that determines the stability properties of the rotating disk is given by the ratio of two length scales. This reduces the influence of the variations of the ionization fraction on the Hall instability and justifies the constant y_e simplification.

The system of equations (6)–(9) yields the dispersion relation

$$\tilde{\omega}^3 - \tilde{\omega}^2\omega_{\rm Hd} - \tilde{\omega}\left[\chi^2 + k^2\left(c_s^2 + V_{\rm A}^2\right)\right] + \omega_{\rm Hd}\left(\chi^2 + k^2\frac{L}{\rho}\frac{\partial\mathcal{P}}{\partial r}\right) = 0, \tag{11}$$

where $\mathcal{P}$ is the total unperturbed pressure, $\tilde{\omega} = \omega - m\Omega$, $k = m/r$, and $\omega_{\rm Hd} = m\ell_i^2\Omega_i/(Lr)$ is the Hall drift frequency. It is a direct result of the assumed form of the perturbation, i.e., equation (5), that the azimuthal and radial components of the perturbed magnetic field as well as the axial component of the perturbed velocity are zero. In addition, the derivatives of the equilibrium profiles have been neglected in deriving the linearized equations above, except in the axial component of Faraday's law (eq. [8]), where due to the Hall term, they are of the same order as the rest of the terms.

2.3. *The Instability against Nonaxisymmetric Perturbations*

In the case of homogeneous density and magnetic field strength ($L \to \infty$), the two roots of equation (11) represent two stable branches of density waves that originate due to both the rotation of the disk and the external magnetic field. However, in the case of density or magnetic field inhomogeneity the roots of equation (11) with real coefficients are real if and only if the following conditions are satisfied:

$$D = \frac{\omega_{\rm Hd}^6}{108}\left[27X^2 + 4X\left(1 - 9Y^2\right) - 4Y^2 + 8Y^4 - 4Y^6\right] \le 0, \tag{12}$$

where

$$X = -\frac{k^2L\nabla_r\mathcal{P}}{\rho\omega_{\rm Hd}^2} + \frac{k^2\left(c_s^2 + V_{\rm A}^2\right)}{\omega_{\rm Hd}^2},$$

$$Y^2 = \frac{\chi^2}{\omega_{\rm Hd}^2} + \frac{k^2\left(c_s^2 + V_{\rm A}^2\right)}{\omega_{\rm Hd}^2}.$$

If the last condition is not fulfilled, equation (11) has two complex-conjugate roots, one of which signifies instability. The conditions for that to happen are

$$X \geq \frac{2}{27}\left(-1+9Y^2+\sqrt{1+9Y^2+27Y^4+27Y^6}\right) \quad (13)$$

or

$$X \leq \frac{2}{27}\left(-1+9Y^2-\sqrt{1+9Y^2+27Y^4+27Y^6}\right). \quad (14)$$

Further insight into the onset of the Hall instability may be gained by denoting the total pressure radial derivative by $(p + B_0^2/8\pi)/L_{\mathcal{P}}$ where $L_{\mathcal{P}}$ is the inhomogeneity length of the total pressure. Next, the well-known relation $c_s = H\Omega$ is recalled, and finally normalizing frequencies to Ω and wavelengths to H, the following dispersion relation is obtained:

$$\tilde{\omega}^3 - q\alpha\tilde{\omega}^2 - \tilde{\omega}\left[\hat{\chi}^2 + q^2\left(1+\beta^{-2}\right)\right] + q\alpha\left[\hat{\chi}^2 + \xi q^2\left(1+\beta^{-2}/2\right)\right] = 0, \quad (15)$$

where

$$\alpha = \frac{1}{\beta}\frac{\ell_i}{L\sqrt{y_e}},$$

$\beta = c_s/V_{\rm A}$, $q = kH$, $\hat{\chi} = \chi/\Omega$, and $\xi = L/L_{\mathcal{P}}$. It is first noted that for $\alpha \to 0$ the MHD regime is recovered and the roots of equation (15) represent the stable combination of the fast magnetosonic waves and the epicyclic oscillations. However, as α is increased (which means that L is decreased relative to the inertial length of the ions) the system enters into the Hall-MHD regime. In that case, elementary analysis of Cardano's solution of cubic equations reveals that the nature of the roots of dispersion equation (15) hinges on the value of μ, which is given by

$$\mu = \frac{\hat{\chi}^2 + \xi q^2(1+\beta^{-2}/2)}{\hat{\chi}^2 + q^2(1+\beta^{-2})}, \quad (16)$$

and the roots of the quadratic equation

$$4\mu S^2 + \left(1 + 18\mu - 27\mu^2\right)S + 4 = 0, \quad (17)$$

where

$$S = \frac{q^2\alpha^2}{\hat{\chi}^2 + q^2(1+\beta^{-2})}.$$

Thus, equation (15) has two complex roots, and hence, the system is unstable in the following two cases:

1. $\mu > 1$, for $S_1 < S < S_2$, where S_1 and S_2 are the roots of equation (17).

In this case ξ must be positive which means that the density and the total pressure change radially in the same direction. It is therefore obvious that regions of instability occur where the total pressure changes more rapidly than the density ($\xi > 1$). This is indeed the case in polytropic disks for which $L/L_p = \gamma > 1$, where L_p is the inhomogeneity length associated with the pressure. Hence, $\xi > \gamma$ and consequently $\mu > 1$, which means that the disk is unstable under the Hall instability if β is such that S is between the two roots of equation (17). Exact values of the growth rate, obtained by the numerical solution of equation (15), are depicted in Figure 1 for Keplerian rotation ($\hat{\chi} = 1$), with $\ell_i/(L\sqrt{y_e}) = 10$ and $\xi = 5/3$, for various values of β. It is found that the disk is unstable for $1 < \beta < 20$.

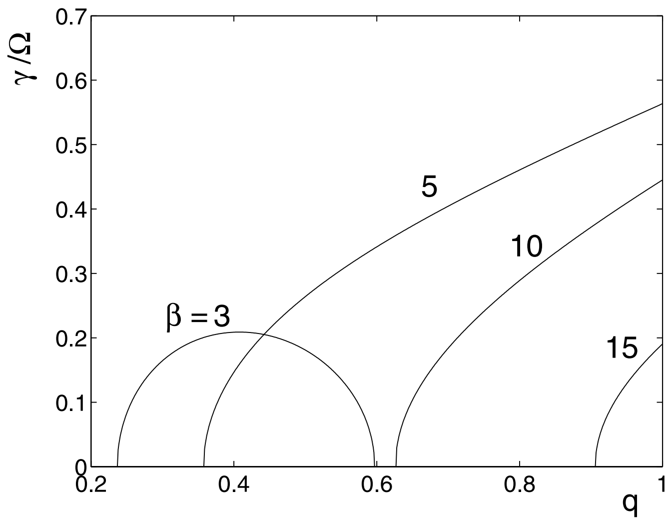

Fig. 1.—Growth rates of the Hall instability for Keplerian rotation, with $\ell_i/(L\sqrt{y_e}) = 10$ and $\xi = 1.66$.

2. $\mu < 0$, for $S > \max(S_1, S_2)$.

In this case it is obvious that the gradients of the density and the total pressure must have opposite signs (i.e., $\xi < 0$). Such situations may occur when radial inflow plays an important role in the dynamics of the evolving disk, such as in young protoplanetary clouds (Hogerheijde 2004) or when radial rings of nonmonotonic density profiles are formed due to gravitational instabilities (Mayer et al. 2005). In these cases, in the limit $\alpha \gg 1$ one of the solutions of equation (15) is approximated by equating the second and fourth terms in equation (15) and is given by

$$\omega = \pm i\gamma, \quad (18)$$

where

$$\gamma = \Omega\sqrt{|\xi|q^2(1+\beta^{-2}/2) - \hat{\chi}^2}. \quad (19)$$

It is clear that in this case the rotation plays a stabilizing role. In addition, since $S \propto 1/\beta$, there is an upper bound on β for instability to occur but not a lower bound. Furthermore, in the limit of small β the growth rate grows without bound as β is decreased. Numerical solutions of equation (15) for Keplerian rotation with $\ell_i/(L\sqrt{y_e}) = 10$ and $\xi = -1.5$ are depicted in Figure 2. The instability exists for $\beta < 9$, and the growth rate is indeed a growing function of $1/\beta$.

The Hall instability for slab geometry was discovered by Brushlinskii & Morozov (1980) and has been investigated in detail by Liverts & Mond (2004). In this geometry the steady state plasma is accelerated due to gradients in the total pressure. Thus, it is the combined effect of spatial density and magnetic field inhomogeneities that drives the Hall instability in slab geometry. As has been shown in Liverts & Mond (2004), the Hall instability results from the merging of the slow branch of the fast

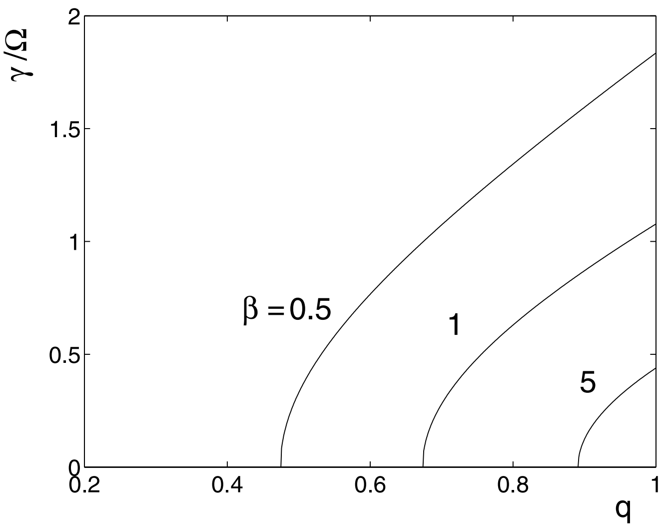

FIG. 2.—Same as Fig. 1, but for $\xi = -1.5$.

magnetosonic waves and the slow quasi-electrostatic mode that exists only due to density inhomogeneity. Thus, the Hall instability provides a powerful mechanism for the azimuthal breaking of radially stratified disks into small fragments of size comparable to the disks' thickness and on a timescale of the order of the rotation period.

3. SUMMARY AND DISCUSSION

This paper examined the instability of weakly ionized, thin disks threaded by an external magnetic field within the Hall-MHD model. The vertical stratification as well as the azimuthal variations of the disks' properties were ignored, whereas radial distribution was considered. In particular, in astrophysical contexts such weakly ionized gaseous nebulae are relevant to the protoplanetary disks. The conditions in protoplanetary disks were discussed in Safronov (1972) and Gurevich & Chernin (1978). At a certain stage of its evolution, the star nebula is believed to have a characteristic disk size up to the order of 30–100 AU, and the total mass of the disk is believed to be less than roughly 0.1 of the mass of the central star. This yields an integrated column density of $\Sigma \approx 3\times10^2$ g cm^{-2}. Assuming that the mean mass of the particles is $m_p = \Lambda m_H$, the number density is given by $n = \Sigma/[\Lambda m_H H(r)] \simeq 2\times10^{14}$ cm^{-3}, where $H(r)$ is the scale thickness of the disk. To estimate the value of the factor Λ it should be noted that within this system lighter elements such as hydrogen and helium were driven out of the central regions by star wind and radiation pressure during a highly active phase, leaving behind heavier elements like Na, Al, and K and dust particles. Thus, in the outer part of the star nebula, ice and volatile gases were able to survive. As a result, the inner planets are formed of minerals, while the outer planets are more gaseous or icy. Concerning thickness estimation, one can use $c_s(r)/\Omega(r)$, which yields $H(r) \approx 0.002$ AU. It should be noted also that due to the very low temperature of the protoplanetary disks the only sources of ionization are nonthermal, e.g., cosmic rays, X-rays, and the decay of radioactive elements. Following Sano & Stone (2002b), the ionization fraction at the midplane of gaseous disk is estimated as $x_e = n_e/n_n \approx 10^{-12}$. So the disk material is a partially ionized plasma where ions and charged small dust grains are well coupled to the neutrals but electrons are not. Following this assumption we can estimate that $\ell_i/\sqrt{y_e}$ is up to the order of 1 AU. It should be noted that such big values are obtained mostly due to the low fraction of ionization; however, one should keep in mind that the existence of positively charged grain particles increases the inertial length (see definition after eq. [3]) and, thus, enhances the effect of the Hall term. In § 2 it has been demonstrated that the Hall term is important if $H(r) < L < \ell_i/\sqrt{y_e}$. Thus, due to the very small values of the ionization fraction in protoplanetary disks, the Hall-MHD model has to be employed when studying the stability of structures with realistic radial density inhomogeneities. Indeed, radial stratification with a length scale $L > H$ may exist in the disk due to such mechanisms as axisymmetric density waves that give rise to alternating high- and low-density rings. On the other hand, L is bounded from below by H due to thermal pressure (Gurevich & Chernin 1978). Hence, as has been further shown in § 2, protoplanetary disks with such radial density distributions are susceptible to strong nonaxisymmetric instabilities whose growth rates are of the order of the rotation period of the disk. Such instabilities result in the breaking of the density rings into fragments that may be identified as planetesimals. Following a widely adopted standpoint, an accumulation of planetesimals may lead to the next stage of evolution of the protoplanetary disk, which is the coalescence of the planetesimals into protoplanets. Such planetesimals may survive the thermal pressure if their characteristic size, i.e., $1/k$, is bigger than the disk's thickness H (Gurevich & Chernin 1978). On the other hand, the linear analysis presented in § 2.2 is valid if $kL < 1$. Combining this last condition and the condition from equation (4) results in the following limitations on k:

$$kH < 1, \quad kH < \sqrt{y_e}\frac{\Omega_i}{\Omega}\frac{\ell_i}{L}, \qquad (20)$$

where the value of β has been taken as 1 for simplicity. Thus, for typical values in protoplanetary disks, namely, $y_e = 10^{-12}$, $\Omega_i = 10^4 B_0/G$ s^{-1}, and $\Omega \approx 1.9\times10^{-7}$ s^{-1}, the right-hand side of the second inequality in equation (20) is of the order of or smaller than unity. It is therefore again the small ionization fraction that enables the onset of the Hall instability in the small magnetic field limit and by thus providing a mechanism for the initiation of the standard scenario of planet formation.

Searching for other possible nonaxisymmetric mechanisms that may compete with the Hall instability naturally leads to the magnetorotational instability (MRI) whose growth rate, as that of the Hall instability, is of the order of the inverse rotation time. Obviously, the Hall effect on the MRI has to be taken into account in the context of weakly ionized disks and in order to carry out the appropriate comparison. Doing so, it has been found (Wardle 1999; Balbus & Terquem 2001; Rudiger & Shalybkov 2004) that MHD unstable MRI modes are stabilized or further destabilized according to whether $\boldsymbol{\Omega}\cdot\boldsymbol{B}$ is positive or negative, respectively. This is due to the fact that the MRI originates from the interaction of the Alfvèn waves and the inertial (Coriolis) modes. This is in contrast to the Hall instability, which represents two other branches of the appropriate dispersion relation, namely, the merging of the slow branch of the fast magnetosonic waves and the slow quasi-electrostatic mode that exists only due to density inhomogeneity, as was pointed out in § 2.3. Hence, the Hall instability exists regardless of the mutual orientation of the magnetic field and the rotation. Furthermore, the Hall instability is insensitive to the rotation shear, but rather depends on the density stratification (radial in the case of the current work). In any case, the comparison between the Hall instability and the Hall modified MRIs cannot be complete, as only analysis of the latter for axial wavenumbers is available in the literature for Keplerian rotation. Some trends, however, may be seen from an equivalent study for Couette flow that indicates that the MRIs are stable in the high-m limit which is just the opposite of the result presented here. Thus, it seems that the Hall instability is currently the only known mechanism to provide a route for the growth of highly nonaxisymmetric perturbations.

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