Wind instability of a foam layer sandwiched between the atmosphere and the ocean

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Wind shortwave instability of a foam layer between the atmosphere and the ocean is examined in order to explain the recent findings of the decrease in momentum transfer from hurricane winds to sea waves. The three-fluid configuration with the high contrasts in densities provides for an effective mechanism to stabilize surface perturbations, and shifts the marginal wavelength to the shortwave part of the spectrum. It is conjectured that such stabilization leads to the observed drag reduction. The foam-layer thickness is found exceeding of which is ineffective for stabilization.

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INTRODUCTION

Results of direct measurements extrapolated from weak to strong winds predict a linear increase in momentum transfer from wind to sea waves. The present study is motivated by recent findings of saturation and even decrease in the drag coefficient ("capping") in hurricane conditions that is accompanied by production of a foam layer on the ocean surface [1]. A possible explanation for the drag coefficient capping is the development of a foam layer at the air-sea interface. The principal role of such an air-water foam layer in energy dissipation and momentum transfer from hurricane wind to sea waves has been first suggested in [2].

Winds generate the ocean surface waves with a wide spectrum of wave lengths. The longest waves of hundreds meters of length attempt to catch up with the wind, while the steeper short waves break out and play a dominant role in drag formation [3-5]. When the wind speed exceeds the storm force (24m/s), wave breaking creates streaks of bubbles near the ocean surface. As the wind exceeds the hurricane force (32m/s), streaks of bubbles combined with patches of foam cover the ocean surface. When the wind speed reaches $50m/s$, a foam layer completely covers the ocean surface [1]. The breaking process does not completely destroy the waves, but rather tears off the tops of the breaking short waves, when their steepness exceeds a critical value determined by nonlinear effects. Thus, in hurricane conditions a fully developed spatially homogeneous foam layer is likely to form.

Nowadays, there is a little hope for comprehensive numerical calculations of the drag coefficient reduction that includes a detailed description of the wave breaking and foam layer production. Instead, several explanations have been proposed within the atmospheric boundary-layer theory (see [1,3-5] and references therein).

However, up to now there is no complete understanding of the phenomenon. Numerous factors influencing the system can be considered as a governing physical mechanism of the phenomenon, e.g. the foam compressibility (which transforms a part of the wind energy into foam-bubble oscillations), the high contrast in foam density compared with air and water densities etc. In the present study the short-wave length Kelvin-Helmholtz instability (KHI) [6-7] of a system of a foam layer sandwiched between the atmosphere and the ocean is investigated in order to qualitatively explain the drag reduction phenomenon. The following simplifications will be adopted: (i) The foam layer interfaces with liquid and gas are planar. (ii) The three fluids that constitute the system, namely, the gas, the foam, and the liquid are incompressible (see discussion in Section III). (iii) Capillary effects at the foam-layer interfaces are ignored.

THE PHYSICAL MODEL

A piecewise constant approximation for the equilibrium densities and longitudinal velocities ρ_j and U_j of the gas, liquid and foam ($j = g, f, l$) is employed:

$$\rho = \rho_l, U = U_l \equiv 0 \qquad \text{for} \quad y < 0,$$

$$\rho = \rho_f, U = U_f \quad \text{for} \quad 0 < y < L_f,$$

$$\rho = \rho_g, U = U_g \equiv U_{g\infty} \qquad \text{for} \quad y > 0, \tag{1}$$

where $U_{g\infty}$ is the known constant value of the wind velocity, while the constant foam layer thickness L_f and velocity U_f are the generally unknown parameters of the foam layer in hurricane conditions. In addition, hydrostatic equilibrium pressure $\partial P_j/\partial y = -g\rho_j$ (g is the gravity acceleration) is assumed in each domain with the appropriate continuity conditions at the two interfaces.

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The equations of motion that govern the dynamics of the system are the corresponding Euler equations in each of the three layers, and kinematic (no voids) as well as pressure continuity are applied at the foam layer interfaces with the liquid and the gas. The equilibrium state is perturbed as:

$$\Phi(x, y, t) = F(y) + F'(x, y, t), \tag{2}$$

where Φ stands for any physical variables, F and F' denote the equilibrium and perturbations. The latter are assumed to be of the form $F' = f'(y)exp(-i\omega t + ikx)$ with real k and complex ω. Thus, the amplitudes f' that satisfy the boundary conditions at $y = \pm\infty$ are given by:

$$f'_g = \tilde{f}_g exp(-ky),\ f'_l = \tilde{f}_l exp(ky),$$

$$f'_f = \tilde{f}_f^{(-)} exp(-ky) + \tilde{f}_f^{(+)} exp(ky), \tag{3}$$

where tilde denotes constant magnitude factors.

Substitution of Eqs. (1)-(3) into the linearized Euler equations and the interface boundary conditions, yields the dispersion relation (quartic in phase velocity C) [8]:

$$2(H_g + H_l) + (E-1)(H_g+1)(H_l+1) = 0, \tag{4}$$

$$E = exp(2kL_f),\ H_l = \frac{\rho_l (U_l - C)^2}{\rho_f (U_f - C)^2} - \frac{(\rho_l - \rho_f)g}{\rho_f k (U_f - C)^2},$$

$$H_g = \frac{\rho_g (U_g - C)^2}{\rho_f (U_f - C)^2} - \frac{(\rho_f - \rho_g)g}{\rho_f k (U_f - C)^2},\ C = \frac{\omega}{k}. \tag{5}$$

To classify the unstable modes of Eq. (4), let us consider the reference two-fluid system by setting $L_f = 0$, or equivalently, either $\rho_f = \rho_l$ or $\rho_f = \rho_g$. Then Eq. (4), is reduced to the standard dispersion relation $H_g + H_l = 0$ for KHI in the two-fluid system [6]:

$$\rho_l (k_0 U_l - \omega_0)^2 + \rho_g (k_0 U_g - \omega_0)^2 = k_0 g(\rho_l - \rho_g), \tag{6}$$

where the subscript 0 denotes the foam free system. Comparing Eq. (4) in the limit $kL_f >> 1$ with the reference Eq. (6), we conclude that it is decomposed on two modes responsible for instability of the water-foam ($H_l + 1 = 0$) and air-foam ($H_g + 1 = 0$) interfaces.

ASYMPTOTIC ANALYSIS

To estimate the effective values of the system parameters which strongly influence the system stability, let us apply the limit of low gas-liquid density ratio $\rho_g/\rho_l = \epsilon^2 << 1$ ($\epsilon^2 \approx 10^{-3}$ for the air-water system). Assuming the equilibrium liquid is at rest ($U_l = 0$), we have from Eq. (6):

$$\omega_0 = \sqrt{gk_0 - \epsilon^2 k_0^2 U_g^2} + O(\epsilon^2 k_0 U_g, \epsilon g k_0/\omega_0, \epsilon^2 \omega_0). \tag{7}$$

It can be concluded that the classical KHI for two-fluid systems is excited at the short wavelength regime:

$$k_0 L_* \sim k_0^* L_* = 1/\epsilon^2,\ \omega_0 L_*/U_* \sim 1/\epsilon,\ C_0/U_* \sim \epsilon. \tag{8}$$

Here and below $L_* = U_*^2/g$, $U_* = U_g$, while the superscript asterisk denotes the marginal values of parameters.

Back to the general case of three-fluid systems, it is assumed that the water content within the foam $\alpha_l \sim 0.05$ is small (the low liquid content is a characteristic feature of the air-water foams). Then α_l is scaled with ϵ

$$\frac{\rho_f}{\rho_*} \approx \alpha_l \sim \epsilon,\ \frac{\rho_g}{\rho_*} = \epsilon^2,\ \frac{\rho_g}{\rho_f} \approx \frac{1}{\alpha_l}\frac{\rho_g}{\rho_l} \equiv \frac{\epsilon^2}{\alpha_l} \sim \epsilon, \tag{9}$$

where $\rho_* = \rho_l$; $\rho_f = \alpha_g \rho_g + \alpha_l \rho_l$ is the foam density; α_g, α_l are the gas and liquid volume fractions within the foam; $\alpha_g + \alpha_l = 1$. Assuming that the three-fluid system operates in the same regime that gives rise to the KHI in the classic two-fluid system, let us adopt the scales

$$kL_* \sim \frac{1}{\epsilon^2},\ \frac{\omega L_*}{U_*} \sim \frac{1}{\epsilon},\ \frac{C}{U_*} \sim \epsilon. \tag{10}$$

As will be seen below the above assumption selects the "water-foam" mode of Eq. (4). Assuming now the scaling for the foam thickness and velocity:

$$U_f/U_* \sim \epsilon^a,\ L_f/L_* \sim \epsilon^b,\ 0 < a < 1,\ 0 < b. \tag{11}$$

we have the estimates for Eq. (4):

$$H_g \sim H_l \sim \epsilon^{1-2a},\ E \sim exp(\epsilon^{b-2}). \tag{12}$$

Inserting the scaling (12) into Eq. (4), and applying once again the principle of the least degeneracy of the three-fluid problem, we obtain $a = 1/2$, $b = 2$, which means:

$$\frac{U_f}{U_*} \sim \epsilon^{1/2},\ \frac{L_f}{L_*} \sim \frac{\lambda_0^*}{L_*} \sim \frac{1}{k_0^* L_*} = \frac{\rho_g}{\rho_*} \sim \epsilon^2, \tag{13}$$

where $\lambda_0^* = 2\pi/k_0^*$. Consequently, following the scaling presented in Eq. (13) it is convenient now to re-scale the wave number and frequency

$$\hat{\omega} = \omega/\sqrt{gk_0^*} \sim \epsilon^0,\ \hat{k} = k/k_0^* \sim \epsilon^0. \tag{14}$$

Then, Eq. (4) to leading order in ϵ yields for the "water-foam" mode

$$\hat{\omega} = \sqrt{\frac{2(\hat{k} - \hat{k}^2) - (E-1)(\hat{k}^2 K_f - \hat{k})(K_f^{-1} + 1)}{2 + (E-1)(K_f^{-1} + 1)}}, \tag{15}$$

$E = exp(2\hat{k}\hat{L}_f)$; $\hat{L}_f$ and K_f are the rescaled foam thickness and the ratio of the foam-to-air dynamic pressure:

$$\hat{L}_f = k_0^* L_f \sim \epsilon^0,\ K_f = \frac{\rho_f U_f^2}{\rho_g U_g^2} \sim \epsilon^0,\ 0 < K_f < 1. \tag{16}$$

It is obvious now that the two dimensionless parameters that determine the properties of the KHI in the three-fluid system are the foam thickness and velocity or equivalently Ri_f and K_f, where Ri_f is the Richardson number

$$Ri_f = \frac{\rho_l - \rho_g}{\rho_g} \frac{gL_f}{U_g^2} \approx \frac{1}{\epsilon^2} \frac{L_f}{L_*} \equiv k_0^* L_f.$$

Two particular limits of the dispersion relation (4) for the "water-foam" mode are readily obtained at small ϵ, namely the foam-free ($H_l + H_g = 0$ at $L_f = 0$) and the foam-saturated ($H_l + 1 = 0$ at $L_f = \infty$) limits. In the first limit the dispersion relation (15) is reduced again to Eq. (7) for the air-water system:

$$\frac{\omega_0}{\sqrt{gk_0^*}} = i\sqrt{\frac{k^2}{k_0^{*2}} - \frac{k}{k_0^*}}, \; Ri_f = k_0^* L_f = 0. \qquad (17)$$

In the second limit, the dispersion relation (15) describes the foam-water system:

$$\frac{\omega_\infty}{\sqrt{gk_\infty^*}} = i\sqrt{\frac{k^2}{k_\infty^{*2}} - \frac{k}{k_\infty^*}}, \; Ri_f = k_0^* L_f = \infty, \qquad (18)$$

which differs from Eq. (17) by replacing k_0^*, ω_0 with $k_\infty^* = k_0^*/K_f$, ω_∞. Comparison of these two limits demonstrates the stabilizing effect of the foam ($K_f < 1$), due to the exponential decrease with kL_f of the marginal wavelength from the foam-free value $\lambda_0^* = 2\pi/k_0^*$ to the foam-saturated value $\lambda_\infty^* = 2\pi/k_\infty^*$. The relation $K_f = k_0^*/k_\infty^*$ allows to express $K_f = \lambda_\infty^*/(\epsilon^2 2\pi L_*)$ through the given wavelength value λ_∞^*. Since $0 < K_f < 1$ this also yields the upper bound for λ_∞^*: $\lambda_\infty^* < \Lambda^* = 2\pi\epsilon^2 L_*$, e.g. for pre-hurricane and developed hurricane wind velocities, $U_* = 32m/s$ and $U_* = 50m/s$, we obtain $\Lambda^* = 0.6m$ and $\Lambda^* = 1.5m$. According to the hurricane observation data [3], the wavelength of the unstable KHI modes is $0.6m < \lambda < 0.9m$ and adopting the values $\epsilon^2 = \rho_g/\rho_l \approx 0.001$, $U_* = 50m/s$, $L_* \approx 250m$, $\lambda \approx \lambda_\infty^*$, we obtain $0.4 < K_f < 0.6$. Figure 1 depicts the imaginary part of the eigenfrequency vs wavenumber. The growth rate $\omega_i/\sqrt{gk_0^*}$ decreases as the foam thickness is increased and approaches its asymptote (given by Eq. (18)) already at the effective value of the foam thickness $k_0^* L_f^{(ef)} \approx 1$ (that is equivalent to the relation $L_f^{(ef)} = \epsilon^2 L_*$). The growth rate is depicted vs the foam-layer thicknesses in Fig. 2. The critical curve ($k = k_\infty^*$, i.e. $k/k_0^* = 2$ at $K_f = 0.5$) with $\omega_i \to 0$ at $k_0^* L_f \to \infty$, separates the subcritical curves ($k > k_\infty^*$) for the foam-saturated two-fluid (foam-water) systems from the supercritical curves ($k < k_\infty^*$). The growth rates of the supercritical waves sharply decrease with the increase of the foam-layer thickness, till total stabilization at a finite value of $k_0^* L_f$ is achieved. For subcritical systems the growth rate strongly drops from the foam-free value at $L_f = 0$ to the foam-saturated level already at $L_f \approx L_f^{(ef)}$.

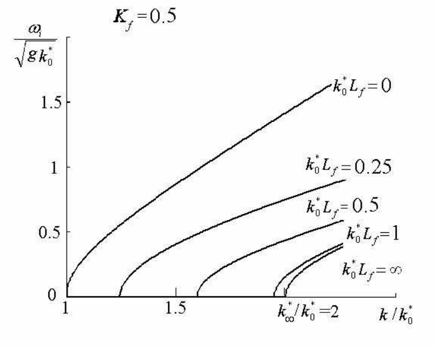

FIG. 1: Dimensionless growth rate $\omega_i/\sqrt{gk_0^*}$ vs wave number, k/k_0^*, for the typical foam-layer thicknesses, $k_0^* L_f$; the dynamic pressure ratio $K_f = 0.5$.

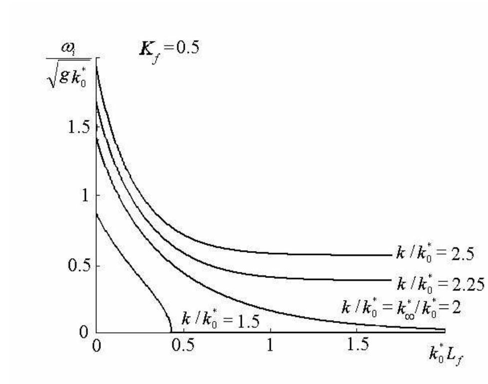

FIG. 2: Dimensionless growth rate $\omega_i/\sqrt{gk_0^*}$ vs foam-layer thickness, $k_0^* L_f$, for the typical wave numbers, k/k_0^*; the dynamic pressure ratio $K_f = 0.5$; the critical curve $k/k_0^* = 1/K_f = 2$ ($k = k_\infty^*$) separates the subcritical ($k > k_\infty^*$) and supercritical ($k < k_\infty^*$) curves.

The threshold wave number k^* satisfies the eigenvalue equation for the three-layer system ($\omega_i = 0$):

$$exp(2k^* L_f) = 1 - \frac{2}{1 + K_f^{-1}} \frac{1 - k^*/k_0^*}{1 - K_f k^*/k_0^*}. \qquad (19)$$

As in the classical two-fluid system, to leading order in ϵ, the waves propagate with phase velocity $C = \omega/k$ without amplification at $k/k^* < 1$, and amplify with zero phase velocity at $k/k^* > 1$. The threshold wave number monotonically decreases with the foam thickness from the foam-free value $k = k_0^*$ at low $k_0^* L_f$ to the foam-saturated value $k^* = k_\infty^*$ at high $k_0^* L_f > 1$.

Assuming the orders of the foam velocity and thickness given by estimations (13), the second "air-foam" mode of Eq. (4) can be obtained replacing the scalings in (10) by

$$kL_* \sim \frac{1}{\epsilon^2}, \; \frac{\omega L_*}{U_*} \sim \frac{1}{\epsilon^{3/2}}, \; \frac{C}{U_*} \sim \epsilon^{1/2}. \qquad (20)$$

Then, Eq. (4) to leading order in ϵ yields that the "air-foam" mode is unstable for all short wavelengthes

$$\frac{\omega}{\sqrt{gk_0^*}}\sqrt{\frac{\rho_f}{\rho_w}}=\frac{k}{k_0^*}\left(\sqrt{K_f}\pm i\sqrt{\frac{E-1}{E+1}}\right). \qquad (21)$$

The growth rate in (21) increases from 0 in the foam-free limit at $L_f=0$ to the foam-saturated limit at $L_f=\infty$.

Finally, the influence of the foam compressibility on the KHI is estimated. Using that the air Mach number $M_g=U_g/C_g$ is small and employing that the foam-to-air sound velocity ratio $C_f/C_g\sim\sqrt{\rho_g/\rho_f}\sim\sqrt{\epsilon}$ [10] is of the same order as $U_f/U_g\sim\sqrt{\epsilon}$ in Eqs. (13), we obtain that $M_f=U_f/C_f$ is of the same order as M_g, namely much less than unity. These estimates demonstrate that in the KHI study foam can indeed be considered as incompressible fluid as it commonly accepted for air [7].

CONCLUSIONS AND DISCUSSION

The analysis of KHI in a three-fluid system is treated asymptotically in two small parameters: gas-liquid density ratio ϵ^2 and liquid content in the foam $\sim\epsilon$.

Two unstable modes were determined, which responsible for instability of the air-foam and water-foam interfaces in the foam-saturated limit ($L_f=\infty$). Although the growth rate for the "air-foam" mode is much larger than that for "water-foam" mode (their ratio $\sim 1/\sqrt{\epsilon}$), the ratio of the magnitude factors at the "air-foam" and "water-foam" modes vanishes exponentially with growing $kL_{f\infty}$ (for the same levels of perturbations of the air-foam interface for both modes)as $exp(-2kL_f)$. These inputs approach to their small foam-saturated limits (at $L_f=\infty$) already at the effective thickness of the foam $L_f^{(ef)}=\epsilon^2 L_*$, i.e. Speculating that non-linear effects saturate the exponential growth of the "air-foam" mode in time, influence of the "air-foam" mode on the water-foam interface can be neglected. With this in mind, the further discussion will concerned to the "water-foam" mode alone and its effect on the water surface.

The dimensionless foam thickness, L_f, and foam-to-air dynamic pressure ratio, K_f, are found to govern the system stability. Since the data available in literature about those two parameters is quite scarce, the value K_f is expressed through the characteristic wavelength, λ_∞^*, which is estimated using the available experimental data, while the effective value L_f is evaluated asymptotically as $L_f^{(ef)}=\epsilon^2 L_*$. When the foam thickness is larger than the effective value $L_f^{(ef)}$, the growth rate reaches its minimal saturated value, and further increasing in L_f is ineffective, as if the foam layer is of infinite thickness. Thus, in hurricane conditions, $U_*=50m/s$, which, as assumed in the modelling, correspond to a complete coverage of the sea surface by the foam, the effective value of the foam thicknesses, $L_f^{(ef)}\approx 0.25m$, is of the order of the experimentally registered values [11]. The foam layer provides for an effective mechanism of the system stabilization against the KHI, due to high contrast densities of the air, foam and water ($\rho_g<<\rho_f<<\rho_l$). Otherwise, e.g. if $\rho_f\approx\rho_l$ or $\rho_f\sim\rho_g$ as is the case for bubbly liquids or spray, respectively, the system will be close to the two-fluid air-water configuration. The results are physically transparent, since in the foam-saturated system the foam layer totally separates the air flow from the sea surface, and the three-fluid system becomes close to a two-fluid foam-water system. Formally this corresponds substituting the foam density and velocity instead of those parameters for the air in the classic air-water KHI model. The marginal wavelengths are shifted to the short-wave part of the spectrum, and the characteristic time of the perturbation growth sharply increases with wavelength from the foam-free to the foam-saturated value.

Thus, the three-fluid configuration, which is characterized by a high contrast in densities, is suggested as a new mechanism to effectively stabilize the water-foam interface. All these effects point to the conclusion that as the foam thickness is increased the rate of momentum transfer from the wind to the sea waves is decreased, and subsequently so is the drag. Justification of this conclusion requires the further investigation of the nonlinear stage of the phenomenon.

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