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ANGULAR MOMENTUM TRANSPORT IN ASTROPHYSICAL DISKS

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ABSTRACT

The evolution of astrophysical disks is dominated by instabilities of gravity perturbations (e.g., those produced by a spontaneous disturbance). We develop a hydrodynamic theory of nonresonant Jeans instability in a dynamically cold subsystem (identified as the gaseous component) of a disk. We show analytically that gravitationally unstable systems, such as disks of rotationally supported galaxies, protoplanetary disks, and, finally, the solar nebula are efficient at transporting mass and angular momentum: already on a timescale of on the order of 2–3 rotational periods an unstable disk sees a large portion of its angular momentum transferred outward, and mass transferred both inward and outward.

Subject headings: galaxies: kinematics and dynamics — planetary systems: formation — solar system: formation

1. INTRODUCTION

The theory of stability of self-gravitating astrophysical disk configurations has now been developed quite thoroughly. Interest in this theory is due to the efforts to solve such problems as formation of spiral arms in galaxies (Lin et al. 1969), planetary formation (Boss 2003), and the fine structure of Saturn's rings (Griv & Gedalin 2003). It has been shown that the structure and evolution of such systems are dominated by instabilities of gravity perturbations (those produced by a spontaneous disturbance or, in rare cases, a companion system). In particular, *unstable* (growing) density waves can be self-excited in the disk via the gravitational Jeans-type instability. In turn, simulations have already demonstrated that hydrodynamic turbulence due to gravitational instability effectively transports angular momentum, thus providing the disk with a source of internal viscosity: anomalously high turbulent viscosity is produced by the disk's instability (Takeda & Ida 2001). The turbulent stresses in computer-generated disks transport the angular momentum outward, as mass flows inward (Laughlin & Bodenheimer 1994; Laughlin & Rózyczka 1996; Gammie 2001; Durisen et al. 2007). Note that a turbulent viscosity could be one source for angular momentum transport in a differentially rotating medium (Lynden-Bell & Pringle 1974). This idea is inherent in the accretion-disk model of Shakura & Sunyaev (1973). Another source is provided by the almost stable waves generated by resonant interactions (Goldreich & Tremaine 1979). Below in this Letter, only *nonresonant* Jeans instabilities as a mechanism for the transfer of angular momentum in a disk are studied. The chief aim in the present theory is to explain the result of numerical experiments mentioned above. Namely, gravitational instabilities produce growing density waves and associated torques, which are potent agents of angular momentum transport.

The route to turbulence and subsequently to accretion in neutral disks such as galactic disks, accretion disks, and protoplanetary clouds has remained one of the outstanding puzzles in astrophysics. As is well known, the accretion of material onto the center is inefficient if the viscosity in the disk is determined by the classical molecular transport coefficients; the presence of developed turbulence is usually postulated to explain the observed features. We investigate hereafter the conditions for excitation of the Jeans instability and the possibility of angular momentum transport arising in various astrophysical disks. We show that the instability can give rise to the *enhanced* outward angular momentum transport (relative to the ordinary molecular viscosity) needed to account for observations of accreting systems.

2. BASIC EQUATIONS

The dynamical response of a cold gas in the presence of the collective self-gravitational field is considered. A Lagrangian description of the motion of a gas element under the influence of a perturbed field is used, looking for time-dependent waves which propagate in a two-dimensional disk. This approximation of an infinitesimally thin disk is a valid approximation if one considers perturbations with a radial wavelength that is greater than the disk thickness $2h$. The time-dependent surface mass density $\sigma(\boldsymbol{r}, t)$, the total gravitational potential of the disk (including the central object, if it exists at all) $\Phi(\boldsymbol{r}, t)$, the pressure $P(\boldsymbol{r}, t)$, and the fluid velocity $\boldsymbol{v}(\boldsymbol{r}, t)$ are written as $X = X_0(r) + X_1(\boldsymbol{r}, t)$, where X stands for any of the above mentioned physical variables, $X_0(r)$ describes the basic flow, $|X_1/X_0| \ll 1$ represents the perturbations, (r, φ, z) are the cylindrical coordinates, and the axis of the disk rotation is along the z-axis. These quantities σ, Φ, and P are then substituted into the equations of motion of the gas, the continuity equation, and the Poisson equation; the second-order terms of the order of σ_1^2, Φ_1^2, and P_1^2 are neglected with respect to the first-order terms. The resulting equations of motion are cyclic in the variables t and φ, and hence by applying the widely used local WKB method one may seek solutions in the form of normal modes by expanding

$$X_1(\boldsymbol{r}, t) = \sum_{\boldsymbol{k}} \tilde{X}_{\boldsymbol{k}} \exp(ik_r r + im\varphi - i\omega_{\boldsymbol{k}} t) + \mathrm{CC}, \qquad (1)$$

where $\tilde{X}_{\boldsymbol{k}} = \mathrm{const}$ is a real amplitude, k_r is the real radial wavenumber, $|k_r| r \gg 1$, m is the nonnegative azimuthal mode number, $\omega_{\boldsymbol{k}} = \Re\omega_{\boldsymbol{k}} + i\Im\omega_{\boldsymbol{k}}$ is the complex frequency of excited waves, subscripts $\boldsymbol{k}$ denote the $\boldsymbol{k}$th Fourier component, and "CC" means the complex conjugate. Evidently X_1 is a periodic function of φ, and hence m must be an integer. A disk is considered to be a superposition of different oscillation modes.

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A disturbance in the disk will grow until it is limited by some nonlinear effect. In the linear theory, one can select one of the Fourier harmonics: $X_1 = \tilde{X} \exp(ik_r r + im\varphi - i\omega t) + \mathrm{CC}$. This solution represents a spiral plane wave with m arms or a ring ($m = 0$). The imaginary part of ω corresponds to the growth ($\Im\omega > 0$) or decay ($\Im\omega < 0$) of the components in time, $\sigma_1, \Phi_1, P_1 \propto \exp(\Im\omega t)$, and the real part to a rotation with constant angular velocity $\Omega_p = \Re\omega/m$. When $\Im\omega > 0$, the medium transfers its energy to the growing wave and oscillation buildup occurs.

The linearized equations of two-dimensional motion of the gas element in the frame of reference rotating with angular velocity Ω at the position r_0 can be written as

$$\frac{\partial v_r}{\partial t} - 2\Omega v_\varphi + 2r_0 x\Omega\frac{d\Omega}{dr} = -\frac{\partial\Phi_1}{\partial r} - \frac{c_s^2}{\sigma_0}\frac{\partial\sigma_1}{\partial r}, \tag{2}$$

$$\frac{\partial v_\varphi}{\partial t} + 2\Omega v_r = -\frac{1}{r_0}\frac{\partial\Phi_1}{\partial\varphi} - \frac{c_s^2}{r_0\sigma_0}\frac{\partial\sigma_1}{\partial\varphi}, \tag{3}$$

where v_r and v_φ are the perturbations in the radial and azimuthal velocities, $c_s = (\partial P/\partial\sigma)_0^{1/2}$ is the local speed of sound, and a local Cartesian coordinate system (x, y) is defined via $r = r_0 + x(r_0, \varphi_0, t)$, $\varphi = \varphi_0 + \varphi_1(r_0, \varphi_0, t) = \Omega t + y/r_0$, $v_r = \partial x/\partial t$, and $v_\varphi = r_0\partial\varphi_1/\partial t = \partial y/\partial t$ (Goldreich & Lynden-Bell 1965). The Eulerian velocities (u_r, u_φ) are obtained as a result of the following relations: $u_r = v_r$ and $u_\varphi = v_\varphi - r_0 x(d\Omega/dr)$ (Lin & Lau 1979). The influence of central object (an inertial halo) enters through $\Omega(r_0)$. Equations (2) and (3) must be solved simultaneously with the continuity equation

$$\sigma_1 = -\int_{-\infty}^{t}\left[\frac{\partial}{\partial r}(\sigma_0 v_r) + \frac{\sigma_0}{r}\frac{\partial}{\partial\varphi}v_\varphi\right]_{r=r_0} dt', \tag{4}$$

where the relatively small term $\sigma_0 v_r/r$ is omitted (Lin & Lau 1979).

A particular solution of the system of equations (2) and (3) is obviously

$$v_r = \frac{\aleph}{\omega_*^2 - \kappa^2}\left(\omega_* k_r + i2\Omega\frac{m}{r}\right), \tag{5}$$

$$v_\varphi = \frac{\aleph}{\omega_*(\omega_*^2 - \kappa^2)}\left[(4\Omega^2 - \kappa^2 + \omega_*^2)\frac{m}{r} - i2\Omega\omega_* k_r\right], \tag{6}$$

where $\omega_* = \omega - m\Omega$ is the Doppler-shifted wave frequency (in a rotating frame), $\kappa = 2\Omega\,[1 + (r/2\Omega)(d\Omega/dr)]^{1/2} \sim \Omega$ is the epicyclic frequency, $\omega_* \neq 0$, $\omega_*^2 - \kappa^2 \neq 0$, and $\aleph = \Phi_1 + c_s^2\sigma_1/\sigma_0$. These solutions describe the perturbed velocities of the gas in the radial and azimuthal directions under the action of the small perturbation, $|v_r|$ and $|v_\varphi| \ll r\Omega$. As is seen, the present theory suggests some systematic motions of the gas element distributed in the form of a spiral-like flow field which is a correction to the basic circular, equilibrium [$r\Omega^2 = \partial\Phi_0/\partial r + (c_s^2/\sigma_0)(\partial\sigma_0/\partial r)$, where the term $\propto c_s^2$ is a small correction] motion.

3. DISPERSION RELATION

From equations (4)–(6) it is straightforward to show that

$$\sigma_1 \approx \frac{\sigma_0\aleph}{\omega_*^2 - \kappa^2}\left(k_r^2 + \frac{4\Omega^2 - \kappa^2 + \omega_*^2}{\omega_*^2}\frac{m^2}{r^2} + \frac{2\Omega}{\omega_*}\frac{m}{rL}\right), \tag{7}$$

where $|L| = |\partial \ln(\Omega\sigma_0\kappa^{-2})/\partial r|^{-1}$ is the radial scale of inhomogeneity, $|L|/r \ll 1$, and the terms $\propto m$ are small corrections. Only the nonresonant low-frequency ($\omega_* \neq 0$, $|\omega_*|^2 \lesssim \kappa^2$) perturbations developing between the Lindblad resonances are considered (Griv et al. 1999).

Equating the density σ_1 (eq. [7]) to the density $\sigma_1 = -|k|\Phi_1/2\pi G$ given by the asymptotic ($k_r^2 \gg m^2/r^2$) solution of the Poisson equation (Lin & Lau 1979), one obtains the generalized Lin-Shu dispersion relation

$$\omega_*^3 - \omega_*\omega_\mathrm{J}^2 - 4\pi\Omega G\sigma_0(m/r|k|L) = 0, \tag{8}$$

where $(k_r r)^2 \gg (k_r L)^2 \gg 1$, $k^2 c_s^2 \ll 2\pi G\sigma_0|k|$, and

$$\omega_\mathrm{J}^2 = \kappa^2 - 2\pi G\sigma_0(k_*^2/|k|) + k_*^2 c_s^2 \tag{9}$$

is the squared Jeans frequency, $k = (k_r^2 + m^2/r^2)^{1/2}$ is the total wavenumber, $k_*^2 = k^2\{1 + [(2\Omega/\kappa)^2 - 1]\sin^2\psi\}$ is the squared effective wavenumber, and $\psi = \arctan(m/rk_r)$ is the perturbation pitch angle.

From equation (8) in the most important frequency range $|\omega_*|^3 \sim |\omega_\mathrm{J}|^3 \gg 4\pi\Omega G\sigma_0(m/r|k|L)$, one determines the dispersion law for the Jeans branch of vibrations:

$$\omega_{*1,2} \approx \pm p|\omega_\mathrm{J}| - 2\pi G\sigma_0\frac{\Omega}{\omega_\mathrm{J}^2}\frac{m}{r|k|L}, \tag{10}$$

where $p = 1$ for gravity-stable perturbations with $\omega_*^2 \approx \omega_\mathrm{J}^2 > 0$, $p = i$ for gravity-unstable perturbations with $\omega_*^2 \approx \omega_\mathrm{J}^2 < 0$, and the term involving L^{-1} is the small correction. In the gravity-unstable case ($\omega_*^2 \approx \omega_\mathrm{J}^2 < 0$), the local equilibrium parameters of the disk determine the pattern speed of growing nonaxisymmetric perturbations (in a rotating frame):

$$\Omega_p \equiv \Re\omega_*/m \approx 2\pi G\sigma_0(\Omega/|\omega_\mathrm{J}^2|)(1/r|k|L) \ll \Omega, \tag{11}$$

where $2\pi G\sigma_0|k| \sim \Omega^2$, $|\omega_\mathrm{J}^2| \sim \Omega^2$, and $rk^2|L| \gg 1$. Because Ω_p does not depend on m, each Fourier component of a perturbation in an inhomogeneous system will rotate with the same constant angular velocity even while the perturbation (for instance the density disturbance) is otherwise growing. The theory states that in homogeneous ($|L| \to \infty$) disks $\Omega_p = 0$.

From equation (9), the disk is unstable to both axisymmetric (radial) and nonaxisymmetric (spiral) perturbations if $c_s < c_\mathrm{T}$, where $c_\mathrm{T} = \pi G\sigma_0/\kappa$ is the usual Safronov-Toomre (Toomre 1964) critical sound speed to suppress the instability of only axisymmetric perturbations.[3] The $m > 0$, i.e., nonaxisymmetric instabilities in a differentially rotating ($2\Omega/\kappa > 1$), disk is more difficult to stabilize; stability is achieved only for sufficiently large sound speed $c_s \gtrsim (2\Omega/\kappa)c_\mathrm{T} \approx 2c_\mathrm{T}$ (Lin & Lau 1979; Griv et al. 1999). Thus, if the disk is thin, $c_s \ll r\Omega$, and dynamically cold, $c_s < c_\mathrm{T}$, then such a model will be gravitationally unstable, and it should almost instantaneously (see below for a time estimate) take the form of a cartwheel (Griv 2005). Clearly, in the latter case of both ring and spiral excitation, the distribution of the surface density along the spiral arms is not uniform, but describes a sequence of maxima that might be identified with forming giant gaseous complexes in galaxies or with giant

[3] At the limit of gravitational stability, the two conditions $\partial\omega_\mathrm{J}^2/\partial k = 0$ and $\omega_\mathrm{J}^2 \geq 0$ are fulfilled. The first condition determines the most unstable wavelength (the modified Jeans-Toomre wavelength) $\lambda_\mathrm{crit} \approx 2c_s^2/G\sigma_0$, corresponding to the minimum on the dispersion curve given by eq. (9). Use of the second condition determines the critical sound speed for the stability of arbitrary but not only axisymmetric perturbations.

planets in protoplanetary disks. This dynamical instability is driven by a strong nonresonant interaction of the gravity fluctuations with the bulk of the particle population, and the dynamics of Jeans perturbations can be characterized as a nonresonant interaction: in equation (7), $\omega_* \pm l\kappa \neq 0$ and $l = 0, 1$. One concludes that Toomre's Q-parameter, $Q = c_s/c_{\rm T}$, of <1 suggests that the disk is likely to be subject to both radial and spiral instabilities and might therefore be clumpy (Griv 2005). Contrarily, if the uncooled disk is thin and warm, $c_s \geq c_{\rm T}$ but $c_s \lesssim (2\Omega/\kappa)c_{\rm T} \approx 2c_{\rm T}$ (or $1 \leq Q \lesssim 2$, respectively), then such a disk will be unstable only with respect to spiral perturbations and cannot therefore fragment. An uncooled hot model with $c_s \gtrsim (2\Omega/\kappa)c_{\rm T} \approx 2c_{\rm T}$ (or $Q \gtrsim 2$) is gravitationally stable.

The growth rate of the instability is relatively high, $\Im\omega_* \approx [2\pi G\sigma_0(k_*^2/|k|)]^{1/2}$, and in general $\Im\omega_* \sim \Omega$; that is, the instability develops rapidly on a dynamical timescale. An important feature of the instability under consideration is the fact that in a rotating frame it is almost aperiodic ($|\Re\omega_*/\Im\omega_*| \ll 1$). According to equation (9), the growth rate of the instability has a maximum at the wavelength $\lambda_{\rm crit} \approx 2c_s^2/G\sigma_0$. At the threshold of the instability, $c_s \approx c_{\rm T}$ and $\lambda_{\rm crit} \approx 2\pi^2 G\sigma_0/\kappa^2 \sim (2\text{–}4)\pi h$. This means that of all the harmonics of the initial gravity perturbation, the perturbation with $\lambda_{\rm crit} \sim 10h$, the associated number of spiral arms $m_{\rm crit}$, and the pitch angle $\psi_{\rm crit}$ will be formed in the time of a single rotation.

4. ANGULAR MOMENTUM TRANSFER

The total torque applied to the disk due to the potential Φ_1 is $\Gamma^{\rm tot} = \Gamma^{\rm grav} + \Gamma^{\rm Reyn}$; $\Gamma^{\rm grav}$ is the z-component of the gravitational torque and $\Gamma^{\rm Reyn}$ is the contribution to the z-torque from the Reynolds stress tensor (Takeda & Ida 2001; Papaloizou & Terquem 2006).

Gravitational torque.—From its definition, the gravitational torque per unit area exerted by the perturbations on the disk is $(1/r)(d\Gamma^{\rm grav}/dr) = -\langle\int d\varphi(\boldsymbol{r} \times \boldsymbol{\nabla}\Phi_1)\sigma_1\rangle$ or

$$\frac{1}{r}\frac{d\Gamma^{\rm grav}}{dr} = -\left\langle \int_0^{2\pi} \sigma_1(r,\varphi')\,\frac{\partial \Phi_1(r,\varphi')}{\partial\varphi'}\,d\varphi' \right\rangle, \qquad (12)$$

where $\langle\cdots\rangle$ denotes the time average over the oscillations. Using equation (7), in terms of the Fourier components defined in equation (1), $\Gamma^{\rm grav} = \sum_{m=1}^{\infty}\Gamma_m^{\rm grav}$, from equation (12) one finds

$$\frac{1}{r}\frac{d\Gamma_m^{\rm grav}}{dr} \approx -8\pi\,\frac{m^2}{rL}\,\frac{\sigma_0\Omega}{\kappa^2\Im\omega_*}\,\Phi_1\Phi_1^* \quad \text{if} \quad \Im\omega_* > 0, \qquad (13)$$

or $d\Gamma_m^{\rm grav}/dr = 0$ if $\Im\omega_* \leq 0$; $\Phi_1\Phi_1^* = |\tilde{\Phi}|^2 \exp(2\Im\omega_* t)$, and Φ_1^* is the complex conjugate potential. Equation (13) is correct only in the main part of the system under study between the inner and outer Lindblad resonances. A special analysis of the solution near corotation ($\omega_* = 0$) and Lindblad ($\omega_* \pm \kappa = 0$) resonances is required. Thus, the points $r_{\rm ILR}$ and $r_{\rm OLR}$ in which $\omega_* \pm \kappa = 0$ are called the points of the inner and outer Lindblad resonances (ILR and OLR, respectively), and they do not play an important role in the present theory. Both the wave-particle and wave-fluid resonances have been investigated (Lynden-Bell & Kalnajs 1972; Goldreich & Tremaine 1979; Griv et al. 2000).

From equation (13) one deduces that the distribution of the angular momentum of a disk will be changed under the action of only the nonaxisymmetric $\propto m$, Jeans-unstable ($\Im\omega_* > 0$) perturbations. Moreover, these growing spiral perturbations can transfer angular momentum only in inhomogeneous disks $[(\partial/\partial r)(\sigma_0\Omega\kappa^{-2}) \neq 0]$. The latter is anticipated, because in homogeneous disks the angular velocity of spiral perturbations $\Omega_p = 0$ (eq. [11]), and therefore there is no exchange of angular momentum in the wave-gas system.

Reynolds torque.—The Reynolds torque is caused by particles' collective motion associated with the wave structure. In a rotating frame, this torque per unit area is

$$\frac{1}{r}\frac{d\Gamma_m^{\rm Reyn}}{dr} = -\frac{1}{r}\frac{\partial}{\partial r}\left[r^2\sigma_0(r)\left\langle\int_0^{2\pi} v_r v_\varphi\, d\varphi'\right\rangle\right] \qquad (14)$$

(Takeda & Ida 2001). Considering the most unstable perturbations with $|\Re\omega_*/\Im\omega_*| \ll 1$ and $\Im\omega \approx 2\Omega \approx \kappa$, and using equations (5)–(6), the rate of change of angular momentum per unit area is given by

$$\frac{1}{r}\frac{d\Gamma_m^{\rm Reyn}}{dr} \approx \frac{\pi}{r}\left(k_r + \frac{m}{r}\right)^2 \frac{\Im\omega_*}{\kappa}\,\Phi_1\Phi_1^*\,\frac{\partial}{\partial r}\frac{r^2\sigma_0}{\Omega^2}, \qquad (15)$$

where $\Phi_1\Phi_1^* = |\tilde{\Phi}|^2 \exp(2\Im\omega_* t)$ and $k_r^2 \gg m^2/r^2$. As is seen, in contrast to the case considered above the distribution of the angular momentum will be changed under the action of both forces, nonaxisymmetric ($m \neq 0$) as well as axisymmetric ($m = 0$) ones. The second conclusion is that the gravitational and Reynolds torques exerted by the growing perturbations are of the same order of magnitude, $\Gamma_m^{\rm grav}/\Gamma_m^{\rm Reyn} \sim (m^2/k_r^2 r^2)(r/|L|) \sim 1$.

Because in self-gravitating disks in equilibrium $L < 0$ and $(\partial/\partial r)(r^2\sigma_0/\Omega^2) > 0$ (Griv 2007), both $d\Gamma_m^{\rm grav}/dr$ and $d\Gamma_m^{\rm Reyn}/dr > 0$: an applied torque increases the angular momentum of the given gas element and thus leads to motion of the gas at a larger radius and thus tends to decrease Ω. [As is known, $\Omega(r)$ in a self-gravitating disk is a decreasing function of r, whereas the angular momentum of a unit mass, Ωr^2, is an increasing function of r.] This takes place in the main part of the disk between the Lindblad resonances where the radial and spiral waves are self-excited via a nonresonant wave-fluid interaction. This in turn cannot be done for all masses because the total orbital momentum must remain constant. Lynden-Bell & Kalnajs (1972) have shown that particles at the ILR give out angular momentum, while those at the corotation resonance and the OLR absorb angular momentum. A system lowers its rotational energy by transferring angular momentum outward. (The study of resonances is beyond the scope of the present Letter.) We speculate that as a result of resonant wave-gas interaction, a group of inner particles with radii $r \approx r_{\rm ILR}$ moves inward loosing orbital momentum. At the same time, both resonant and nonresonant particles with radii $r_{\rm ILR} < r \leq r_{\rm OLR}$ move outward absorbing orbital momentum.

5. DISCUSSION

According to equations (13) and (15), the angular momentum transfer efficiency of Jeans-unstable density waves depends on their spatial and temporal form. Let us evaluate the torque for a realistic model of the disk. For that purpose, only the fastest growing mode with $m \approx 1$, $k_* = k_{\rm crit}$, and $\Im\omega_* \sim \Omega$ is considered. Estimating $8\pi m^2\Phi_1\Phi_1^* \sim \Phi_0^2$ (an astrophysicist might well consider a perturbation with Φ_1/Φ_0 of 1/10 to be quite small) and $\Phi_0 \sim r^2\Omega^2/2$, where Φ_0 is the basic potential from equation (13), one obtains $|(1/r)(d\Gamma_m^{\rm grav}/dr)| \sim \sigma_0 r^3\Omega^2/4|L|$. The angular momentum of the given gaseous element is $\mathcal{P} = \sigma_0 r^2\Omega$. Then the characteristic time of the angular momentum redistribution is $t \sim \mathcal{P}/|(1/r)(d\Gamma_m^{\rm grav}/dr)| \sim (4|L|/r)\Omega^{-1} \sim (2\text{–}3)\Omega^{-1}$. Thus, already on a timescale on the order of 2–3 rotation periods the

Jeans-unstable disk sees a large portion of its angular momentum transferred outward, and mass transferred both inward and outward. We *do* find evidence of angular momentum transport great enough to be astrophysically important. The distribution of angular momentum in astrophysical disks may be due to internal gravitational instabilities that transfer angular momentum between different portions of the disk. Accretion (and merging) events are keys in the formation of galactic and protoplanetary disks. We emphasize here that enhanced accretion generated by spontaneous, that is, intrinsic gravitational instabilities may contribute significantly to the evolution of these systems.

The current theory can explain the results of simulations by Laughlin & Bodenheimer (1994), Laughlin & Rózyczka (1996), Gammie (2001), Boss (2003), Durisen et al. (2007), and others: the evolution of gravitationally (Jeans-) unstable disks in which magnetic fields play no important role proceeds in the direction of increasing central mass concentration and of extending outer portions. The bulk of angular momentum is transferred radially outward whereas an inner medium moves inward, loosing a large part of its angular momentum. The distribution of mass and angular momentum in gravitating astrophysical disks is altered by both the ring and spiral evolution (eqs. [13] and [15]). The outward transfer of orbital momentum allows the central parts of disks to contract without breaking up, and the remnant disk is the reservoir for forming gaseous clouds, stars, or planets. Particularly, the main result of *N*-body simulations by Takeda & Ida (2001) is explained: in the presence of the gravitationally unstable density waves angular momentum transfer is dominated by both gravitational torque due to the spiral structure and the Reynolds torque due to the spiral and ring structures.

The model considered here also has evident application to the problems of material supply for galactic nuclear activity (Hopkins & Hernquist 2006) and black hole accretion (Ebisawa & Kawaguchi 2006). The standard accretion-disk theories by Shakura & Sunyaev (1973), Lynden-Bell & Pringle (1974), and Paczyński (1978) have explained angular momentum transport in that they assumed that it is due to turbulent stresses; turbulent viscosity driven by local disk instabilities was assumed to transfer angular momentum outward, thus making the accretion of matter onto the central object possible. There is one important difference between our model and the standard models, namely in the present one the physical cause for the turbulence is included self-consistently by considering a combined system of the gasdynamic and Poisson equations.

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