

# Zero-temperature geometric spin dephasing on a ring in the presence of an Ohmic environment

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**Abstract** – We study zero-temperature spin dynamics of a particle confined to a ring in the presence of spin-orbit coupling and Ohmic electromagnetic fluctuations. We show that the dynamics of the angular position  $\theta(t)$  are decoupled from the spin dynamics and that the latter is mapped to certain correlations of a spinless particle. We find that the spin correlations in the  $z$ -direction (perpendicular to the ring) are finite at long times, *i.e.* do not dephase. The parallel (in-plane) components for spin  $\frac{1}{2}$  do not dephase at weak dissipation but they probably decay as a power law with time at strong dissipation.

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**Introduction.** – Due to recent advances in semiconductor technology, it became possible to isolate and manipulate spins of individual electrons [1,2]. For efficient spin manipulation, however, slow spin decay is needed. Spin decay in mesoscopic devices is generated by two major sources: hyperfine interaction with nuclear spins [3] is responsible for spin decay in most materials. However, spin-orbit (SO) coupling can also induce spin relaxation, and under certain conditions, phonon-[4] or electromagnetic-field-induced SO relaxation [5] can dominate the decay [6,7]. As shown in ref. [5] two-photon (or two-phonon) processes lead to *geometrical spin relaxation* even in the absence of external field and, as pointed out recently, this mechanism can become even dominant in hole-doped systems [8,9].

Here we make an attempt to understand whether the above-mentioned geometrical spin relaxation can survive even at  $T=0$  temperature. Although Ohmic electromagnetic fluctuations were found to lead to a vanishing spin relaxation rate at  $T=0$  [5], the results of ref. [5] are not conclusive, since they allow for *non-exponential* relaxation, common in Ohmic systems. To address this issue

more rigorously, we consider a ring geometry. Studying a ring is, however, not of pure theoretical interest; high-quality semiconductor rings [10,11] can in fact also be used as quantum spin qubits [11], and the usefulness of these devices depends on spin dephasing, a topic under active experimental study [6–8,12,13].

There are two types of spin-orbit coupling in two-dimensional electron systems: the Rashba interaction induced, *e.g.*, by an electric field perpendicular to a two-dimensional (2D) layer [14], and the Dresselhaus coupling induced by bulk inversion asymmetry [15]. Our aim is to study how these couplings influence spin coherence for an electron confined to a ring, in the presence of Ohmic fluctuations. We shall first derive the appropriate Hamiltonian for a confined electron, and show that the presence of the spin does not influence the orbital motion of the confined electron, which is governed exclusively by fluctuations of the external electric field. The dynamics of the spin, on the other hand, is determined by the orbital motion of the electron, and has a topological character. We find that for weak dissipation the spin does not dephase, but certain spin components are reduced by fluctuations. For strong dissipation, however, we find that certain components of the spin probably relax even

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at  $T=0$  temperature, due to the disordering of the orbital degrees of freedom [16]. The relaxation we find is, however, not exponential but of a power law, typical of confined particles at temperature  $T=0$  [17,18].

**Hamiltonian.** – Let us start by projecting a 2D spin-orbit Hamiltonian on a ring, a procedure which is not entirely trivial [19,20]. In addition to the kinetic terms, the 2D Hamiltonian consists of a potential  $V_0(r)$  that confines the particle to a ring of radius  $R \pm \delta R$ , with  $\delta R \ll R$ . We write the total Hamiltonian in polar coordinates as  $\mathcal{H}_0 + \mathcal{H}'$ , where

$$\begin{aligned} \mathcal{H}_0 &= -\frac{\hbar^2}{2m_e} \left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right] + V_0(r), \\ \mathcal{H}' &= \frac{p_\theta^2}{2m_e r^2} + \alpha_0 (\mathbf{S}_x p_y - \mathbf{S}_y p_x) + \beta_0 (\mathbf{S}_x p_x - \mathbf{S}_y p_y). \end{aligned} \quad (1)$$

Here  $p_\theta = -i\hbar\partial/\partial\theta$ ,  $\mathbf{S}$  are spin operators,  $m_e$  is the electron mass and  $p_x$  and  $p_y$  denote the  $x$  and  $y$  components of the momentum. The  $\alpha_0$  term is the Rashba coupling, while  $\beta_0$  denotes the Dresselhaus coupling. Labeling the radial eigenstates of  $\mathcal{H}_0$  by  $|n\rangle$  and their energies by  $E_n$ , our aim is to project  $\mathcal{H}'$  on the subspace,  $|0\rangle$ , while keeping terms up to order  $O(\delta R)$ , a procedure that involves some subtleties. First we rewrite  $\mathcal{H}' = \mathcal{H}_1 + \mathcal{H}_2$  by introducing  $\mathbf{S}_r^\pm \equiv \cos\theta \mathbf{S}_x \pm \sin\theta \mathbf{S}_y$  and  $\mathbf{S}_\theta^\pm \equiv \cos\theta \mathbf{S}_y \mp \sin\theta \mathbf{S}_x$ ,

$$\begin{aligned} \mathcal{H}_1 &= \frac{p_\theta^2}{2mr^2} + \frac{\alpha_0}{2r} \{\mathbf{S}_r^+, p_\theta\} - \frac{\beta_0}{2r} \{\mathbf{S}_\theta^-, p_\theta\}, \\ \mathcal{H}_2 &= i\alpha_0 \hbar \mathbf{S}_\theta^+ \left( \partial_r + \frac{1}{2r} \right) - i\beta_0 \hbar \mathbf{S}_r^- \left( \partial_r + \frac{1}{2r} \right). \end{aligned} \quad (2)$$

As noticed by Meijer *et al.* [19], for any state  $\psi(r)$  that is radially localized near  $R$ , one has  $\langle \psi | 2\partial_r + \frac{1}{r} | \psi \rangle = 0$ . Therefore, to first order in the SO coupling,  $\mathcal{H}_2$  does not give a contribution to the projected Hamiltonian. Nevertheless, as previously overlooked [20],  $\mathcal{H}_2$  cannot be ignored: localization on a scale  $\delta R$  implies  $\partial_r \sim 1/\delta R$  and hence 2nd-order perturbations in  $\mathcal{H}_2$  do give a contribution,  $\sim \mathcal{H}_2^2/E_n = O(1)$ , since  $E_n \sim 1/(\delta R)^2$ . The next-order contributions scale as  $\mathcal{H}_2^3/E_n^2 = O(\delta R)$ , and vanish in the limit  $\delta R \rightarrow 0$ , similar to all higher-order terms in the perturbation series.

Perturbation theory [21] to 2nd order yields therefore the projected spin- and angle-dependent effective Hamiltonian

$$\mathcal{H}_{\text{ring}} = \langle 0 | \mathcal{H}_1 | 0 \rangle - \sum_{n \neq 0} \frac{\langle 0 | \mathcal{H}_2 | n \rangle \langle n | \mathcal{H}_2 | 0 \rangle}{E_n - E_0} + O(\delta R). \quad (3)$$

The sum in eq. (3) can be evaluated analytically by making use of a sum rule [22,23], and the second term of eq. (3) simply becomes  $\frac{1}{2} m_e (\alpha_0 \mathbf{S}_\theta - \beta_0 \mathbf{S}_r')$ .

Introducing the vector  $\mathbf{h}(\theta)$  via  $\mathbf{h} = (h_x, h_y, h_z) \equiv (\alpha \cos\theta - \beta \sin\theta, \alpha \sin\theta - \beta \cos\theta, 0)$ , and with the dimensionless Rashba and Dresselhaus couplings defined as

$\alpha \equiv mR\alpha_0$  and  $\beta \equiv mR\beta_0$ , we can finally rewrite our effective ring Hamiltonian in the  $\delta R \rightarrow 0$  limit as

$$\mathcal{H}_{\text{ring}} = \frac{\hbar^2}{2m_e R^2} [p_\theta + \mathbf{h}(\theta) \cdot \mathbf{S}]^2. \quad (4)$$

We remark, in particular, that the term  $\sim \alpha\beta \sin 2\theta$  in the effective Hamiltonian of ref. [20] is exactly canceled by the 2nd-order terms. As a consequence, eq. (4) possesses a conserved ‘‘momentum’’,  $\hat{Q} \equiv p_\theta + \mathbf{h}(\theta) \cdot \mathbf{S}$ .

**Spectrum.** – The eigenstates and eigenenergies of (4) can be analytically computed for  $\beta=0$ , when the system is rotationally invariant and therefore  $J_z = p_\theta + S_z$  is also conserved. The Hamiltonian can then be written as

$$\mathcal{H}_{\text{ring}} = \frac{\hbar^2}{2m_e R^2} [J_z - \mathbf{n}(\theta) \cdot \mathbf{S} \sqrt{1 + \alpha^2}]^2, \quad (5)$$

with  $\mathbf{n}(\theta) = (-h_x(\theta), -h_y(\theta), 1)/\sqrt{1 + \alpha^2}$  a unit vector. The energy spectrum and the eigenvalues can then easily be found by constructing common eigenstates of the two commuting operators,  $J_z$  and  $\mathbf{n}(\theta) \cdot \mathbf{S}$ . For  $S=1/2$ ,  $\mathbf{n}(\theta) \cdot \mathbf{S}$  and  $J_z$  have eigenvalues  $\mathbf{n}(\theta) \cdot \mathbf{S} = \sigma/2$  and  $J_z = m + \sigma/2$ , respectively, with  $\sigma = \pm$  and  $m$  an integer. The spectrum is  $\epsilon_{m,\sigma} = \frac{1}{2m_e R^2} [m + \sigma(\frac{1}{2} - \frac{1}{2}\sqrt{1 + \alpha^2})]^2$ , and the eigenstates are of the form  $\int_\theta \frac{e^{im\theta}}{\sqrt{2\pi}} |\theta\rangle \otimes |\pm \mathbf{n}(\theta)\rangle$ , with  $|\pm \mathbf{n}\rangle$  denoting spin coherent states, defined through the usual relation,  $\mathbf{\Omega} \cdot \mathbf{S} |\mathbf{\Omega}\rangle = S |\mathbf{\Omega}\rangle$  [24]. The wave functions can be explicitly expressed as

$$\begin{aligned} \psi_{m,+}(\theta) &= e^{im\theta} \left( \cos \frac{\bar{\alpha}}{2}, -e^{i\theta} \sin \frac{\bar{\alpha}}{2} \right), \\ \psi_{m,-}(\theta) &= e^{im\theta} \left( e^{-i\theta} \sin \frac{\bar{\alpha}}{2}, \cos \frac{\bar{\alpha}}{2} \right), \end{aligned} \quad (6)$$

with  $\bar{\alpha}$  defined as  $\bar{\alpha} \equiv \arctan(\alpha)$ . The states  $\psi_{\pm m, \pm}$  are related by time reversal, and their energies equal  $E_{m,+} = E_{-m,-}$ . For  $\alpha < \sqrt{3}$  the ground state has  $m=0$  (see footnote 1).

**Dissipation.** – Having understood the properties of an isolated ring, we now couple the motion of the particle to the coordinate  $\xi$  of a dissipative environment, *i.e.*, we consider the total Hamiltonian as  $\mathcal{H} = \mathcal{H}_{\text{ring}} + V(\theta, \xi)$ . Throughout most of this paper we shall assume that  $V(\theta, \xi)$  describes the coupling to a Caldeira-Leggett (CL) environment, appropriate for small rings in an Ohmic (metallic) environment<sup>2</sup>. Then  $\xi$  represents the random force generated by the environment,  $V = \xi_- e^{i\theta} + \xi_+ e^{-i\theta}$ , and the  $T=0$  Fourier transform of the environment correlations is  $\langle \xi_- \xi_+ + \xi_+ \xi_- \rangle_\omega = \hbar^2 \eta |\omega|$ , with  $\eta$  the dimensionless friction coefficient.

The corresponding equations of motion for  $\theta(t)$  are

$$\dot{\theta} = \frac{p_\theta + \mathbf{h} \cdot \mathbf{S}}{m_e R^2}, \quad \ddot{\theta} = -\frac{1}{m_e R^2} \partial_\theta V(\theta, \xi). \quad (7)$$

<sup>1</sup>Here we used the phase convention of ref. [24]. This construction can be generalized for larger spins.

<sup>2</sup>In a dirty metal environment, *e.g.*, one needs the ring’s radius to be smaller than the mean free path.

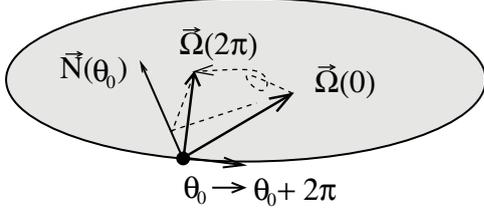


Fig. 1: Evolution of the spin (coherent state) while the electron makes a circle,  $\theta_0 \rightarrow \theta_0 + 2\pi$ . The initial state is rotated around an axis  $\mathbf{N}(\theta_0)$  by an angle  $\Gamma_{\theta_0}$ .

Hence, as a consequence of the simple form of  $\mathcal{H}_{\text{ring}}$ , eq. (4), the dynamics of  $\theta$  in the dissipative environment are not affected by the spin-orbit couplings. This decoupling allows us to describe the  $\theta(t)$  evolution by a path integral, where for each trajectory the spin dynamics follow from eq. (4),

$$\frac{d\mathbf{S}}{dt} = \dot{\theta} \mathbf{h}(\theta) \times \mathbf{S} \quad \Rightarrow \quad \frac{d\mathbf{S}}{d\theta} = \mathbf{h}(\theta) \times \mathbf{S}. \quad (8)$$

Viewing  $\theta$  as a “time” variable, these dynamics correspond to a spin precession around a “time”-dependent magnetic field  $\mathbf{h}(\theta)$ . Note that switching to “Schrödinger” picture, the spin coherent states have a simple  $\theta$  evolution, too. Apart from a phase, they evolve as  $|\Omega(\theta)\rangle$ , where  $\Omega(\theta)$  is the vector solution of (8), *i.e.*  $\frac{d\Omega}{d\theta} = \mathbf{h}(\theta) \times \Omega$ . In particular, the vector  $\mathbf{n}(\theta)$  can also be shown to satisfy this equation.

In terms of the spin operators, eq. (8) is solved as a simple linear mapping  $S_i(\theta) = R_{ij}(\theta, \theta_0) S_j$ , with  $R_{ij}(\theta, \theta_0)$  a rotation matrix. The rotation matrix  $R_{2\pi}(\theta_0) = R(\theta_0 + 2\pi, \theta_0)$ , corresponding to the particle going once around the ring by  $2\pi$ , is of special interest. We denote by the unit vector  $\mathbf{N}(\theta_0)$  its axis of rotation and by  $\Gamma_{\theta_0}$  the corresponding rotation angle (see fig. 1). In particular, for  $\beta = 0$  we find that the angle  $\Gamma_{\theta_0}$  is independent of the initial value,  $\Gamma = 2\pi(1 - \sqrt{1 + \alpha^2})$ , and is typically incommensurate with  $2\pi$ .

**Mapping to a spinless system.** – For a given evolution,  $\theta_0 \rightarrow \theta$ , we can obtain the evolution of the spin part of the wave function from eq. (8), which is described by a unitary operator,  $U_{\text{spin}}(\theta, \theta_0)$ . Here the Hamiltonian to describe the  $\theta$  (“time”) evolution of the spin is  $H_s = \mathbf{h}(\theta) \cdot \mathbf{S}$ . We proceed to study the case  $\beta = 0$ . Then, as in the standard NMR rotating field problem, the spinor transformation  $\psi' \equiv e^{i(\theta - \theta_0) S_z} \psi$  to the “rotating frame” cancels the “time” ( $\theta$ ) dependence, and amounts in replacing  $H_s \rightarrow \mathbf{h}(\theta_0) \cdot \mathbf{S} - S_z = -\sqrt{1 + \alpha^2} \mathbf{n}(\theta_0) \cdot \mathbf{S}$ . For  $S = 1/2$  this leads to the evolution operator,  $U_{\text{spin}}(\theta, \theta_0) = e^{-i \frac{\theta - \theta_0}{2} \sigma_z} e^{i \sqrt{1 + \alpha^2} \frac{\theta - \theta_0}{2} \mathbf{n}(\theta_0) \cdot \boldsymbol{\sigma}}$ . Using now the expression of  $\hat{Q}$  we find that the  $\theta$  evolution of the spin states has a particularly simple form,

$$U_{\text{spin}}(\theta, \theta_0) \psi_{m,\pm}(\theta_0) = e^{iq_{m,\pm}(\theta_0 - \theta)} \psi_{m,\pm}(\theta), \quad (9)$$

where  $q_{m,\pm} = m \pm \frac{1}{2} \mp \frac{1}{2} \sqrt{1 + \alpha^2}$  denote the eigenvalues of the momentum  $\hat{Q}$ . After a  $2\pi$  rotation the state  $\psi_{m,\pm}$  picks up an incommensurate phase,  $2\pi q_{m,\pm}$ . Note that the semiclassical evolution involves a similar incommensurate angle,  $\Gamma$ , as discussed below eq. (8) (see also fig. 1).

Making use of the decoupling of orbital and spin degrees of freedom, we can construct a mixed path integral formalism (to be detailed in ref. [23]), where the spin is treated in an evolution operator formalism, while the orbital motion of the particle is developed in a path integral formalism. The full evolution for a given environment history is then obtained as

$$\psi_{m,\pm}(\theta_t, t) = \sum_n \int_0^{2\pi} d\theta_0 \int_{\theta_0}^{\theta_t + 2\pi n} \mathcal{D}\theta e^{iS_P(\theta, \xi)} U_{\text{spin}}(\theta_t + 2\pi n, \theta_0) \psi_{m,\pm}(\theta_0). \quad (10)$$

Note that  $\theta$  in this equation is a non-compact variable, and an additional integration over the environment configurations has to be carried out in the end. Importantly, the action  $S_P(\theta, \xi) = \int_0^t [\frac{1}{2} m_e r^2 \dot{\theta}^2 - V(\theta, \xi)]$  describes a particle on the ring in the presence of dissipation for a given environment history, and is independent of the spin evolution.

For  $\beta = 0$  we can make use of eq. (9) and obtain a particularly simple path integral representation for the spin evolution. Consider spin correlations with an initial density matrix  $|\sigma\rangle\langle\sigma|$  built from one of the two Kramers degenerate ground states of  $m = 0$  and  $\sigma = \pm$ , having momenta  $\hat{Q} = q_{0,\pm} = \pm G$  with  $G = \frac{1}{2} - \frac{1}{2} \sqrt{1 + \alpha^2}$ . Using eqs. (10) and (9) for the forward and backward spin evolutions, we find an exact mapping of the spin correlations onto a superposition of equilibrium correlations of spinless particles on a ring with a flux  $\Phi = \pm G$  (in units of quantum flux):

$$P_{\alpha, \Phi}(t_{21}) = \langle e^{-ia\theta(t_2)} e^{ia\theta(t_1)} \rangle_{\Phi}. \quad (11)$$

Here again,  $\theta(t)$  is a non-compact variable within  $(-\infty, \infty)$  to be used within the path integral representation of the spinless problem. Note that the bath still couples to  $e^{\pm i\theta}$  hence we expect that (11) depends only on the non-integer part of  $\Phi$ . For  $\langle S_x(t) S_x(0) \rangle$  we obtain the following identity for an initial density matrix,  $|+\rangle\langle+|$ :

$$C_{++}^x(t) = \frac{1}{4} \sin^2 \bar{\alpha} (P_{1,G}(t) + P_{-1,G}(t)) + \cos^4 \frac{\bar{\alpha}}{2} P_{-2G,G}(t) + \sin^4 \frac{\bar{\alpha}}{2} P_{2-2G,G}(t). \quad (12)$$

For  $C_{--}(t)$  the same result holds with all subscripts of  $P_{\alpha,q}$  reversing sign. For the  $S_z$  correlations, on the other hand, we obtain

$$C_{++}^z(t) = \cos^2 \bar{\alpha} + P_{-1-2G,G}(t) \sin^2 \bar{\alpha}, \quad (13)$$

and for  $C_{--}^z(t)$  the same holds with  $P_{1+2G,-G}$ . Notice that the degeneracy point,  $\alpha = \sqrt{3}$ , corresponding to fluxes  $\Phi = \pm \frac{1}{2}$  represents a special case, and is not studied here.

While the correlation function of the  $z$ -component of the spin,  $C^z(t)$ , obviously contains a constant non-decaying piece, the correlation function  $C^x(t)$  contains only phase correlation functions  $P_{a,\Phi}$  with  $a \neq 0$ . There is some evidence that these correlations decay in time. In particular  $P_{1,0} \sim 1/t^2$  from the XY lattice model [16] and from small  $\eta$  expansion [25]. Correlations with incommensurate  $a$  were studied in a related system of dissipative Josephson junctions [26], and found to decay algebraically. Further evidence for algebraic decay is found for large  $\eta$ , as discussed below. To further appreciate these correlations we have evaluated the path integrals in (11) analytically for  $\eta = 0$ , and surprisingly, we find  $P_{\mp 2G, \pm G}^{\eta=0}(t) = 1$ . As a consequence, for  $\eta = 0$  the correlation function  $C^x$  contains a piece which does not oscillate. As discussed below, though reduced, this part seems to survive for very weak dissipation,  $\eta \ll 1$ , while it apparently decays algebraically for strong dissipation,  $\eta \gg 1$ .

**Strong-dissipation limit.** – In the strong-dissipation limit,  $\eta \gg 1$ , we can describe the evolution of the phase through a Langevin equation, and an expansion in  $1/\eta$  is possible (for details, see refs. [17,18,25]). In this limit of strong dissipation the motion of the particle becomes semiclassical. In technical terms, the forward and backward propagating paths of the particle on the Keldysh contour,  $\Theta_{\pm}(t)$ , remain close to each other, and the (small) quantum part  $\hat{\Theta}(t) \equiv \Theta_+(t) - \Theta_-(t)$  appears as a Gaussian noise term. The correlation functions  $P_{a,q}$  with  $a \neq 0$  are found to decay algebraically. Large- $\eta$  perturbation theory yields that  $P_{a,\Phi} \sim t^{-a^2/\pi\eta}$  and the  $x$ -component of the spin also decays algebraically, while the  $z$ -component remains finite and does not decay. This result, however, holds only for up to times  $\ln t < O(\eta)$  beyond which effects of renormalization of  $\eta$  cannot be neglected. Therefore, the long-time behavior of  $P_{a,\Phi}$  for an incommensurate  $a$  is not established. In fact, RG methods show that  $\eta \gg 1$  flows to a value of  $O(1)$  [25,27]. In a recent work [18] we have shown that in the presence of a weak DC electric field there is a critical  $\eta_c = 1/2\pi$  such that  $\eta > \eta_c$  flows to  $\eta_c$  which would indicate  $P_{a,\Phi} \sim t^{-2a^2}$ . In some sense the fluctuating spin corresponds to a time-dependent flux, *i.e.* an electric field, though the correspondence is not precise.

**Weak dissipation.** – The rather different behavior of  $S_{x,y}$  and  $S_z$  should already be manifest in the weak-dissipation limit, where we can perform perturbation theory in the strength of the dissipation,  $\eta$ . To do perturbation theory, we restrict ourselves to the case  $S = 1/2$  and  $\beta = 0$ , and use Abrikosov's pseudofermion method [28] to represent each spinor  $\psi_{m\sigma}$  of (6) by a pseudofermion operator,  $f_{m\sigma}$ . In this language the ring Hamiltonian becomes  $\mathcal{H}_{\text{ring}} = \sum_{m,\sigma} \epsilon_{m\sigma} f_{m\sigma}^\dagger f_{m\sigma}$ , while the interaction is expressed as

$$V = \sum_{m,\sigma} (\xi_- f_{m\sigma}^\dagger f_{m-1\sigma} + \xi_+ f_{m\sigma}^\dagger f_{m+1\sigma}), \quad (14)$$

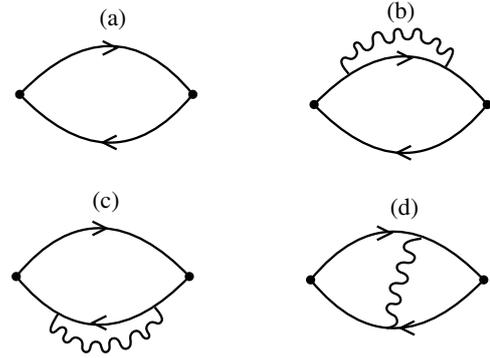


Fig. 2: Zero- and leading-order ( $\sim \eta$ ) corrections to the spin susceptibility. Continuous lines and wavy lines represent the pseudofermion and bosonic propagators, respectively, while the dots represent spin vertices. Logarithmic divergencies in the last three diagrams above exactly cancel.

and standard field-theoretical methods can be used to evaluate physical quantities. A renormalization group analysis of the vertex function and the pseudofermions' self-energy reveals that, although ultraviolet logarithmic divergencies appear in both quantities, they cancel and the dissipation parameter  $\eta$  is in leading order, nevertheless, *exactly marginal*, and the mass of the particle remains also unrenormalized [23].

In this perturbative regime, fingerprints of a non-exponential spin decay should appear in the susceptibility,  $\chi$ , which, in the absence of spin decay, should contain a Curie part. To compute  $\chi$ , we first express the impurity spin operator in terms of pseudofermions as

$$\mathbf{S}_i = \sum_{m,\sigma,m',\sigma'} \mathcal{S}_{m,\sigma,m',\sigma'}^i f_{m\sigma}^\dagger f_{m'\sigma'}, \quad (15)$$

with the matrix elements simply determined from the wave functions (6), as  $\mathcal{S}_{m,\sigma,m',\sigma'}^i = \langle \Psi_{m,\sigma} | \mathbf{S}_i | \Psi_{m,\sigma} \rangle$ . The leading corrections to  $\chi$  are shown in fig. 2. These diagrams contain logarithmic singularities. Diagram (b), *e.g.*, —together with the corresponding counterterm diagram, not shown in this figure— gives

$$\chi_x^{(b)} = \frac{\cos^4(\bar{\alpha}/2)}{4T} \left( -\frac{\eta}{2\pi} \right) \ln \frac{\Lambda^2}{\Delta^2(1-4G^2)}, \quad (16)$$

with  $\Lambda$  a high energy cut-off, and  $\Delta = 1/2m_e R^2$  the characteristic finite-size energy of electrons moving along the ring. Remarkably, however, all these ultraviolet singularities exactly cancel, and one finally obtains just a finite renormalization of the perpendicular Curie susceptibility,

$$\chi_{x,y} = \frac{\cos^4(\bar{\alpha}/2)}{4T} \left[ 1 - \frac{\eta}{2\pi} \left[ \frac{1}{G} \ln \left( \frac{1+2G}{1-2G} \right) - 4 \right] + O(\eta^2) \right].$$

The prefactor  $\cos^4(\bar{\alpha}/2)$  is identical to the coefficient of  $P_{-2G,G}(t)$  in eq. (12), and accounts for  $g$ -factor renormalization in the isolated ring. The correction  $\sim \eta$ , on the

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other hand, represents the environment-induced renormalization of the  $x$  and  $y$  components of the spin ( $g$ -factor). The above perturbative result and the survival of the Curie susceptibility indicates that the term  $P_{-2G,G}(t)$  decays to a reduced but non-zero value for small  $\eta$ .

In contrast to the  $x$  and  $y$  components, the  $z$ -component of the susceptibility,  $\chi_z$ , is found to remain unrenormalized by  $\eta$  to leading order in the dissipation. These results imply that, for weak Ohmic dissipation, the only effect of dissipation is to slightly and anisotropically renormalize the  $g$ -factor, but apart from that the spin behaves as a free spin, and does not decay.

**Case of  $\beta \neq 0$ .** – So far we discussed only the case  $\beta = 0$ . We show now that the system with both  $\alpha, \beta$  finite is equivalent to the Hamiltonian (14). Assume a state  $|q\rangle$  that is an eigenstate of  $\hat{Q}|q\rangle = q|q\rangle$ . This state generates a ladder of, states,  $|m+q\rangle \equiv e^{im\theta}|q\rangle$ , with integer  $m$  by

$$\hat{Q}e^{im\theta}|q\rangle = (m+q)e^{im\theta}|q\rangle. \quad (17)$$

Since  $\hat{T}^{-1}\hat{Q}\hat{T} = -\hat{Q}$ , a sequence of time-reversed states is also generated by the time reversal operator,  $\hat{T}: \hat{Q}\hat{T}|m+q\rangle = -(m+q)\hat{T}|m+q\rangle$ . All these states are orthogonal since they correspond to different energy eigenvalues, and the environment couples the  $m$  and  $m \pm 1$  states, exactly as for  $\beta = 0$ . The only difference is that  $E_{0\uparrow}(\alpha, \beta)$ , which is not known analytically, changes the factor  $\frac{1}{2} - \frac{1}{2}\sqrt{1+\alpha^2}$  in  $E_{m\uparrow}, E_{m\downarrow}$ . Hence eq. (14) is a correct representation also of the  $\beta \neq 0$  case<sup>3</sup>.

**Conclusions.** – We derived the effective Hamiltonian of an electron confined to a ring within a 2-dimensional electron gas, in the presence of SO coupling, and subject to a dissipative environment. We have shown that the orbital motion of the particle decouples from the spin evolution, and correspondingly, spin decay has a geometric character [5]. For an Ohmic environment, we mapped the spin relaxation problem to that of a spinless particle on a ring pierced by a magnetic flux (eqs. (12), (13)). We find that the  $z$ -component of the ground-state spin is not affected by dissipation. The  $x$  and  $y$  in-plane spin components are, on the other hand, reduced by dissipation, but we find no dephasing for spin  $\frac{1}{2}$  and weak dissipation. However, these components seem to dephase at large dissipation.

We should remark that these latter results are based on the assumption of Ohmic dissipation. The situation may, however, change for sub-Ohmic dissipation or  $1/\omega^\gamma$  noise, present in many systems. In this case, the decoupling of the spin and orbital motion and thus eqs. (12), (13) remain valid; however, for sub-Ohmic dissipation  $\eta$  is a *relevant* perturbation, and even a small dissipation could possibly lead to the decay of the  $x$  and  $y$  spin components. This possibility, however, needs to be further explored.

<sup>3</sup>The matrix elements of the spin operators are nevertheless different, changing, *e.g.*, the overall coefficient in eq. (15).

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## REFERENCES

- [1] ELZERMAN J. M. *et al.*, *Nature*, **430** (2004) 431.
- [2] NOWACK K. C., KOPPENS F. H. L., NAZAROV YU. V. and VANDERSYPEN L. M. K., *Science*, **318** (2007) 1430.
- [3] KHAETSKII A. V., LOSS D. and GLAZMAN L., *Phys. Rev. Lett.*, **88** (2002) 186802.
- [4] KHAETSKII A. V. and NAZAROV Y. V., *Phys. Rev. B*, **61** (2000) 12639; **64** (2001) 125316.
- [5] SAN-JOSE P., ZARAND G., SHNIRMAN A. and SCHÖN G., *Phys. Rev. Lett.*, **97** (2006) 076803.
- [6] MEUNIER T. *et al.*, *Phys. Rev. Lett.*, **98** (2007) 126601.
- [7] AMASHA S. *et al.*, *Phys. Rev. Lett.*, **100** (2008) 046803.
- [8] GERARDOT B. *et al.*, *Nature*, **451** (2008) 441.
- [9] TRIF M., SIMON P. and LOSS D., *Phys. Rev. Lett.*, **103** (2009) 106601.
- [10] FUHRER A., IHN T., ENSSLIN K., WEGSCHEIDER W. and BICHLER M., *Phys. Rev. Lett.*, **91** (2003) 206802.
- [11] IHN T., FUHRER A., ENSSLIN K., WEGSCHEIDER W. and BICHLER B., *Physica E*, **26** (2005) 225 and references therein.
- [12] PFUND A., SHOROBALKO I., ENSSLIN K. and LETURC R., *Phys. Rev. Lett.*, **99** (2007) 036801.
- [13] KOPPENS F. H. L. *et al.*, *Phys. Rev. Lett.*, **99** (2007) 106803.
- [14] RASHBA E. I., *Sov. Phys. Solid State*, **2** (1960) 1109; BYCHKOV Y. A. and RASHBA E. I., *J. Phys. C*, **17** (1984) 6039.
- [15] DRESSELHAUS G., *Phys. Rev. B*, **100** (1955) 580.
- [16] SPOHN H. and ZWERGER W., *J. Stat. Phys.*, **94** (1999) 1037; in this work only the decay of imaginary time correlations is considered. However, the  $1/\tau^2$  found in imaginary time is strongly suggestive of a corresponding  $1/t^2$  decay in real time.
- [17] GOLUBEV D. S., SCHÖN G. and ZAIKIN A. D., arXiv:cond-mat/0208548; SCHÖN G. and ZAIKIN A. D., *Phys. Rep.*, **198** (1990) 237.
- [18] ETZIONI Y., HOROVITZ B. and LE DOUSSAL P., *Phys. Rev. Lett.*, **106** (2011) 166803.
- [19] MEIJER F. E., MORPUNGO A. F. and KLAPWIJK T. M., *Phys. Rev. B*, **66** (2002) 033107.
- [20] SHENG J. S. and CHANG K., *Phys. Rev. B*, **74** (2006) 235315.
- [21] LANDAU L. D. and LIFSHITZ E. M., *Quantum Mechanics: Non-Relativistic Theory*, Vol. **3**, 3rd edition (Pergamon) 1991.
- [22] KOHN W., *Phys. Rev.*, **133** (1964) A171.

- [23] HOROVITZ B., LE DOUSSAL P. and ZARÁND G., unpublished.
- [24] AUERBACH A., *Interacting Electrons and Quantum Magnetism* (Springer-Verlag, New York) 1994.
- [25] HOROVITZ B. and LE DOUSSAL P., *Phys. Rev. B*, **82** (2010) 155127; the small  $\eta$  expansion at imaginary times can be extended to finite flux  $\Phi \neq \frac{1}{2}$  with  $P_{1,\Phi} \sim 1/t^2$ .
- [26] LUKYANOV S. L. and WERNER P., *J. Stat. Mech.* (2007) P06002.
- [27] HOFSTETTER W. and ZWERGER W., *Phys. Rev. Lett.*, **19** (1997) 3737.
- [28] ABRIKOSOV A. A., *Physics*, **2** (1965) 5; 21.

The same result is obtained through a pseudofermion calculation [23].