

# Particle renormalizations in presence of dissipative environments

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A charged particle of mass  $M$  on a ring of radius  $R$  is coupled to a dirty metal environment. With Monte-Carlo methods we evaluate the curvature of the Aharonov-Bohm oscillations and find a quantum phase transition at a critical  $R_c$ . At low temperatures  $T$  the curvature has the form  $1/M^*R^2$  with an  $R$  independent  $M^* > M$  in the  $R > R_c$  phase, while  $M^*$  rapidly approaches  $M$  in the  $R < R_c$  phase. The approach to  $T = 0$  defines diverging length scales  $\sim T^{-\eta}$  with  $\eta \approx 1$  and  $\eta \approx \frac{1}{4}$  in the large and small  $R$  phases, respectively.

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The problem of interference in presence of a dissipative environment is fundamental for a variety of experimental systems. Interference has been monitored by Aharonov-Bohm (AB) oscillations in mesoscopic rings<sup>1,2,3</sup> or in quantum Hall edge states<sup>4</sup> in presence of noise from gates or other metal surfaces. Cold atoms trapped by an atom chip are sensitive to the noise produced by the chip<sup>5,6,7</sup>. In particular giant Rydberg atoms are studied<sup>8</sup> whose huge electric dipole is highly susceptible to such noise.

An efficient tool for monitoring the effect of the environment, as proposed by Guinea<sup>9</sup>, is to find the AB oscillation amplitude as function of the radius  $R$  of the ring. This amplitude is measured by the curvature<sup>10,11,12</sup> of the ground state energy  $E_0$  at external flux  $\phi_x = 0$ , i.e.  $1/M^*R^2 = \partial^2 E_0 / \partial \phi_x^2 |_0$ , defining an effective mass  $M^*$ . For free particles of mass  $M$  this curvature is the mean level spacing  $1/MR^2$ . The particle can be coupled to a variety of environments, with three systems of particular interest: (i) a Caldeira-Legget (CL) bath<sup>9</sup>, (ii) a charged particle in a dirty metal environment<sup>9,13</sup> and (iii) a particle with an electric dipole in a dirty metal environment<sup>14</sup>. System (i) has been studied with a large variety of methods, all showing that the AB amplitude is exponentially suppressed  $\sim e^{-\pi^2 \gamma R^2}$ , i.e. a new length scale  $\sim 1/\sqrt{\gamma}$  is generated by the coupling  $\gamma$  to the environment<sup>9</sup>. System (ii) has been studied by renormalization group (RG) methods<sup>9,15</sup> finding  $M^* \sim R^\mu$  with a small  $\mu$ , a Monte Carlo (MC) numerical method gave<sup>13</sup>  $\mu = 1.8$  for strong coupling, while a variational scheme<sup>14</sup> gave  $\mu = 0$ . System (iii) was also studied within the variational scheme<sup>14</sup>, leading to  $\mu = 0$  as well.

In the present work we use MC methods to analyze mostly system (ii). We find that the energy cutoff used in a previous study<sup>13</sup> is insufficient and a higher cutoff  $\omega_c$  is needed. The low  $T$  data shows a quantum critical point at  $R_c$ . At  $R > R_c$  we find  $M^* > M$  and  $R$  independent, i.e.  $\mu = 0$ , while at  $R < R_c$  we find that  $M^*$  rapidly approaches  $M$ . We also find that a free particle with mass  $M^*$  reproduces at  $R > R_c$  the finite  $T$  curvature at  $T \ll \omega_c$ . The approach to  $T = 0$  defines diverging length scales  $\sim T^{-\eta}$  with  $\eta \approx 1$  and  $\eta \approx \frac{1}{4}$  in the large and small  $R$  phases, respectively. A related study shows that similar scales correspond to a dephasing process<sup>16</sup>.

The time dependent angular position  $\theta_m(\tau)$  of a particle on the ring has in general a winding number  $m$  so that  $\theta_m(\tau) = \theta(\tau) + 2\pi m T \tau$  where  $\theta(0) = \theta(1/T)$  has periodic boundary condition and  $T$  is the temperature. In presence of an external flux  $\phi_x$  (in units of the flux quantum  $hc/e$ ) the partition sum has the form

$$Z = \sum_m e^{2\pi i m \phi_x} \int \mathcal{D}\theta e^{-S^{(m)}}$$

$$S^{(m)} = \frac{1}{2} M R^2 \int_0^{1/T} \left( \frac{\partial \theta}{\partial \tau} + 2\pi m T \right)^2 d\tau +$$

$$\alpha \int_0^{1/T} \int_0^{1/T} \frac{\pi^2 T^2 K[\theta(\tau) - \theta(\tau') + 2\pi m T(\tau - \tau')]}{\sin^2 \pi T(\tau - \tau')} d\tau d\tau' \quad (1)$$

where the effect of environments, in each of the 3 cases, is<sup>9,13,14</sup>

$$K(z) = \begin{aligned} & \sin^2 z/2; & \alpha &= \gamma R^2 & (i) \\ & = 1 - [4r^2 \sin^2 \frac{z}{2} + 1]^{-1/2}; & \alpha &= \frac{3}{8k_F^2 l^2} & (ii) \\ & = 1 - [4r^2 \sin^2 \frac{z}{2} + 1]^{-3/2}; & \alpha &= \frac{p^2}{e^2 l^2} \frac{3}{8k_F^2 l^2} & (iii). \end{aligned} \quad (2)$$

Case (i) is the CL system where  $\gamma$  is the coupling to a harmonic oscillator bath; case (ii) is a charge coupled to a dirty metal where  $k_F$  is the Fermi wavevector,  $l$  is the mean free path in the metal, and  $r = R/l$ ; case (iii) is an electric dipole of strength  $p$  coupled to a dirty metal. We note that at  $r \lesssim 1$  the models (ii),(iii) correspond qualitatively to the microscopic model in describing the reduction of  $K(z)$  with  $r$ .

We are interested in the effect of the environment on the visibility of quantum interference as measured by the particle. As a measure of this visibility we consider the curvature of the Aharonov-Bohm oscillations

$$\frac{1}{M^*(T)R^2} = \frac{\partial^2 F}{\partial \phi_x^2} |_{\phi_x=0} \quad (3)$$

where  $F = -T \ln Z$ . It is useful to consider a free particle

$\alpha = 0$ , for which

$$\left(\frac{M}{M^*(T)}\right)_{\alpha=0} = 2\pi^2 t \sum_m m^2 e^{-\pi^2 m^2 t} / \sum_m e^{-\pi^2 m^2 t} \equiv f(t) \quad (4)$$

where  $t = 2MR^2T$ . This identifies the thermal length  $L_T \sim 1/\sqrt{MT}$ .

In the interacting system a high energy cutoff can be identified by considering  $\tau \rightarrow \tau'$  (corresponding to high frequencies  $\omega$ ) so that expansion of  $K(z)$  and Fourier transform yield

$$S^{(m)} \rightarrow \frac{1}{2} \int \frac{d\omega}{2\pi} [MR^2\omega^2 + 2\pi\alpha K''(0)|\omega|] |\theta(\omega)|^2 + (2\pi m)^2 [\frac{1}{2}MR^2T + \alpha K''(0)]. \quad (5)$$

The cutoff  $\omega_c$  is identified when the kinetic  $\sim \omega^2$  and  $\sim |\omega|$  interaction terms are comparable, i.e.

$$\omega_c = \frac{2\pi\alpha K''(0)}{MR^2}. \quad (6)$$

This  $\omega_c$  replaces a possibly higher environment cutoff, since significant renormalizations start only below  $\omega_c$  where the linear  $|\omega|$  dispersion leads to  $\ln \omega$  terms in perturbation theory and to the need for either RG treatment, or an equivalent variational scheme<sup>14</sup>. Note that  $K''(0) = \frac{1}{2}; r^2; 3r^2$  in the 3 models above, hence  $\omega_c = \pi\gamma/M$  in case (i), while  $\omega_c \sim \alpha/MI^2$  in cases (ii) and (iii).

For the MC numerical method we need to discretize the time axis into a Trotter number  $N_T$  of segments, i.e. the time interval of each segment is  $\Delta\tau = 1/(TN_T)$ . The discrete action is

$$S^{(m)} = \frac{1}{2} [MR^2 N_T T + \alpha K''(0)] \sum_n (\theta_{n+1} - \theta_n + \frac{2\pi m}{N_T})^2 + \frac{\alpha\pi^2}{N_T^2} \sum_{n \neq n'} \frac{K(\theta_n - \theta_{n'} + 2\pi m(n - n')/N_T)}{\sin^2(\pi(n - n')/N_T)}. \quad (7)$$

The  $\frac{1}{2}\alpha K''(0)$  term comes from the  $n = n'$  interaction term by expanding  $K(z)$  around  $z = 0$ . A key issue in our MC study is the choice of energy cutoff  $1/\Delta\tau$  and the corresponding Trotter number  $N_T = 1/(T\Delta\tau)$ . The correct choice is such that the free kinetic term dominates over the single  $n = n'$  interaction term, i.e.  $N_T \gtrsim \omega_c/T$ ; this corresponds to an energy cutoff of  $\omega_c$ , as anticipated above. A previous MC study on the charge problem<sup>13</sup> has chosen  $N_T$  in the range  $1/t$  to  $4/t$ , i.e. an energy cutoff of  $\approx 1/MR^2$ . For large  $r$  this cutoff is much smaller than  $\omega_c$  and is therefore insufficient.

Eqs. (1,3) identify  $1/M^*(T)R^2 = 2\pi^2 T \langle m^2 \rangle |_{\phi_x=0}$  so that the MC evaluates the fluctuations in winding number  $\langle m^2 \rangle$  at external flux  $\phi_x = 0$ . The procedure is to start with some  $m$ , update  $\theta_n$  at a time position  $n$  to  $\theta'_n$  and accept or reject the change according to the MC rule with probability  $\exp[S^{(m)}\{\theta_n\} - S^{(m)}\{\theta'_n\}]$ . After the  $N_T$  points are successively updated, the winding number is

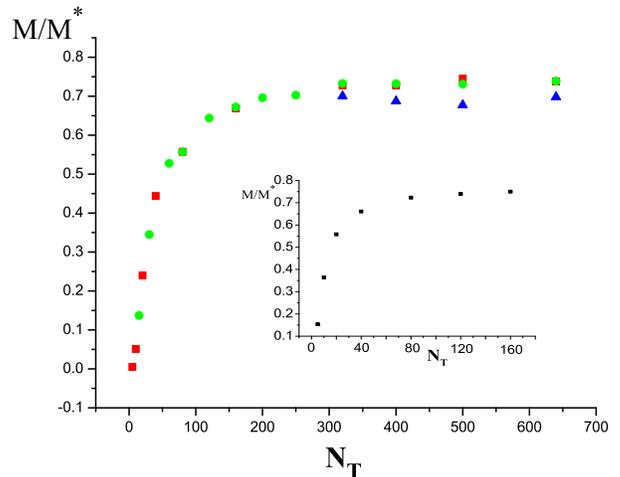


FIG. 1: Trotter number dependence of the effective mass for the dipole case with  $r = 5, t = 0.2, \alpha = 0.02$ , using (i) all  $N_T$  points in the double sum Eq. (7) – red squares, (ii) For points  $|n - n'| > 0.03N_T$  sum is coarse grained (see text) – green circles, (iii) the whole sum is coarse grained – blue triangles. Inset: The charge case with  $r = 5, t = 0.2, \alpha = 0.019$  using all  $N_T$  points in the sums.

shifted to  $m' = m \pm 1$  and the shift is accepted or rejected with the probability  $\exp[S^{(m)}\{\theta_n\} - S^{(m')}\{\theta_n\}]$ . An update of  $\theta_n$  is done randomly with a step size that produces an acceptance ratio of about 50%<sup>11</sup>.

The inset in Fig. 1 shows the  $N_T$  dependence of  $M/M^*$  for the charge problem with  $r = 5, t = 0.2, \alpha = 0.019$ . A choice for  $N_T$  in the range  $1/t - 4/t$  is clearly insufficient; saturation sets in around  $N_T \approx 100$  which is of order of  $\omega_c/T = 30$ . In the following we choose our  $N_T$ , in the charge problem, to be  $N_T = 40\alpha r^2/t = 10\omega_c/(\pi T)$ , i.e.  $N_T = 95$  for the inset parameters. For the dipole case, where  $\omega_c$  is 3 times higher we choose  $N_T = 120\alpha r^2/t = 10\omega_c/(\pi T)$ . Fig. 1 shows that for  $r = 5, t = 0.2, \alpha = 0.02$  (red squares) saturation indeed sets in near  $N_T = 300$ .

This high value of  $N_T$  restricts realistic MC studies. We have noticed, however, that this high  $N_T$  is necessary only in the vicinity of  $n = n'$  in the double sum of (7), where the summand is rapidly varying. Hence the double sum is taken over all points only in the vicinity of the singularity, i.e. for  $|n - n'| < 0.03N_T$ . For points that are further separated we coarse grain the sum with fewer points, corresponding to an effective  $N_T = 1/t$ .

The results of this procedure are shown by the green circles in Fig. 1, and are in agreement with the full calculation that includes all  $N_T$  points. The double sum has then  $\approx \frac{1}{2}10^{-3}N_T^2 + \frac{1}{2}t^{-2}$  terms, much less than the  $\frac{1}{2}N_T^2$  terms of the full calculation. We also show data where the double sum is coarse grained at all points, including those near  $n = n'$ , by blue triangles. Here the double sum

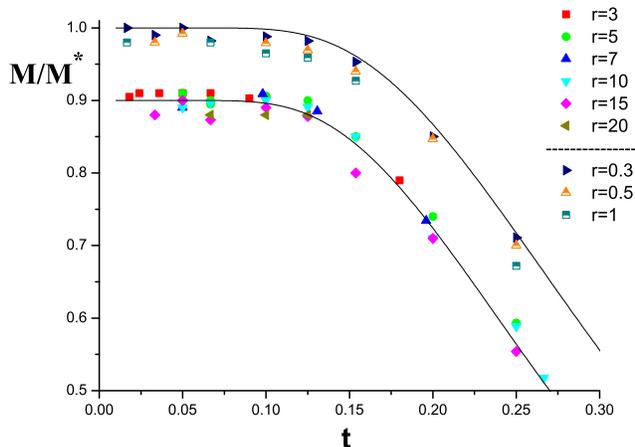


FIG. 2: AB curvature as function of reduced temperature with  $\alpha = 0.019$ . All  $r \geq 3$  values fit the renormalized form  $0.9f(t/0.9)$  – the lower curve. At  $r \leq 1$  the data approaches  $f(t)$  of a free particles – the upper curve.

has only  $\frac{1}{2}t^{-2}$  terms; this data has significant deviations from the full calculation.

We proceed to present our results on  $M/M^*(T)$ . At low temperatures we evaluate  $\langle m^2 \rangle$ , and the average involves typically many values of  $m$ . To estimate errors we evaluate the correlation function for a given run and deduce a correlation length  $\xi$ . We discard the initial  $10^4$  MC iterations and then evaluate the standard deviation  $\sigma$  of the average data; the error is then<sup>17</sup>  $\sigma\sqrt{2\xi+1}$ .

We typically find a short correlation length of a few units and we run till an error of  $\sim 2\%$  is achieved; the number of iterations is then  $\approx (1-2) \cdot 10^5$ , where each iteration is an update of  $N_T$  values of the  $\theta_n$ . At high temperatures  $t > 1$ , where  $M/M^* \lesssim 10^{-3}$ , the probability of  $m \neq 0$  becomes extremely small so that just  $m = \pm 1$  determine the outcome<sup>11</sup>. Hence we evaluate  $\langle m^2 \rangle = 2\langle e^{S_1 - S_0} \rangle_0$ , averaging with  $e^{-S_0}$ . In this method we find a rather long correlation length of  $\sim 10^3$ , yet there is no need to vary  $m$  and a 2% accuracy can be achieved after  $\approx (1-2) \cdot 10^5$  iterations.

In Fig. 2 we show our data at low temperatures,  $t < 0.3$ . We discuss first the data for  $r \geq 3$ , where we observe saturation at  $M/M^* \approx 0.9$ , independent of  $r$ . The possible dependence of  $M^*(r)$  at  $T = 0$  has been of interest as a means of monitoring anomalies in the ground state<sup>9,13</sup>. Previous studies proposed  $M^* \sim r^\mu$  with either<sup>9,15</sup> a small  $\mu$  or<sup>13</sup>  $\mu = 1.8$  for  $\alpha r > 1$  or<sup>14</sup>  $\mu = 0$ . Our MC shows that  $\mu \lesssim 0.05$  and is consistent with the  $\mu = 0$  prediction<sup>14</sup>. In view of the saturation of  $M/M^*$  at  $3 < r < 20$  we expect this saturation to persist at higher  $r$ , though the numerics cannot exclude other asymptotic forms. The more likely  $\mu = 0$  result shows that the AB curvature  $\sim 1/R^2$  is the same as for

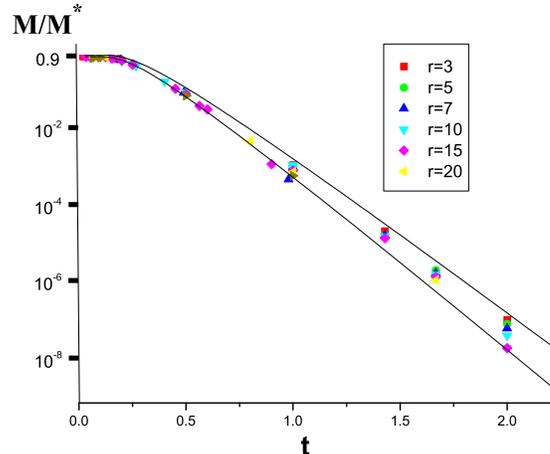


FIG. 3: AB curvature including high temperatures with  $\alpha = 0.019$ . All data fall in between the upper line  $f(t)$  and the lower line  $0.9f(t/0.9)$ .

free particles, i.e. the ground state has no anomaly. Furthermore, Fig. 2 shows that  $M^*$  determines the finite temperature behavior, as long as  $T \ll \omega_c$ . Thus if we replace  $M \rightarrow M^* = M/0.9$  in Eq. (4) we obtain the lower curve  $0.9f(t/0.9)$  in Fig. 2 which is a good fit to the data. The thermal length is then  $L_T \sim 1/\sqrt{M^*T}$ .

We turn now to discuss the  $r \leq 1$  data in Fig. 2 which shows at low  $t$  saturation near  $M^* = M$ . In fact, in the narrow range of  $1 < r < 3$  we find a continuous change of  $M/M^*$  with a critical point at  $r_c \approx 2.5$ . A possible interpretation is that at low  $r$  the level spacing  $1/2MR^2$  becomes larger than  $\omega_c$  leading to weaker renormalization at  $r \lesssim 1/\sqrt{4\pi\alpha}$ . This, however, does not account for a phase transition; furthermore, our preliminary data at higher  $\alpha$  indicates a higher  $r_c$ .

In Fig. 3 we show our  $r \geq 3$  data up to  $t = 2$ . The data falls in between two lines:  $0.9f(t/0.9)$  and  $f(t)$ . The lower curve  $0.9f(t/0.9)$  corresponds to the renormalized system and fits data with  $T \ll \omega_c$ , i.e.  $t \ll 4\pi\alpha r^2$ . For a fixed  $t$  as  $r$  decreases  $T$  approaches  $\omega_c$  and the data approaches the upper curve which is the unrenormalized free particle form  $f(t)$ .

We therefore parameterize our data by a function  $x(r, t)$  such that  $M/M^* = f(tx)/x$ . In the  $r \geq r_c$  phase we expect  $x(r, t)$  to increase towards  $x = 1/0.9$  as  $T$  decreases while for  $r \leq r_c$  we expect the opposite trend, i.e.  $x$  decreases into a weakly renormalized value near  $x = 1$  as  $T$  decreases. Fig. 4 shows that  $x(r, t)$  has a scaling form that is markedly distinct in the two phases. In the  $r > r_c$  phase the scaling variable is  $t/r$  while in the  $r < r_c$  it is  $r^2t$ . The scaling for  $r \geq r_c$  is consistent with a high  $t$  expansion<sup>13</sup> that yields  $x = 1 + 4\alpha r/(\pi t)$ , though we have not tested scaling with  $\alpha$ .

Temperature affects both renormalizations: due to the

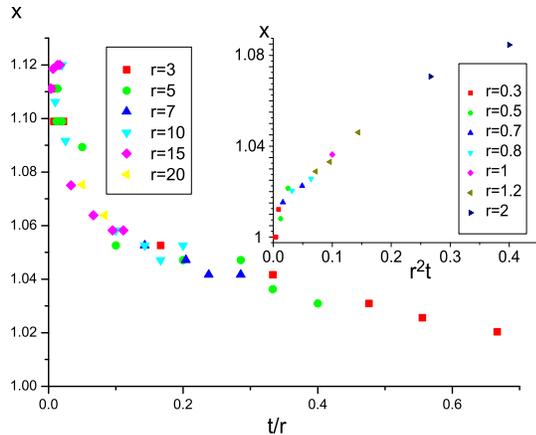


FIG. 4: Scaling of the  $x$  variable in  $M/M^* = f(tx)/x$ . Note the distinct scaling forms of the large and small  $r$  phases.

bath, controlled by  $T/\omega_c$ , and due to the free particle spectra, controlled by  $t = 2MR^2T$  via Eq. (4). To focus on the bath renormalization, it is useful to study scaling of  $r$  with  $t$  fixed, rather than with  $T$  fixed. In both phases, increasing  $r$  at a fixed  $t$  leads to a larger  $M^*$ , which is expected since more degrees of freedom become coupled. However, since the scaling variables are  $t/r$  and  $r^2t$  in the two phases, a remarkable result follows that the  $t$  dependence is opposite. In particular, at  $T = 0$  the renormalization parameter  $x$  is maximal for  $r > r_c$  while it is minimal (near  $x = 1$ ) for  $r < r_c$ . We can interpret  $r_c$  as an unstable fixed point, with RG flow for  $r > r_c$  to  $r \rightarrow \infty$  leading to fully renormalized  $M^*$ , while for  $r < r_c$  the flow is to  $r \rightarrow 0$  (possibly  $\alpha$  is also renormalized at very low  $t$ ) leading to a weakly renormalized  $M^*$  at  $T = 0$ .

To support our claim for a phase transition we note that a  $T = 0$  perturbation<sup>13</sup> in  $\alpha$  can be extended to finite small  $t$ ,

$$M/M^* = 1 - 2\alpha \sum_{n=1} a_n + 4t\alpha \sum_{n=1} a_n/n^2 \quad (8)$$

where  $K(z) = \sum_{n=1} a_n \sin^2(nz/2)$ . At  $T = 0$  Eq. (8) shows a crossover of  $M/M^*$  from 1 to  $1 - 4\alpha$  at large  $r$ , similar to the MC data. However,  $M/M^*$  approaches its  $T = 0$  value from above at all values of  $r$ , inconsistent with our data at low  $r$ .

Recalling that  $t = 2MR^2T$ , we conclude that the data shows length scales  $r_M \sim T^{-\eta}$  with  $\eta \approx 1$  and  $\eta \approx \frac{1}{4}$  in the large and small  $r$  phases, respectively. At scales  $r > r_M$  the system approaches its  $T = 0$  fixed point, which depends on the initial  $r$ . Similar length scales, with the same exponents as above, were recently identified as dephasing lengths<sup>16</sup>.

In conclusion, we have found an unexpected phase transition between two phases of model (ii) with distinct  $T$  dependence and renormalization properties. The  $r > r_c$  ground state corresponds to an  $r$  independent  $M^*$  (at least for  $r < 20$ ) while at  $r < r_c$   $M^*$  approaches rapidly  $M$  as  $r$  decreases. The approach to these ground states is via distinct scaling laws, corresponding to length scales that diverge as  $\sim T^{-\eta}$  with  $\eta \approx 1$  and  $\eta \approx \frac{1}{4}$  in the two phases, respectively.

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