Decoupling and decommensuration in layered superconductors with columnar defects

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We consider layered superconductors with a flux lattice perpendicular to the layers and random columnar defects parallel to the magnetic field $B$. We show that the decoupling transition temperature $T_d$ at which the Josephson coupling vanishes, is enhanced by columnar defects by an amount $\delta T_d / T_d \sim B^2$. Decoupling by increasing field can be followed by a reentrant recoupling transition for strong disorder. We also consider a commensurate component of the columnar density and show that its pinning potential is renormalized to zero above a critical long-wavelength disorder. This decommensuration transition may account for a recently observed kink in the melting line.

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The phase diagram of layered superconductors in a magnetic field $B$ perpendicular to the layers is of considerable interest in view of recent experiments on high-temperature superconductors.\textsuperscript{1–4} Columnar defects (CD) induced by heavy-ion bombardment provide an additional interesting probe.\textsuperscript{7} In particular, the irreversibility line at low temperatures is enhanced,\textsuperscript{5,6} while within the liquid phase an onset of enhanced $z$-axis correlation\textsuperscript{7–9} was observed. Recent data\textsuperscript{10} indicate that CD produce a porous vortex matter in which ordered vortex crystallites are embedded in the “pores” of a rigid matrix of vortices pinned on the CD. A sharp kink in the melting curve signals an abrupt change from melting enhanced by the matrix at high fields to a more weakly enhanced melting at lower fields. Theoretical studies on CD have shown a “localization” transition within a bose glass phase.\textsuperscript{11,12} Recent simulations\textsuperscript{13} have been interpreted in terms of a Bragg-Bose glass with positional order which sets in as field increases. Also a “recoupling” crossover transition\textsuperscript{14} was studied in the vortex liquid phase. In the absence of CD, theoretical studies have shown layer decoupling due to thermal fluctuations\textsuperscript{15–17} or due to disorder.\textsuperscript{17,18} At this phase transition, the Josephson coupling between layers vanishes at long scales, i.e., the critical current perpendicular to the layers vanishes and superconducting correlations in the $z$ direction (perpendicular to the layers) become short range. Decoupling involves, in principle, also proliferation of point defects—vacancies and interstitials (VI).\textsuperscript{19} The flux lattice is present even in the decoupled phase with the $z$-axis positional correlations maintained by magnetic couplings. In the case of point disorder, this phase would thus still exhibit Bragg-glass-type order without dislocations. An increase in the critical current at a “second peak” transition has been interpreted as due to an apparent discontinuity in the tilt modulus at decoupling.\textsuperscript{20} Plasma resonance data\textsuperscript{21,22} have shown a significant jump at this transition, consistent with the decoupling scenario. Whether this transition is driven by decoupling alone, rather than by a sudden dislocation proliferation,\textsuperscript{23} remains to be investigated.

In the present work, we consider the effects of CD within the flux lattice phase, neglecting VI whose role is discussed below. We find that the decoupling transition temperature $T_d(B)$ is enhanced by CD. In particular, for strong disorder, the low-field form $T_d(B) \sim B$ becomes $T_d(B) \sim B$ at strong fields, hence decoupling followed by a reentrant transition into a coupled state, i.e., recoupling, is possible with increasing field. These predictions can test whether the second peak transition is of a decoupling type. We also allow for a finite component of the CD density, which is commensurate with the flux lattice, a component that usually needs to be specifically prepared.\textsuperscript{24} We find that long-wavelength disorder renormalizes the commensurate coupling to zero, i.e., decommensuration, above a critical value of disorder. We propose that the matrix component of the porous vortex matter provides a commensurability potential for the embedded crystallites. At high fields, this enhances the crystallite melting temperature, while below the decommensuration transition, the crystallites decouple from the matrix, leading to a weaker enhancement of melting.

We study the classical partition function of $L/d$ Josephson coupled layers, where $L \rightarrow \infty$ is the total length in the $z$ direction perpendicular to the layers and $d$ is the interlayer spacing. The elastic energy of the transverse displacement fields $u(q,k)$ in the absence of Josephson coupling can be written as\textsuperscript{25,26}

$$H_{el} = \frac{1}{2L} \sum_{q,k,a} \int |(c_{66} q^2 + c_{44}^0 (k)^2) u^a (q,k)|^2,$$

where a replica index $a = 1, 2, \ldots, n$ is needed below for the disorder average. The elastic constants are\textsuperscript{25,26} $c_{44}^0 (k) = \pi l (8 d a_0^2 k^2 \lambda_{ab}^2) [1 + (2 a_0^2 k^2 / 4 \pi)]$ and $c_{66} = \pi l (16 d a_0^2)$, where $k$ is the wave vector in the $z$ direction, $l = (2d) \sin (kd/2)$, $\pi l = \Phi_0^2 d / (4 \pi^2 \lambda_{ab}^2)$, $\lambda_{ab}$ is the magnetic penetration length parallel to the layers, $a_0^2$ is the area of the flux lattice unit cell, and $\Phi_0$ is the flux quantum, i.e., $\Phi_0 = B a_0^2$. Note that the Josephson coupling induces an additional term in $c_{44}^0$,\textsuperscript{26} as also shown below. The decoupling transition of the pure system (for weak Josephson coupling) is given by\textsuperscript{16,17}
and our principal aim is to obtain the corresponding $T_d$ in the presence of correlated disorder. Consider a distribution of CD whose positions within a layer are random and uncorrelated. Each of the CD has a radius $b_0$ and their average areal density $n_{CD}$ is low, $n_{CD}b_0^2 \ll 1$. A flux line has a core of radius $\xi_0$ that usually satisfies $\xi_0 < b_0$. Once a flux line is partially inside a CD, it gains its core energy $E_c$ per layer. The pinning potential per unit area is then $U_{pin}(r) = (E_c/\xi_0)\sum(p(r - r'))$ with the sum on the CD positions and $p(r)$ is a shape function, e.g., $p(r)=1$ for $r < b_0$ and vanishes for $r > b_0$. The variance, neglecting CD overlaps, is therefore,

$$U_{pin}(r)U_{pin}(r') = E_c^2 n_{CD}(b_0/\xi_0)^4 \delta^2(r-r').$$

The average with respect to a flux density involves an additional factor $(\xi_0/a_0)^4$ due to the decomposition of a sharply peaked flux into harmonics with reciprocal vectors $Q$. The replica average at temperature $T$ is then\(^{27}\)

$$\mathcal{H}_{dis} = -\sum_{a,b} \left\{ \frac{1}{L} \sum_k \int \frac{d^2q}{(2\pi)^2} qyL \delta_{k,0} \theta^a(q,k)u^b(\mathbf{r}-\mathbf{q}) \right\} + W \sum_{a,n} \int d^2r \cos[\mathbf{Q} \cdot (\mathbf{u}_{n}(\mathbf{r}) - \mathbf{u}_{n'}(\mathbf{r}))] / 2T,$$  

where $W = E_c^2 n_{CD}(b_0/a_0)^4$ and only the shortest most relevant \(^{27}\) $\mathbf{Q}$ is retained. The cos term above involves vectors $\mathbf{Q}$ and $\mathbf{u}_n$ which in the averages below yield $(\mathbf{Q} \cdot \mathbf{u})^2 = \mathbf{Q}^2(\mathbf{u}_n^2 + \mathbf{u}^2)/2$; where $u_i$ is the longitudinal displacement that is neglected for now as it has no effect on the decoupling transition. The parameter $s$ measures a long-wavelength random torque coupled to a local bond angle \(^{28}\) $\gamma = (\partial_x u_y - \partial_y u_x)/2$, since for transverse modes $(\nabla u)^2 = 4\gamma^2$.

The long-range Bragg glass properties depend on the nonlinear cos term in Eq. (4). If this cos is expanded, it yields $\sum_{a,b} \int d^2ru^a(\mathbf{r}, k=0)u^b(\mathbf{r}, k=0)$, i.e., a $k=0$ quadratic term that has no effect on the decoupling transition. It is, therefore, essential to treat the Bragg glass nonlinearities properly.

We also allow for a commensurate term of the CD density of the form

$$\mathcal{H}_{com} = -y_c(2d/Q^2) \sum_{n,a} \int d^2r \cos[\mathbf{Q} \cdot \mathbf{u}_{n}(\mathbf{r})].$$

Consider next the Josephson phase, i.e., the relative superconducting phase of two neighboring layers. Each flux line can be viewed as a collection of point singularities, or pancake vortices, positioned one on top of the other in consecutive layers. Around each pancake vortex, the superconducting phase follows the angle $\alpha(r)$ that changes by $2\pi$ in a complete rotation. The Josephson phase involves then a nonsingular component $\theta_n(r)$ and a singular contribution from pancake vortices. The latter are positioned at $\mathbf{R}_n + \mathbf{u}_{n'}$ in the $n$th layer and at $\mathbf{R}_n + \mathbf{u}_{n+1}$ in the $(n+1)$th flux line. The total Josephson phase is then

$$\theta_n(r) + \sum_l \left[ \alpha(r - \mathbf{R}_n - \mathbf{u}_{n'}) - \alpha(r - \mathbf{R}_n - \mathbf{u}_{n+1}) \right]$$

$$= \theta_n(r) + \sum_l (\mathbf{u}_{n'} - \mathbf{u}_{n+1}) \nabla \alpha(r - \mathbf{R}_n),$$

where the expansion is justified in the Bragg glass, since the correlation length in the $z$ direction is $\gg d$. We define (including now the replica index) $\theta_n^a(q,k) = -2\pi d e^{i(kd/2)}u^a(q,k)_{\mathbf{k}/(qa_0^2)}$ so that the Josephson phase is $\theta_n^a(q,k) + b_n^a(q,k)$. Fluctuations of the $\theta_n(r)$ field involve the Josephson energy as well as magnetic-field terms,

$$\mathcal{H}_J = \frac{1}{2L} \sum_{k,a} \int \frac{d^2q}{(2\pi)^2} G_{J}^{-1}(q,k) |\theta_n^a(q,k)|^2$$

$$- y_J \sum_{n,a} \int d^2r \cos[\theta_n^a(r) + b_n^a(r)],$$

where $G_J(q,k) = 4\pi d^2(\lambda_{ab}^2 + k^2_\perp)/(\tau q^2)$. The full Hamiltonian is then $\mathcal{H} = \mathcal{H}_{dis} + \mathcal{H}_{com} + \mathcal{H}_J$.

We proceed to solve this system by the variational method allowing for replica symmetry breaking (RSB) \(^{27,30}\). The form of the variational Hamiltonian $\mathcal{H}_0$ is obtained by expanding the cos terms and replacing $y_j$, $y_z$, and $W$ by variational parameters $\gamma_j$, $\gamma_z$, and $d_{ab}(k)$, respectively. The Josephson term involves a $\theta_{ab}(k)$ cross term that is eliminated by a shift $\overline{\theta}(q,k) = \theta_n^a(q,k) - u^a(q,k)\gamma_j(2\pi k_z/\lambda_{ab}^2) \exp(ikd/2)/(G_{J}^{-1} + z_J/d)$. Hence (repeated indices are summed),

$$\mathcal{H}_0 = \frac{1}{2L} \sum_k \int \frac{d^2q}{(2\pi)^2} \left[ G_{ab}^{-1}(q,k) u^a(q,k) u^b(q,k) \right]$$

$$+ [G_{J}^{-1}(q,k) + z_J/d] |\overline{\theta}(q,k)|^2,$$

$$G_{ab}^{-1}(q,k) = [c_{66}^2 + c_{44}(q,k)k^2 + \gamma_z] \delta_{ab}$$

$$- sL \frac{q^2}{T} \delta_{k,0} - \sigma_{ab}(k).$$

The effect of the nonsingular $\theta_n$ is to shift $c_{44}(q,k)$ of Eq. (1) into $c_{44}(q,k)$

$$c_{44}(q,k) = c_{44}(q,k) + \frac{B^2}{4\pi(1 + \lambda_{ab}^2 q^2 + \lambda_{ab}^2 k^2)},$$

where $\lambda_{ab}^2 = \Phi_{ab}^2/(16\pi^2\gamma_z d)$. Note that the limit $\lambda_{ab} \rightarrow \infty$ at decoupling must be taken before $q \rightarrow 0$.

The variational method minimizes the free energy $F_0 + \langle \mathcal{H} - \mathcal{H}_0 \rangle_0$, where the free energy $F_0$ and the average $\langle \cdots \rangle_0$ correspond to $\mathcal{H}_0$. This yields
\[
\sigma_{ab}(k) = (WQ^2/2d^2T) \int_0^L \frac{dz}{\cos kz \exp(-B_{ab}(z)/2)}
\]
\[= -\delta_{ab} \sum_z \exp(-B_{ac}(z)/2), \tag{11}\]
\[
z_f = y_j \exp\left\{ - (T/2) \int_{q,k} \left[ \frac{k^2}{2} \frac{2\pi d}{a^2} \frac{G_f^{-1}(q,k)}{G_f^{-1}(q,k) + \zeta_f/d} \right]^2 \right\}, \tag{12}\]
\[
z_c = y_c \exp\left\{ -(TQ^4/2) \int_{q,k} G_{aa}(q,k) \right\}, \tag{13}\]
where \(\int_{q,k} = \int d^2qdk/(2\pi)^3\) and \(B_{ab}(z)\) is given by
\[
B_{ab}(z) = TQ^2 \int \frac{d^2qdk}{(2\pi)^3} \left[ G_{aa}(q,k) - \cos(kz)G_{ab}(q,k) \right].
\]
Since the disorder is \(z\) independent, the off-diagonal terms \(B_{a\neq b}(z)\) are \(z\) independent so that \(\sigma_{a\neq b}\) has only a \(k=0\) component; hence RSB is present only at \(k=0\). It is convenient to define \(G_f^{-1}(q,k) = \Sigma_g^{-1}(q,k)\) so that for \(k \neq 0\)
\(G_c(q,k) = G_{aa}(q,k)\). The RSB solution reduces here to a one-step form, hence \(G_c(q,k)\) can be written with self-energies in the form
\[
G_c^{-1}(q,k) = c_{66}^2q^2 + c_{44}(q,k) + \Sigma_g(1 - \delta_{k,0}) + I(k), \tag{14}\]
\[
\Sigma_g + z_c = (WQ^4/16\pi d^2c_{66}) \exp(-B_+/2). \tag{15}\]
Here, \(B_+ = TQ^2 \int_{q,k} G_{ld}(q,k)\) is a Debye-Waller factor that is dominated by large \(q\), so that \(c_{44}^0(q,k)\) can be used to obtain
\[
|B_+| \leq \frac{16\pi T \ln(c_{66}^2)}{\tau} \frac{\pi^2}{\rho d a_0^2 \lambda_{ab}^2} \left[ \Sigma_g + I(\pi/d) + z_c \right], \tag{16}\]
while the function \(I(k)\) satisfies for \(T \ll \tau\)
\[
I(k) = 4\pi c_{66} G_c(\Sigma_g + z_c) \int \frac{d^2q}{(2\pi)^2} \left[ (c_{66}^2 q^2 + \Sigma_g)^{-1} - G_c(q,k) \right]. \tag{17}\]
Note that for \(k \to 0\) this yields \(I(k) \sim |k|\), while \(I(k) \sim \Sigma_g + z_c\) for large \(k\), up to logarithmic terms. A condition for melting can be estimated by a Lindemann number \(c_L = 0.15\) (Ref. 11) so that \(\langle u_n^2(r) / a_0^2 \rangle = B_+/4\pi d^2 \approx c_L^2\). Hence, \(\tau\) is a measure of the melting temperature and for \(T \ll \tau\), \(B_+\) is small. We note that it is essential to keep the nonsingular phase \(\theta\) to obtain the correct structure factor \(G_c(q,k)\) in Eq. (14).

The decoupling transition is determined by the vanishing of \(z_f\). Equation (12) can be written in the form
\[z_f = y_j \exp\left\{ - T/2 \int_{q,k} \left[ \frac{\zeta_f}{a_0^2 q} \right]^2 \right\} \times G_c(q,k) \tag{18}\]
The integral is dominated by \(k \approx q\), so for \(k > 1/\lambda_{ab}\) the effect of the \(\theta\) field via \(G_f(q,k)\) is negligible for \(a_0^2 / 8\pi \lambda_{ab}^2 \approx 1\). The \(q\) integration is then dominated by \(c_{44}^0(q,k)\) (while \(c_{66}^2\) serves as a cutoff) leading to a \(\ln z_f\).
\[z_f \sim y_j \exp\left\{ T/8\pi \frac{\zeta_f}{a_0^2 q} \frac{\left[ \Sigma_g + I(k) \right] / k^2}{\ln z_f} \right\}, \tag{19}\]
hence,
\[
T_d = \frac{4{a_0}^4}{d^2} \left( \int k c_{44}^2 \frac{1}{\ln z_f} \right)^{-1}, \tag{20}\]
where an effective elastic modulus is \(c_{44}^0(k) = c_{44}^0(0) + \left[ \Sigma_g + I(k) / k^2 \right]\). The \(z_c\) term is obvious here by an expansion of \(H_{\text{om}}\) (Eq. 5). However, the \(\Sigma_g + I(k)\) term cannot be derived by an expansion of \(H_{\text{dis}}\) and a full RSB treatment is required. \(\Sigma_g\) acts then as \(z_c\) and leads to a divergence of \(c_{44}^0(k)\) as \(k \to 0\), as already noted in Ref. 27. Since large \(k\) dominates the integral in Eq. (18), we use \(I(k) \approx \Sigma_g + z_c\) so that
\[
T_d = T_0 \left[ 1 + (\Sigma_g + z_c) \frac{8d a_0^2 a_0^2}{\pi \ln(a_0/d)} \right], \tag{19}\]
where \(T_0 = \rho a_0^2 / 4\pi d a_0^2\), from Eq. (2), is the transition temperature in the pure system. Since \(Q \sim 1/a_0\) and \(W \sim a_0^{-4}\), we have from Eqs. (15) \(\Sigma_g + z_c \sim a_0^{-6}\). Thus, the change in \(T_d\) due to columnar defects is \(\delta T_d / T_0 \sim a_0^{-4} \sim B^2\), up to \(\ln B\) terms. From Eqs. (15) and (19), we obtain our first principal result
\[
\delta T_d / T_0 = \frac{2(4\pi)^2 E_c n_{CD} b_0^4 \lambda_{ab}^2}{\pi^2 a_0^2 \ln(a_0/d)} \approx 10^2 \left( \frac{b_0}{a_0} \right)^4 n_{CD} \lambda_{ab}^2, \tag{20}\]
where \(E_c \approx 0.2\tau\) (Ref. 11). For strong disorder and strong fields, the CD can dominate and then \(T_d \approx T_0 \sim B\) and then a recoupling would occur at a higher field, assuming this field is still below melting.

Next we address the commensurability term, which, unlike the Josephson coupling, depends also on the longitudinal \(u_i(q,k)\) component. We, therefore, add longitudinal energy terms: first, an elastic energy of the form (1) with \(c_{66}\) and \(c_{44}\) replaced by \(c_{11}\) and \(c_{44}^0\), respectively; and second, the usual long-wavelength disorder coupled to \(\mathbf{V} \cdot \mathbf{u}\) (Ref. 27) which yields the form of the first term of Eq. (4) with \(s\) replaced by \(s^i\). Since \(\sigma_{ab}\) originates from the \(W\) term in Eq. (4), \(\Sigma_g\) and \(I(k)\) are common to both longitudinal and transverse parts, while the location of the one-step solution changes by a factor \(c_{11} / (c_{11} + c_{66})\). Since \(c_{66} / c_{11}\)
$= a_1^2/16 \pi \lambda_{ab}^2 \approx 1$, the effect on $\Sigma_1$ is small, yet the structure factor for the longitudinal modes [analog of Eq. (14)] is significantly modified by the same $\Sigma_1$ and $f(k)$.

The equation for $z_c$ depends also on the $k=0$ component of $G_{ab}(q,k)$ which involves the long-wavelength disorder parameters $s$, $s'$. Using inversion methods for $G_{ab}$ (Refs. 27,30), we obtain our second principal result

$$z_c \approx -y_c \left( \frac{z_c}{\Sigma_1 + z_c} \right)^{1/2} (z_c)^{q^2/(16 \pi \lambda_{ab}^2)} + s' q^2/(16 \pi \lambda_{ab}^2). \quad (21)$$

Hence, at some critical $s$, $s'$ [where the powers of $z_c$ on both sides of Eq. (21) equal], the commensurability potential is renormalized to zero. We note that this renormalization is driven by $k=0$ terms, i.e., the same derivation is valid for a 2D system with point disorder.

Long-wavelength disorder can generate dislocations at $s > c_1^2 \rho_0 / 16 \pi$, i.e., below decoupling. Furthermore, on very long scales, dislocations will be induced by short-wavelength CD disorder as the system is effectively two-dimensional. We limit our discussion to a Bragg glass domain that ignores these very long scale effects.

The decoupling description neglects point defects, i.e., the nucleation of VI. The latter were studied in the absence of Josephson coupling and were shown to be generated by point disorder, leading to logarithmically correlated disorder for VI. Disordered CD, however, induce only a $k=0$ component of disorder which has exponentially decreasing correlations $\sim (q^2 + \lambda_2)^{-2} (q^2 + \Sigma_1)^{-1}$, hence the defect transition is not affected by the CD. The true decoupling, which allows for both Josephson phase fluctuations and for point defects, lies near the above decoupling for not too small Josephson coupling.

Finally, we address the data on the melting curve showing a kink at fields $B_k \gg B_\phi$. Within the proposed porous vortex model, we suggest that the “vortex matrix,” pinned by the random CD, forms a commensurate potential. The lowest harmonic of this potential which couples to the flux periodicity has wave vector $Q$ [harmonics with $Q^2 > Q$ have a $(z_c/(z_c + \Sigma_1))^{q^2/2Q^2}$ factor in Eq. (21), forcing $z_c = 0$ solution]. Since $s$ is a second-order effect, we consider $s' = W/B - B_\phi B^2$, hence decoumentation occurs at $B \sim B_\phi$, the bare proportionality constant being, however, too small to account for the data. The parameters $s$, $s'$ are relevant parameters within RG (Ref. 28) so that their renormalized values can be large. The main result is then that elasticity dominates at large $B$, while disorder dominates at low $B$, driving $z_c \approx 0$, in qualitative agreement with the data. We propose then to search for an additional phase transition line in the solid phase, corresponding to decoumentation, which meets the melting curve at $B_k$.

In conclusion, we have shown that columnar defects enhance the decoupling transition so that $\delta T_d / T_B \sim B^2$. However, in contrast, the melting temperature involves the same ratio within a logarithm [see Eq. (16)]; hence at weak disorder the enhancement is also $\sim B^2$, while at strong disorder it is only a weak $\ln B$ effect. The $B^2$ enhancement at strong disorder can, therefore, be useful in identifying a decoupling transition. Furthermore, for strong CD disorder a possibility of a reentrant transition has been found, i.e., with increasing field, decoupling is followed by recoupling. We have also studied effects of a commensurate CD density and shown that its potential vanishes above a critical value of long-wavelength disorder. This decoumentation transition may account for the unusual kink in the melting curve data.

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