

## Landau-level mixing and extended states in the quantum Hall effect

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We study the energies of extended states for two coupled Landau bands within a network model as a function of  $\Delta$ , the bare energy separation of extended states in the absence of level mixing. As  $\Delta$  increases the energy separation of the extended states becomes less than the bare energy separation, i.e., at high-level separation, level repulsion changes into level attraction. The level attraction leads to a minimum in the energy of the lower state at a finite magnetic field, consistent with level floating at low fields. For a spin-split Landau level we predict level attraction at high magnetic fields.

The existence of extended states is a necessary condition for the quantum Hall effect. In strong magnetic fields it is well established that the energies of extended states are on the corresponding Landau levels  $E_n = (n + \frac{1}{2})\hbar\omega_c$ , where  $\omega_c$  is the cyclotron frequency. On the other hand, it is believed that all states of a disordered two-dimensional (2D) system should be localized in the absence of a magnetic field.<sup>1</sup> Therefore a "floating scenario" has been suggested by Khmel'nitskii<sup>2</sup> and Laughlin<sup>3</sup> in which at low magnetic fields the energies of extended states increase, or float above the Fermi level, and all occupied states become localized. Numerical studies of a few Landau bands<sup>4-7</sup> focused on critical exponents, except for an early work by Ando<sup>8</sup> that supports the floating scenario. Ando's work used  $\delta$  function impurities, which is, however, not suitable when the impurity concentration is too low, since bound states of the  $\delta$  potential shift the extended state even for a single Landau band.<sup>8</sup> It is therefore of interest to study a system with smooth potentials, which is valid for both strong and intermediate magnetic fields.

In a number of recent experiments<sup>9-12</sup> a transition from an insulator at low magnetic fields  $B$  to a quantum Hall conductor at finite  $B$  was demonstrated, consistent with the floating scenario. A recent experiment by Glozman, Johnson, and Jiang<sup>13</sup> relates energies of extended states to positions of peaks in the longitudinal conductance  $\sigma_{xx}$ , which are then measured as a function of magnetic field. It was demonstrated that, indeed, the lowest extended state floats up at low  $B$ .

In the present work we use a two-channel version of Chalker and Coddington's network model,<sup>14</sup> which was previously used to study a spin-split Landau level.<sup>4-6</sup> Electrons move along unidirectional links that form closed loops in analogy with semiclassical motion on contours of constant potential. Nodes correspond to regions in space where two classical contours approach one another, i.e., nodes are saddle points in the potential. At nodes tunneling must complement the semiclassical motion so that scattering at nodes couples states on neighboring links. The assumption that each link carries current only in one direction implies that the wave packets are sufficiently localized in the transverse direction, i.e., the magnetic length is small in comparison with the spacing of nodes or with the correlation length of the potential fluctuations. The network model is therefore

a strip whose width has  $M/2$  links with two channels per link and scattering of states on two neighboring links is allowed at  $M/4$  nodes.

The transfer matrix at each node is a  $4 \times 4$  matrix that transfers four states (on two links) on the left to four states on the right and has the form<sup>5,14</sup>

$$\mathbf{T} = \begin{pmatrix} \mathbf{U}_1 & 0 \\ 0 & \mathbf{U}_2 \end{pmatrix} \begin{pmatrix} \mathbf{C} & \mathbf{S} \\ \mathbf{S} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{U}_3 & 0 \\ 0 & \mathbf{U}_4 \end{pmatrix}. \quad (1)$$

Propagation along links is described by blocks  $\mathbf{U}_i$ , each  $\mathbf{U}_i$  depends on random phases  $\phi_i$  ( $i=1-4$ ) and on a mixing angle  $\varphi$  between channels

$$\mathbf{U} = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix} \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{pmatrix}. \quad (2)$$

The  $2 \times 2$  matrices  $\mathbf{C}, \mathbf{S}$  describe the scattering at each node within each channel, i.e.,  $\mathbf{C}, \mathbf{S}$  are diagonal.

To parametrize scattering at nodes we consider nodes as saddle-point potentials  $V_{\text{SP}}(x, y) = -Ux^2 + Uy^2 + V_0$  in a magnetic field  $B$ . (The results below can be easily extended to allow anisotropy in the  $x^2, y^2$  coefficients.) The scattering states are well known<sup>15</sup>—they are labeled by the energy  $E$  and by an integer quantum number  $n \geq 0$ . The transmission  $T$  is diagonal in  $n$  and is given by

$$T = \frac{1}{1 + \exp(-\pi\epsilon)}, \quad (3)$$

where  $\epsilon \equiv [E - (n + \frac{1}{2})E_2 - V_0]/E_1$ , and

$$E_1 = \frac{1}{2\sqrt{2}}\omega_c \left[ \left\{ \frac{64U^2}{m^2\omega_c^4} + 1 \right\}^{1/2} - 1 \right]^{1/2}, \quad (4)$$

with  $\omega_c = eB/mc$ . The oscillator frequency  $E_2$  is

$$E_2 = \frac{1}{\sqrt{2}}\omega_c \left[ \left\{ \frac{64U^2}{m^2\omega_c^4} + 1 \right\}^{1/2} + 1 \right]^{1/2}. \quad (5)$$

Our two channels correspond to either two Landau levels with  $n=0,1$  or to two spin states with  $n=0$  and  $V_0 = \pm \frac{1}{2}g\mu B$ , where  $g$  is the electron  $g$  factor and  $\mu$  is the Bohr magneton. The application to  $n=0,1$  Landau levels assumes that mixing of states with  $n$ 's differing by

$\Delta n=2$  is much smaller than for those with  $\Delta n=1$ . Mixing or transition rates can be evaluated for a potential of the form  $(1/2)\tilde{U}(x^2+y^2)-\tilde{U}y^3/\lambda$ , where  $\lambda$  is a measure of the correlation length of the random potential. For states near the local minimum we find that mixing is  $\sim \exp[-\Delta n(m\omega_c^2/\tilde{U})\ln(\lambda/\ell)]$ , where  $\ell$  is of the order of the magnetic length. We therefore expect our results for the  $n=0$  state, within the two-channel model, to be valid down to  $m\omega_c^2/U \approx 1$  (assuming  $\tilde{U} \approx U$ ) with a lower limit for longer-range potential fluctuations. The results for the  $n=1$  state are, however, not directly relevant.

We parametrize the two states by  $\epsilon$  and  $\epsilon-\Delta$ , where  $\Delta=E_2/E_1$  for two Landau levels or  $\Delta=g\mu B/E_1$  for two spin states. The transmission Eq. (3) determines then the following transfer matrix in Eq. (1):

$$\mathbf{C} = \begin{pmatrix} \sqrt{1 + \exp[-\pi(\epsilon - \Delta)]} & 0 \\ 0 & \sqrt{1 + \exp[-\pi\epsilon]} \end{pmatrix},$$

$$\mathbf{S} = \begin{pmatrix} \exp[-\pi(\epsilon - \Delta)/2] & 0 \\ 0 & \exp[-\pi\epsilon/2] \end{pmatrix}, \quad (6)$$

apart from phases that are absorbed in the  $\mathbf{U}_i$  matrices. [Note that the conventional  $\theta$  parameter<sup>4,5,14</sup> for the  $\epsilon$  channel is determined by  $\epsilon = -(2/\pi)\ln(\sinh\theta)$ .]

We note that unlike discrete Landau levels, the saddle-point potential allows for a continuous energy  $E$  for each discrete state  $n$ . Furthermore, for  $\omega_c \gg \gamma$   $E_1 \rightarrow 0$  and  $E_2 \rightarrow \hbar\omega_c(n+1/2) + V_0$  correspond to the usual discrete Landau levels. In the opposite limit of  $\omega_c \ll \gamma$  the integer  $n$  corresponds to a quantum number of the harmonic potential  $Uy^2$ .

In the absence of level mixing we know from results of the one-channel system<sup>14</sup> that  $\epsilon=0$  corresponds to extended states. This defines ‘‘bare’’ extended states at  $E_{\text{ex}} = E_2(n+1/2) + V_0$ , which can be considered as Landau-level extended states in the presence of the saddle-point potential. The bare energy splitting of the  $n=0,1$  states is  $E_2$ , which is magnetic-field dominated at  $\omega_c(m/U)^{1/2} > 1$  and is potential dominated at  $\omega_c(m/U)^{1/2} < 1$ , i.e., remains finite as  $\omega_c \rightarrow 0$ . The latter region is acceptable for a network model if the correlation length of the potential fluctuation is long compared with the magnetic length, so that locally the saddle-point potential determines a finite splitting.

The topology of the network implies that reflection at one node becomes transmission at the next mode. The system is then, on average, invariant under  $90^\circ$  rotation if at the next-neighbor node the transmission and reflection (of each channel) are interchanged, i.e.,  $\epsilon \rightarrow -\epsilon$  and  $\epsilon - \Delta \rightarrow -\epsilon + \Delta$ . The system is therefore symmetric under  $\epsilon \rightarrow -\epsilon$ ,  $\Delta \rightarrow -\Delta$  and the extended states at  $\epsilon_i$  ( $i=1,2$ ) satisfy  $\epsilon_i(-\Delta) = -\epsilon_i(\Delta)$ . A further translation of energies by  $\Delta$  returns the system to itself except for  $1 \leftrightarrow 2$  interchange,  $\epsilon_2(-\Delta) + \Delta = \epsilon_1(\Delta)$ ; hence  $\epsilon_{1,2}$  are constrained by the condition  $\epsilon_1 + \epsilon_2 = \Delta$ .

For a strip with  $M/2$  links in the  $y$  direction we build the  $M \times M$  transfer matrix to describe the transfer of  $M$  channels so that each node has the form of Eq. (1) and periodic boundary conditions in the  $y$  direction are used. The  $M \times M$  transfer matrices are then multiplied to generate Lyapunov expo-

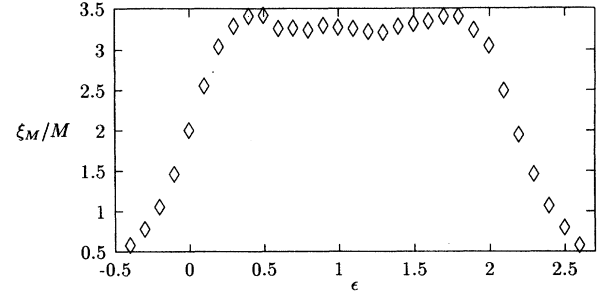


FIG. 1. Renormalized localization length  $\xi_M/M$  as a function of  $\epsilon$  for  $\Delta=2.2$  and  $M=32$ .

nents for a strip with length typically of 60 000; the smallest exponent is the inverse localization length. Finally, we use finite-size scaling<sup>14</sup> for fitting  $\xi_M/M$  onto a single curve  $f[\xi_\infty(\epsilon)/M]$  for various  $M$  and  $\epsilon$ . The localization length  $\xi_\infty(\epsilon)$  of the 2D system identifies critical energies  $\epsilon_i$  and a critical exponent  $\nu$  by  $\xi_\infty(\epsilon) \sim |\epsilon - \epsilon_i|^{-\nu}$ .

Our simulations used  $M=16,32$ , though for  $\Delta=0$  and  $\Delta=2.2$  we used also  $M=64$ , which affected critical energies by 3%; the  $\Delta=0$  are also within 5% from the result of Wang *et al.*<sup>6</sup> We choose the mixing parameter  $\sin\varphi$  to be uniformly distributed in the interval  $[0,1]$ ; we checked that other distributions in  $\sin\varphi$  lead to similar results. The raw data for one particular  $\Delta$  (see Fig. 1) represent the characteristic features of the system. One can see that  $\xi_M(\epsilon)$  is indeed a symmetric function around  $\Delta/2=1.1$ . Furthermore, for  $\Delta \geq 1$  we obtain two pronounced maxima of  $\xi_M/M$ , which are near the critical energies; we also find  $\nu=2.5 \pm 0.5$ .

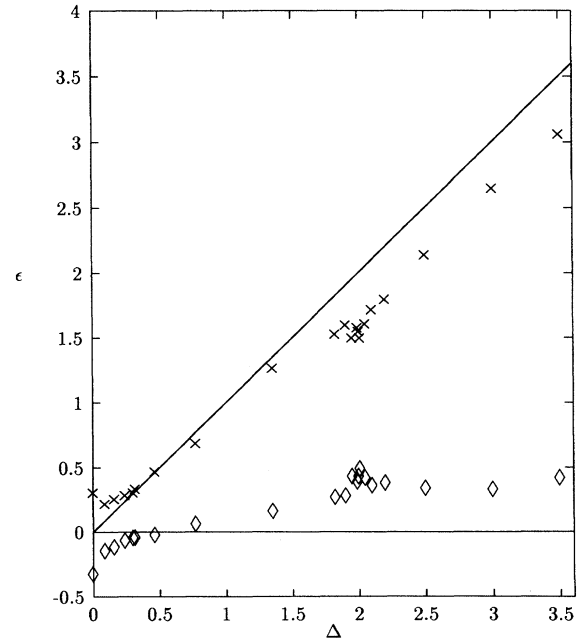


FIG. 2. Critical values  $\epsilon$  as functions of  $\Delta$ . Full lines are energies of the bare extended states  $\epsilon=0$  and  $\epsilon=\Delta$ .

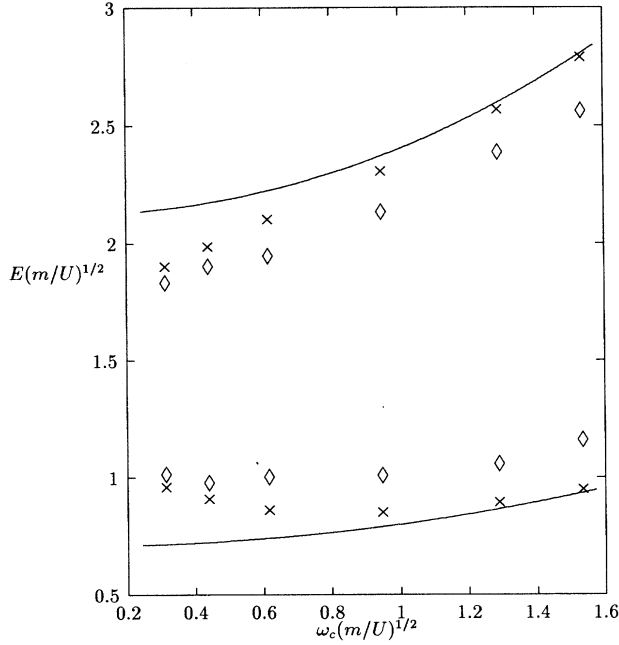


FIG. 3. Energies of two extended states as a function of magnetic field.  $\diamond$  represents first and second extended states with full mixing  $\sin\varphi \in [0,1]$ ,  $\times$  represents first and second extended states with reduced mixing,  $\sin\varphi \in [0, \exp(-m\omega_c^2/U)]$ . Full lines are energies of the bare extended states.

The critical values  $\epsilon_{1,2}$  are presented as a function of  $\Delta$  in Fig. 2; we could not calculate  $\xi_M/M$  for  $\Delta > 3.5$  because of round-off error. The most remarkable aspect of the data is the crossing of  $\epsilon_i(\Delta)$  with the bare extended states at  $\Delta \approx 0.5$ . Usually one expects that mixing affects mainly extended states and then leads to level repulsion. This expectation implies that  $\epsilon_1(\Delta)$  approaches zero from below and  $\epsilon_2(\Delta)$  approaches  $\Delta$  from above, as implied by Fig. 2 of Ref. 4. Contrary to this expectation, we find that above  $\Delta \approx 0.5$  there is level attraction. We note also that, curiously, there is a small bump at  $\Delta = 2$ , which is just within our numerical accuracy.

We present now results for the energies of extended states as a function of magnetic field by relating  $\Delta$  to  $B$  via Eqs. (5,6). As noted above,  $E_2$  is always finite leading to  $\Delta \geq 2$  ( $\Delta \rightarrow 2$  for  $B \rightarrow 0$ ), so that we are always in the level attraction regime. The results are shown in Fig. 3 with the  $\diamond$  symbol and the full lines are the bare extended state energies. Our data show a minimum at  $\omega_c(m/U)^{1/2} \approx 0.5$  in the lower state, consistent with floating, which is a result of level attraction due to Landau-level mixing. Allowing for mixing with the  $n=2$  Landau level may cause floating of the  $n=1$  state as well, but will have a small effect on the floating of the  $n=0$  state, as discussed above.

The assumption of full mixing, i.e.,  $\sin\varphi \in [0,1]$  in Eq. (2), is not valid for strong magnetic fields where tunneling between Landau levels should be suppressed. We model this situation in a reduced mixing model where  $\sin\varphi \in [0, \exp(-m\omega_c^2/U)]$ . The results are shown in Fig. 3, with the

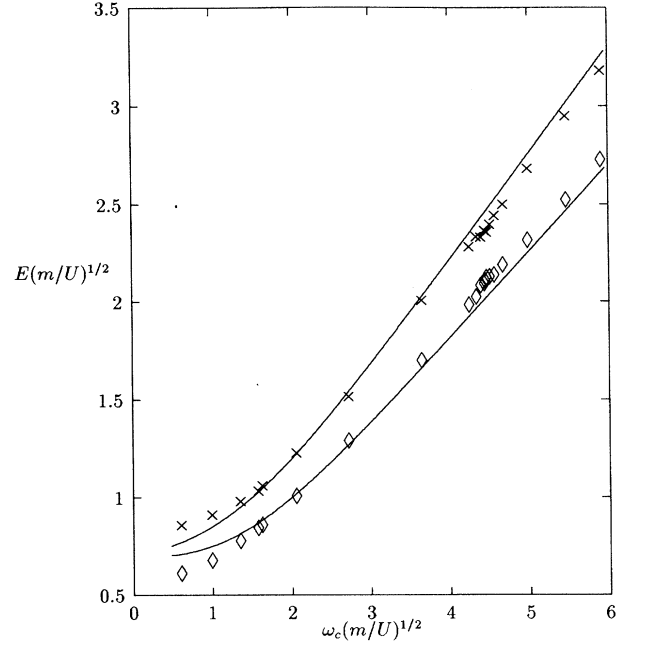


FIG. 4. Energies of two extended states in the spin-split model as a function of magnetic field with first extended state ( $\diamond$ ) and second extended state ( $\times$ ). Full lines are energies of the bare extended states with Zeeman splitting ( $\frac{1}{2}E_2 \pm \frac{1}{2}g\mu B$ ).

$\times$  symbol. The minimum in the lower level is now more pronounced and is at a higher field,  $\omega_c(m/U)^{1/2} \approx 1$ .

Finally, in Fig. 4 we present application of our data to extended state energies for two spin states, i.e., a spin-split Landau band where  $\Delta = g\mu B/E_1$ . We find that at high fields level attraction appears; i.e., the separation of extended state energies is smaller than the Zeeman splitting of  $g\mu B$ . Using potential curvatures for sheet-doped GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  (Ref. 16) we expect this to happen at fields of 1–10 T, which can be experimentally tested.

In conclusion, we have shown that level mixing within the network model leads to a minimum in the lower level, consistent with floating. For  $N$  Landau levels our symmetry argument shows that the extended state energies come in pairs, the averages of which are the same as for the bare states, i.e.,  $\epsilon_i + \epsilon_{N+1-i} = (N-1)\Delta$  with  $i=1, \dots, N$ . Hence we expect the energies of the lower half states to increase at low fields, consistent with the floating scenario. For a spin-split Landau level we predict level attraction at strong fields, which can be experimentally tested.

*Note added.* After submission of this manuscript we became aware of the paper by Shahbazyan and Raikh,<sup>17</sup> who show that, in a high  $\Delta$  expansion, levels indeed attract and the attraction vanishes as  $1/\Delta$ . Since in our results (Fig. 2) the attraction is fairly constant in the range  $2 < \Delta < 3.5$ , the  $1/\Delta$  behavior could be valid at higher  $\Delta$ .

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