

Optoacoustic Solitons in Bragg Gratings

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Optical gap solitons, which exist due to a balance of nonlinearity and dispersion due to a Bragg grating, can couple to acoustic waves through electrostriction. This gives rise to a new species of “gap-acoustic” solitons (GASs), for which we find exact analytic solutions. The GAS consists of an optical pulse similar to the optical gap soliton, dressed by an accompanying phonon pulse. Close to the speed of sound, the phonon component is large. In subsonic (supersonic) solitons, the phonon pulse is a positive (negative) density variation. Coupling to the acoustic field damps the solitons’ oscillatory instability, and gives rise to a distinct instability for supersonic solitons, which may make the GAS decelerate and change direction, ultimately making the soliton subsonic.

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Introduction.—Light and sound tend to have widely disparate frequencies, wavelengths, and velocities. As a consequence, interaction between optical and acoustic signals is often weak. The interaction is important, e.g., for Brillouin scattering, diffraction of light by ultrasonic waves [1]. Brillouin scattering in optical fibers—coupling of longitudinal or transverse acoustic modes to optical waves through electrostriction—has been analyzed extensively, though generally not for solitons [2–6]. Sound waves acting on trains of optical solitons in fibers have been studied [7,8], but these interactions were essentially between acoustic continuous waves and their optical counterparts, subject to periodic modulation. Brillouin scattering has been employed in acousto-optic filters, in which light diffracts off an acoustic grating [6]. Brillouin scattering in fibers can lead to pulse compression [9] or formation of trains of very narrow optical solitons [10]. Interaction of phonons with optical nonlinear Schrödinger solitons, with nonlinearity due to electromagnetically induced transparency, was analyzed in Ref. [11]. Reference [12] considered solitons in Bragg gratings, with electrostriction the only nonlinearity; the correct soliton solution is a limiting case of the results below.

In this work, we consider light and acoustic waves in a cubic ($\chi^{(3)}$) nonlinear medium with a Bragg grating, a fixed spatially periodic variation in the index of refraction [13]. Bragg gratings are typically written in the cladding of an optical fiber, though they have been induced by copropagating continuous waves [14]. Fiber Bragg gratings without electrostriction are known to support optical gap solitons, nonlinear solitary waves with frequencies in the band gap created by the Bragg grating [13,15,16]. Gap solitons can have arbitrarily small velocity, which has motivated great interest in them for stopping light, though creating very slow gap solitons experimentally has been

difficult. Early experimental realizations of gap solitons had velocities approximately half the speed of light [16]. Various methods have been proposed for creating slow gap solitons: building up slow gap solitons directly by Raman amplification [17], obtaining quiescent solitons from collisions of fast-moving gap solitons [18], and retarding fast solitons by passing them through gradually varying (“apodized”) Bragg gratings [19]. In a recent experimental breakthrough, optical gap solitons were slowed to a sixth the speed of light using apodized Bragg gratings [20].

Electrostriction, compression of the medium due to variations of the intensity of light, allows light to drive acoustic waves; conversely, acoustic waves feed back into the optical field through the dependence of the refractive index on the material density. Gap solitons can be arbitrarily slow; at velocities close to the speed of sound, an *optoacoustic resonance* may dramatically enhance the interplay between light and sound. We derive a generalization of the standard optical gap soliton, including an acoustic component. The dynamics of these “gap-acoustic solitons” (GASs) are significantly different from optical gap solitons without the acoustic dressing when the velocities are slow, and slightly different when velocities are large. Our results demonstrate that if an optical gap soliton is slowed to a velocity on the order of the speed of sound, the inherent electrostrictive effects can do the work of slowing the soliton to significantly below the speed of sound, even creating stopped light. The results also suggest that GASs may be controlled by continuous acoustic waves or pulses.

We assume one transverse electromagnetic mode and one acoustic mode, either because the waveguide has exactly one transverse optical mode and one acoustical mode, or because coupling to other modes is negligible. We have derived effectively (1 + 1)-dimensional coupled mode

equations using standard methods [13,21] for light in a waveguide, adding a phonon field and acousto-optic coupling induced by electrostriction [1–5,7–9]:

$$0 = ik'_0 u_t + iu_z + \kappa v + \frac{2\pi(\omega_0/c)^2}{k_0 A} (\chi_s |u|^2 + \chi_x |v|^2) u + \chi_{es} w u, \quad (1a)$$

$$0 = ik'_0 v_t - iv_z + \kappa^* u + \frac{2\pi(\omega_0/c)^2}{k_0 A} (\chi_x |u|^2 + \chi_s |v|^2) v + \chi_{es} w v, \quad (1b)$$

$$0 = w_{tt} - \Gamma w_{tzz} - \beta_s^2 w_{zz} + \lambda(|u|^2 + |v|^2)_{zz}. \quad (1c)$$

Here z is the coordinate along the waveguide and t is time; $u(z, t)$ and $v(z, t)$ are slowly varying envelopes of the counterpropagating electromagnetic waves [total electric field $E(z, t) = u(z, t) \exp[i(-\omega_0 t + k_0 z)] + v(z, t) \times \exp[i(-\omega_0 t - k_0 z)] + \text{c.c.}$], with carrier frequency ω_0 and wave numbers $\pm k(\omega_0) \equiv \pm n(\omega_0) \omega_0 / c$, $n(\omega)$ being the index of refraction, $k'_0 \equiv dk/d\omega|_{\omega=\omega_0}$ the reciprocal group velocity, and κ the Bragg reflectivity. The self- and cross-phase modulation coefficients are χ_s and χ_x , and A is the effective cross-section area of the waveguide modes. The acoustic field $w(z, t)$ is the density of the medium, offset by a constant W_0 . Equations (1) assume that the acoustic wavelengths are much larger than the optical one, so sound waves are not subject to Bragg reflection. The form of the electrostrictive interaction involves the average square electric field $\langle E^2 \rangle = 2(|u|^2 + |v|^2)$ [see Eqs. (9.3.11-12) in Ref. [1]]. The electrostrictive coefficient λ , which comes from the dependence of the energy density on the index of refraction $n(\omega, W_0 + w)$, is $\lambda \equiv (2\pi)^{-1} (W_0 + w) n dn/dw$ [1]; in a waveguide, the coefficient may be strongly affected by the modal structure [2,4,5]. The other electrostrictive coefficient, $\chi_{es} = (\omega_0/c) dn/dw$, comes from local changes in the wave vector [$k = n(\omega_0, W_0 + w) \omega / c$], which depends on the index of refraction, which in turn depends on the acoustic field [1,4,8,21]. The speed of sound is denoted by β_s . Phonon viscosity Γ is included for completeness, though current Bragg waveguides are not long enough for it to be significant. Mass $M = A \int_{-\infty}^{\infty} w(z, t) dz$ and number of photons $N = [n(\omega_0)^2 / 4\pi\hbar\omega_0] A \int_{-\infty}^{\infty} (|u|^2 + |v|^2) dz$ are conserved; momentum $P = [n(\omega_0)^2 / 4\pi\omega_0] A \int_{-\infty}^{\infty} [i(uu_z^* + vv_z^*) - (\chi_{es}/k'_0 \lambda) r_z r_t] dz$, where $r \equiv \int^z w(z', t) dz'$ is an acoustic potential, is invariant for $\Gamma = 0$, but decays when viscosity is nonzero.

Equations (1) have resemblances to the Zakharov system [22,23]: both contain dispersive high-frequency waves, coupled to nondispersive acoustic modes. The dissimilarity is the source of dispersion—a Bragg grating rather than intrinsic material dispersion. Looking at Eqs. (1) as a Zakharov system with a Bragg grating helps to identify realizations in additional physical systems: surface and bulk acoustic waves in solids, where the grating (for the surface mode) is a surface superstructure; coupled vibron and acoustic waves in polymer molecules, with the grating (for the vibrons) provided by an intramolecular structure; and coupled Langmuir and ion-acoustic waves in plasmas (the original physical Zakharov system), with a grating in the form of a dust crystal [24].

Soliton solutions.—We found a family of GAS solutions to Eqs. (1) with zero phonon viscosity ($\Gamma = 0$),

$$u = \sqrt{\kappa\gamma(1 + \beta k'_0)} \alpha \sin Q \operatorname{sech}(\zeta \sin Q - iQ/2) \times \exp[i\theta(\zeta) - i\tau \cos Q], \quad (2a)$$

$$v = -\sqrt{\kappa^*\gamma(1 - \beta k'_0)} \alpha \sin Q \operatorname{sech}(\zeta \sin Q + iQ/2) \times \exp[i\theta(\zeta) - i\tau \cos Q], \quad (2b)$$

$$w = \frac{\lambda|\kappa\alpha^2|}{\beta_s^2 - \beta^2} \frac{4\gamma \sin^2 Q}{\cosh(2\zeta \sin Q) + \cos Q}, \quad (2c)$$

with $\tau \equiv \gamma|\kappa|[t/k'_0 - (\beta k'_0)z]$, $\zeta \equiv \gamma|\kappa|(z - \beta t)$, $\gamma \equiv [1 - (\beta k'_0)^2]^{-1/2}$, and

$$\theta(\zeta) \equiv \frac{4|\alpha|^2(\beta k'_0)}{1 - (\beta k'_0)^2} \left(\frac{2\pi(\omega_0/c)^2}{k_0 A} \chi_s + \frac{\chi_{es} \lambda}{\beta_s^2 - \beta^2} \right) \times \tan^{-1}[\tanh(\zeta \sin Q) \tan(Q/2)], \quad (3a)$$

$$|\alpha|^{-2} \equiv \frac{2\pi(\omega_0/c)^2}{k_0 A} \left(\chi_x + \chi_s \frac{1 + (\beta k'_0)^2}{1 - (\beta k'_0)^2} \right) + \frac{2\chi_{es} \lambda}{(\beta_s^2 - \beta^2)[1 - (\beta k'_0)^2]}. \quad (3b)$$

The solitons are characterized by two parameters: β , the soliton velocity, and Q , which takes values $0 < Q < \pi$ and determines the soliton's full width at half-maximum, $(|\kappa|\gamma \sin Q)^{-1} \cosh^{-1}(2 + \cos Q)$, peak intensity, and frequency [in the rest frame, $\gamma|\kappa|(k'_0)^{-1} \cos Q$]. These solutions also hold for quiescent solitons ($\beta = 0$) with nonzero phonon viscosity ($\Gamma > 0$). The velocity may take any value up to the group velocity of light, $|\beta| < 1/k'_0$, except for a *velocity-spectrum gap* at and above the speed of sound, $|\beta| \notin [\beta_s, \beta_{cr}]$, with

$$\beta_{cr}^2 = \frac{1}{2(k'_0)^2} \frac{\chi_x + \chi_s}{\chi_x - \chi_s} + \frac{\beta_s^2}{2} \pm \sqrt{\left(\frac{1}{2(k'_0)^2} \frac{\chi_x + \chi_s}{\chi_x - \chi_s} - \frac{\beta_s^2}{2} \right)^2 - \frac{k_0 A}{2\pi(\omega_0/c)^2} \frac{2\chi_{es} \lambda / (k'_0)^2}{\chi_x - \chi_s}}. \quad (4)$$

For the typical case, $\chi_x = 2\chi_s > 0$, the negative sign applies. For subsonic ($|\beta| < \beta_s$) solitons [supersonic solitons ($\beta_s < \beta_{cr} < |\beta| < 1/k'_0$)], the density perturbation around the soliton is positive [negative], i.e., compression [rarefaction]. The light components (u, v) vanish as the velocity approaches the speed of sound from below, $|\beta| \rightarrow \beta_s$; both light and sound

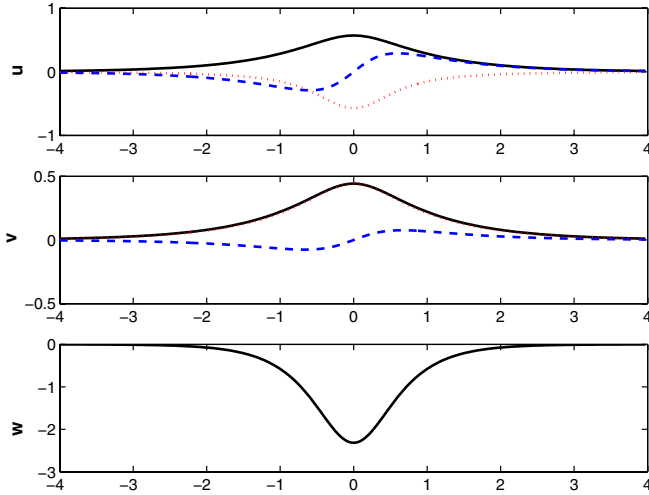


FIG. 1 (color online). Three components of a supersonic gap-acoustic soliton (right- and left-traveling envelopes of the electromagnetic waves and the acoustic field), for $k'_0 = \kappa = 2\pi(\omega_0/c)^2(k_0A)^{-1}\chi_x = \chi_{cs} = 1$, $\chi_s = \chi_x/2$, $\beta = 0.25$, $\beta_s = 0.2$. $\Gamma = 0$, and $\lambda = 0.1$. Dashed, dotted, and solid lines show, respectively, the real and imaginary parts and absolute values of the fields.

(w) amplitudes diverge when the velocity approaches β_{cr} from above. Figure 1 shows a moderately supersonic soliton, with frequency in the middle of the band gap ($Q = \pi/2$), and velocity $\beta = (1.25)\beta_s = 0.25$.

With electrostriction, $\lambda \neq 0$, Eqs. (1) do not admit solitons without an acoustic component, but purely acoustic waves (solutions to the free D'Alembert equation for w) are possible. Equations (1) do not admit stable breathers, as oscillations of a localized mode will generate emission of acoustic waves, dissipating the oscillations.

Stability.—We tested soliton stability by numerical simulation of Eqs. (1), using a split-step fast Fourier transform scheme [21]. Simulations were carried out systematically for three values of the soliton coefficient, $Q = \pi/3$ (in the middle of the top half of the band gap), which, for gap solitons without electrostriction ($\lambda = 0$) [15], is well inside the stable region; $Q = \pi/2$ (midpoint of the band gap), which is stable but close to the instability border; and $Q = 2\pi/3$ (in the middle of the bottom half of the band gap), which has an oscillatory instability [25]. We took ten values of the electrostrictive coefficient λ , ranging over 4 orders of magnitude, and the limit with zero instantaneous Kerr nonlinearity $\chi_s = \chi_x = 0$. The self- to cross-phase ratio was $\chi_s/\chi_x = 1/2$. The speed of sound was taken to be $\beta_s = 0.2/k'_0$, which is faster than physically realistic, but more clearly illustrates the dynamics. Initial soliton velocities were taken over a range from zero to twice the speed of sound. To impose consistent perturbations, all numerical simulations had initial light amplitudes 1% greater than those of the exact soliton solutions.

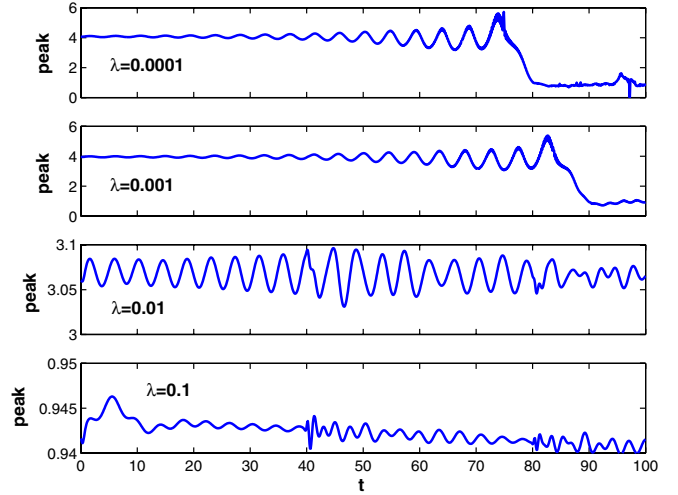


FIG. 2 (color online). Time evolution of the peak power of the GAS [$\max_z(|u(z,t)|^2 + |v(z,t)|^2)$], for electrostrictive coefficients $\lambda = 0.0001, 0.001, 0.01$, and 0.1 . The solitons are taken with zero velocity and intrinsic parameter $Q = 2\pi/3$, which is unstable in the standard gap soliton model ($\lambda = 0$). The speed of sound is $\beta_s = 0.2$.

Like gap solitons without electrostriction [25], GASs are subject to oscillatory instabilities, which grow until they destroy the solitons. However, electrostriction decreases the growth rate of the instability, as shown in Fig. 2. Similarly, numerical simulations showed that the growth rate of the oscillatory instability decreases for velocities closer to the speed of sound. A general trend is that larger phonon components produce greater damping of the oscillatory instability. The damping may be due to emission of acoustic waves, and that the GAS's acoustic component creates an effective trapping potential which suppresses disintegration of the optical components.

All solitons that are stable without electrostriction remained stable with electrostriction $\lambda \neq 0$, if the velocity was subsonic $|\beta| < \beta_s$. All supersonic ($\beta_s < \beta_{cr} < |\beta| < 1/k'_0$) GASs were found to be unstable if numerically integrated for long enough. This supersonic instability is qualitatively different from the oscillatory instability, and is unknown for optical gap solitons without electrostriction. The supersonic instability may be connected to the fact that for slightly supersonic solitons, the soliton momentum decreases with velocity above the speed of sound $\frac{dP}{d\beta} < 0$, which is a known instability criterion [23]. Supersonically unstable GASs tend to retain their integrity through growth of the instability. The closer the supersonic soliton's velocity to the speed of sound, the sooner the instability takes effect; this may be explained by the divergence of the momentum slope $\frac{dP}{d\beta}$ at the critical velocity. The changes in velocity are fairly abrupt, and are accompanied by emission of phonons, which carry off momentum. The solitons may change speed and direction a few times before eventually settling to a stable subsonic form.

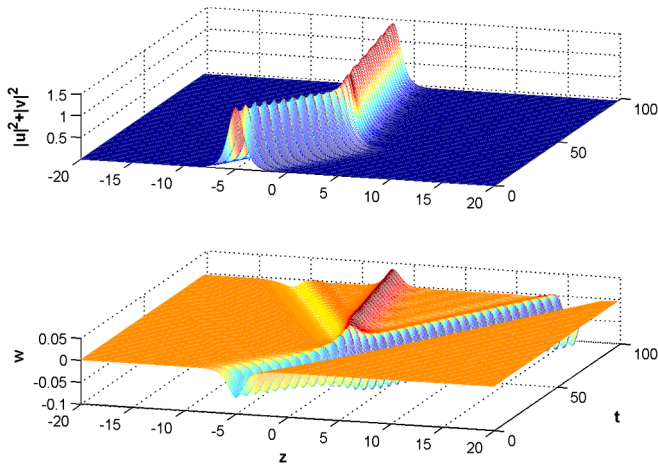


FIG. 3 (color online). Instability of a supersonic GAS, for $Q = \pi/3$ and initial velocity $\beta = 0.25$, which is 125% the speed of sound $\beta_s = 0.2$. The electrostrictive coefficient is $\lambda = 0.001$.

When the electrostrictive coefficient is larger, and the phonon field larger, the supersonic instability tends to be stronger: it occurs sooner, and the soliton is more likely to be destroyed. Destruction seems to be associated with the decelerating GAS spending significant time in the forbidden velocity region ($\beta_s \leq |\beta| < \beta_{cr}$). Figure 3 shows an unstable supersonic GAS, in which the soliton retains its integrity, settling to a stable subsonic GAS.

Summary.—Electrostriction couples acoustic waves to light in a waveguide with a Bragg grating, and the resulting system supports “gap-acoustic” solitons, a generalization of optical gap solitons. Electrostriction damps the gap soliton’s (known) oscillatory instability, and gives rise to another instability, which occurs when the soliton’s velocity exceeds the speed of sound. In contrast to the oscillatory instability, which destroys the gap soliton, the supersonic instability tends to leave the soliton intact, transforming it into a stable subsonic soliton. Thus, electrostriction adds a new level of complexity to the structure of optical gap solitons, and may facilitate the creation of ultraslow optical pulses. Electrostriction and acoustic waves are present in virtually all materials, so an understanding of physically realistic optical gap solitons, especially slow ones, must entail a grasp of the effects of electrostriction.

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- [1] R.W. Boyd, *Nonlinear Optics* (Academic, San Diego, 2003).
- [2] P.J. Thomas *et al.*, Phys. Rev. B **19**, 4986 (1979).
- [3] R.M. Shelby, M.D. Levenson, and P.W. Bayer, Phys. Rev. B **31**, 5244 (1985).
- [4] E.L. Buckland and R.W. Boyd, Opt. Lett. **21**, 1117 (1996); Opt. Lett. **22**, 676 (1997).
- [5] A. Fellegara, A. Melloni, and M. Martinelli, Opt. Lett. **22**, 1615 (1997); E.M. Dianov, M.E. Sukharev, and A.S. Biriukov, Opt. Lett. **25**, 390 (2000); S. Afshar *et al.*, Opt. Lett. **30**, 2685 (2005).
- [6] E. Granot *et al.*, Appl. Opt. **44**, 4959 (2005); E. Granot *et al.*, J. Opt. Soc. Am. B **23**, 1250 (2006).
- [7] K. Smith and L.F. Mollenauer, Opt. Lett. **14**, 1284 (1989); P.J. Hardman *et al.*, Opt. Lett. **21**, 393 (1996); P. Maák *et al.*, Appl. Phys. B **82**, 283 (2006).
- [8] E.M. Dianov *et al.*, Opt. Lett. **15**, 314 (1990).
- [9] D.T. Hon, Opt. Lett. **5**, 516 (1980).
- [10] I. Bongrand *et al.*, Opt. Lett. **26**, 1475 (2001).
- [11] S.V. Sazonov, J. Exp. Theor. Phys. **101**, 979 (2005); JETP Lett. **81**, 201 (2005).
- [12] A.A. Zabolotskii, Opt. Spectrosc. **97**, 936 (2004).
- [13] C.M. de Sterke and J.E. Sipe, Prog. Opt. **33**, 203 (1994).
- [14] G. Van Simaey, S. Coen, M. Haelterman, and S. Trillo, Phys. Rev. Lett. **92**, 223902 (2004).
- [15] W. Chen and D.L. Mills, Phys. Rev. Lett. **58**, 160 (1987); D.N. Christodoulides and R.I. Joseph, Phys. Rev. Lett. **62**, 1746 (1989); A.B. Aceves and S. Wabnitz, Phys. Lett. A **141**, 37 (1989); A.B. Aceves, Chaos **10**, 584 (2000).
- [16] B.J. Eggleton *et al.*, Phys. Rev. Lett. **76**, 1627 (1996); B.J. Eggleton, C.M. de Sterke, and R.E. Slusher, J. Opt. Soc. Am. B **16**, 587 (1999).
- [17] H.G. Winful and V. Perlin, Phys. Rev. Lett. **84**, 3586 (2000).
- [18] W.C.K. Mak, B.A. Malomed, and P.L. Chu, Phys. Rev. E **68**, 026609 (2003).
- [19] W.C.K. Mak, B.A. Malomed, and P.L. Chu, J. Mod. Opt. **51**, 2141 (2004).
- [20] J.T. Mok *et al.*, Nature Phys. **2**, 775 (2006).
- [21] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic, San Diego, 1989).
- [22] V.E. Zakharov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)].
- [23] A.S. Davydov, Phys. Scr. **20**, 387 (1979); L. Stenflo, Phys. Scr. **33**, 156 (1986); H. Hadouaj, B.A. Malomed, and G.A. Maugin, Phys. Rev. A **44**, 3925 (1991); **44**, 3932 (1991); G.A. Maugin, H. Hadouaj, and B.A. Malomed, Phys. Rev. B **45**, 9688 (1992).
- [24] S.A. Khrapak and G. Morfill, Phys. Plasmas **8**, 2629 (2001).
- [25] B.A. Malomed and R.S. Tasgal, Phys. Rev. E **49**, 5787 (1994); I.V. Barashenkov, D.E. Pelinovsky, and E.V. Zemlyanaya, Phys. Rev. Lett. **80**, 5117 (1998); A. De Rossi, C. Conti, and S. Trillo, Phys. Rev. Lett. **81**, 85 (1998).