Statistics of atomic populations in output coupled wave packets from Bose-Einstein condensates: Four-wave mixing

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The statistics of the atomic population distributions in nonlinear matter-wave processes, such as four-wave mixing of matter waves (or output coupled wave packets produced by Bragg scattering) from Bose-Einstein condensates (BECs), are determined. Fluctuations of the populations of atoms in the four-wave mixing wave packet can be due to (a) fluctuations of the laser fields that produce the separate momentum wave packets of the BEC, (b) quantum fluctuations arising from finite temperature effects, and (c) the quantum-mechanical nature of the mean-field BEC wave function. We focus on the latter source of fluctuations. The distribution of the number of atoms in the four-wave mixing wave packet is binomial and reduces to a Gaussian distribution for strong conversion. We calculate the skewness and kurtosis of the distribution. The differences in the nature of the fluctuations in nonlinear phenomena for atoms (matter waves) and photons are discussed.

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I. INTRODUCTION

Recently, the theory of multiwave mixing of matter waves formed from Bose-Einstein condensates (BECs) was presented [1], and the first experimental observation of coherent four-wave mixing (4WM) in which three sodium matter waves mix to produce a fourth was reported [2]. The dependence of the generated matter wave on the densities of the three input wave packets showed a clear signature of the nonlinear 4WM process. Here, we generalize the theory to describe the statistics of the atomic population distributions that are expected to occur in such nonlinear matter wave experiments. Our results are also applicable for determining the statistics of the atomic distribution in any process in which the separation of the condensate into distinct subsystems occurs, e.g., the use of a sequence of optical light pulses to produce high-momentum component wave packets by Bragg scattering [3,4].

Reference [2] measured the expectation values of the number of atoms in the 4WM wave packet, but did not report on the statistics of the atomic distributions in the various wave packets produced by the dynamics. Such statistics are interesting to determine theoretically and experimentally. Measurement of the distribution of the atomic populations in the various wave packets will provide additional information on the nonlinear 4WM process. There are three potential sources for the fluctuations of the number of atoms in the wave packets: (a) fluctuations of the laser fields that produce the separate momentum wave packets of the BEC, (b) quantum fluctuations arising from finite temperature effects, and (c) the quantum-mechanical nature of the mean-field BEC wave function. Clearly, the statistics of the photon light field used to produce the high-momentum wave packets that in turn produce the 4WM wave packet can also introduce fluctuations in the number of atoms in the 4WM wave packet. If the light field is intense and well reproducible from shot to shot, fluctuations arising through the light fields should be negligible. The theory for BECs has been developed beyond mean-field [6,7], and can be applied to determine the fluctuations of the number of atoms in the 4WM wave packet and the statistics of this problem for finite temperature at a higher level of approximation. For temperatures significantly below the critical temperature, these effects should be negligible. Therefore, we focus on the latter source of fluctuations, and assume that the other sources of fluctuations are small. Here, we consider the statistics only within the context of mean-field theory [5].

II. MEAN-FIELD DESCRIPTION OF FOUR-WAVE MIXING IN BECs

At the mean-field level of description of a BEC, all the translational modes of the BEC are described by a single mean-field orbital, and the wave function is symmetric under interchange of the Bose particles. The N-particle wave function Ψ of the zero-temperature Bose condensate is given by the symmetric product

$$\Psi(t)=\prod_{j=1}^{N} \psi(\mathbf{x}_j,t), \tag{1}$$

where j is the particle index, and the mean-field orbital ψ is the same for all particles. For the 4WM experiment under consideration [2], the initial wave function is obtained by turning off the confining harmonic potential, letting the condensate wave packet ballistically expand for some time and applying a set of optical pulses to create wave packets with momenta $\hbar\mathbf{k}_1$, $\hbar\mathbf{k}_2$, and $\hbar\mathbf{k}_3$ [1,2,8]. Immediately after application of the optical Bragg pulses which produce the high-momentum components of the BEC by a far-detuned stimulated Raman-type transition, to an excellent approximation [9], ψ is comprised of three BEC wave packets [1,8],

$$\psi(\mathbf{x},t=0)=\sum_{i=1}^{3} a_i(0)\,\phi(\mathbf{x},0)\exp(i\mathbf{k}_i\cdot\mathbf{x}), \tag{2}$$

where $\phi(\mathbf{x},0)$ is ballistically expanded solution to the Gross-Pitaevskii equation, and the amplitudes $a_i(0)$, determined by the intensities of the Bragg pulses, specify the probability amplitudes for the three initial wave packets, where $\Sigma_{i=1}^{3}|a_i(0)|^2=1$. The normalization is chosen so that the norm of ψ is unity (the norm of ϕ is also unity).

The nonlinear Schrödinger equation (the Gross-Pitaevskii equation) determines the dynamical evolution of the time-dependent wave function $\psi(\mathbf{x},t)$ [5],

$$i\hbar\frac{\partial\psi}{\partial t}=[T_{\mathbf{x}}+V(\mathbf{x},t)+U_0|\psi|^2]\psi, \tag{3}$$

where

$$T_{\mathbf{x}}=\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$$

is the kinetic energy operator, $V(\mathbf{x},t)=0$ since no potential is imposed on the atoms after the confining potential is dropped, $U_0=(4\pi a_0\hbar^2/m)N$ is the atom-atom interaction strength, proportional to the *s*-wave scattering lengths a_0, and N is the total number of atoms in the condensate. For times t larger than the collision time of the wave packets, t_{col}, when the wave packets are separated in space, the wave function $\psi(\mathbf{x},t)$ can be written as

$$\psi(\mathbf{x},t)=\sum_{i=1}^{4}\phi_i(\mathbf{x}-\mathbf{x}_i(t),t)\exp(i\mathbf{k}_i\cdot\mathbf{x}), \tag{4}$$

where $\mathbf{x}_i(t)$ $i=1,2,3,4$, is the center of the ith wave packet, $\mathbf{x}_i(t)=(\hbar\mathbf{k}_i/m)t$, and $\mathbf{k}_4=\mathbf{k}_1-\mathbf{k}_2+\mathbf{k}_3$. The fourth wave packet is the 4WM wave packet, absent initially, and created exclusively due to the nonlinear interaction in the dynamics of three wave packet collision.

For $t>t_{col}$, the expectation value of the population of the ith wave packet is independent of t and given by $p_i(t)=p_i=\langle N_i\rangle/N=\int_{V_{\mathbf{x}_i}}d\mathbf{x}\langle\psi(\mathbf{x})|\psi(\mathbf{x})\rangle$, where the integration region $V_{\mathbf{x}_i}$ is selected to include the region around the ith wave packet. Alternatively, p_i can be defined in momentum space as $p_i(t>t_{col})=p_i=\langle N_i\rangle/N=\int_{V_{\mathbf{k}_i}}d\mathbf{k}\langle\psi(\mathbf{k})|\psi(\mathbf{k})\rangle$ where the integration region $V_{\mathbf{k}_i}$ is selected to include the region around the ith wave packet in momentum space. Note that $p_i(0)=|a_i(0)|^2$ immediately after the light pulses are applied.

III. PROBABILITY DISTRIBUTION FOR FOUR-WAVE MIXING

Substituting the wave function ψ in Eq. (4) into Eq. (1) and expanding the product, we obtain the multiatom wave function $\Psi(t)$ in the form of a sum over 4^N terms. Each of these terms represents a definite distribution of atoms between various 4WM peaks. Due to the indistinguishability of particles, all terms that differ only by the permutation of the particle indices are equivalent. The number of these equivalent terms is equal to the appropriate multinomial coefficient, $N!/(N_1!N_2!N_3!N_4!)$. Further, the sum of equivalent terms is proportional to the symmetrized and normalilzed N boson Fock state which in the occupation number representation is denoted as $|N_1,N_2,N_3,N_4\rangle$, hence:

$$\Psi(t>t_{col})=\sum_{N_1,N_2,N_3,N_4}\delta_{N,N_1+N_2+N_3+N_4}\times\left(\frac{N!\,p_1^{N_1}p_2^{N_2}p_3^{N_3}p_4^{N_4}}{N_1!N_2!N_3!N_4!}\right)^{1/2}|N_1,N_2,N_3,N_4\rangle. \tag{5}$$

The probability distribution $P_N(N_1,N_2,N_3,N_4)$ for finding N_i atoms in the wave packet $\psi_i(\mathbf{x}-\mathbf{x}_i,t)$, $i=1,4$, given that the total number of atoms in the BEC is N, is a modulus square of the inner product of the wave function in Eq. (5) with $|N_1,N_2,N_3,N_4\rangle$ [10]:

$$\begin{aligned}P_N(N_1,N_2,N_3,N_4)&=|\langle N_1,N_2,N_3,N_4|\Psi(t>t_{col})\rangle|^2\\&=\delta_{N,N_1+N_2+N_3+N_4}\frac{N!}{N_1!N_2!N_3!N_4!}\\&\quad\times p_1^{N_1}p_2^{N_2}p_3^{N_3}p_4^{N_4}.\end{aligned} \tag{6}$$

Further simplification results if we are interested in determining only the number of atoms in the newly created 4WM wave packet, irrespective of the distribution of the atoms in wave packets 1, 2 and 3 (as long as the total number of atoms equals N). In this case our distribution function, defined by Eq. (6), may be reduced to the binomial distribution:

$$\begin{aligned}P_N(N_4)&=\sum_{N_1,N_2,N_3}\delta_{N,N_1+N_2+N_3+N_4}P_N(N_1,N_2,N_3,N_4)\\&=\frac{N!}{(N-N_4)!N_4!}p_4^{N_4}(1-p_4)^{N-N_4}.\end{aligned} \tag{7}$$

The results of several interesting experiments are embodied within the probability distribution (6). Experimental tests are likely to entail measurement of moments of the probability distribution. We now give explicit expressions for the most common moments and convenient expressions for generating arbitrary moments. The calculation of moments follows readily by use of the generating function,

$$Q\equiv\sum_{N_1,N_2,N_3,N_4}\delta_{N,N_1+N_2+N_3+N_4}\frac{N!}{N_1!N_2!N_3!N_4!}\times p_1^{N_1}p_2^{N_2}p_3^{N_3}p_4^{N_4}(p_1+p_2+p_3+p_4)^N. \tag{8}$$

We define new variables $c_i\equiv\ln(p_i)$, so that

$$\begin{aligned}Q&=\sum_{N_1,N_2,N_3,N_4}\delta_{N,N_1+N_2+N_3+N_4}\frac{N!\,e^{[c_1N_1+c_2N_2+c_3N_3+c_4N_4]}}{N_1!N_2!N_3!N_4!}\\&=(e^{c_1}+e^{c_2}+e^{c_3}+e^{c_4})^N.\end{aligned} \tag{9}$$

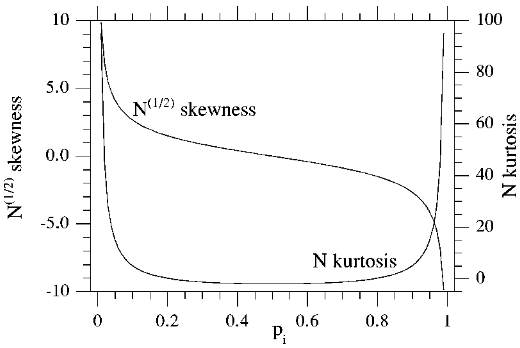

FIG. 1. Distribution functions $P_N(N_4)$ vs number of atoms N_4 for different mean number of particles in the 4WM wave packet.

Moments are generated by taking derivatives with respect to the c_i:

$$\langle N_i^p N_j^q \cdots \rangle \equiv \sum_{N_1,N_2,N_3,N_4} N_i^p N_j^q \cdots P_N(N_1,N_2,N_3,N_4) = \left(\frac{\partial^{p+q+\cdots}}{\partial c_i^p \partial c_j^q \cdots} Q \right) \Bigg|_{e^{c_1}+e^{c_2}+e^{c_3}+e^{c_4}=1}. \quad (10)$$

The condition $e^{c_1}+e^{c_2}+e^{c_3}+e^{c_4}=p_1+p_2+p_3+p_4=1$ is to be invoked *after* derivatives are taken. The general expression for moments of the population within wave packet i can be written as

$$\langle N_i^k \rangle = Np_i(1+[N-1]p_i(2+[N-2]p_i(\cdots(k+[n-k])))). \quad (11)$$

From the general expression (10), several useful particular results follow. The average population in each wave packet is

$$\langle N_i \rangle = Ne^{c_i} = Np_i, \quad (12)$$

and the covariances are

$$\langle (N_i-\langle N_i\rangle)(N_j-\langle N_j\rangle)\rangle = \begin{cases} Np_i(1-p_i), & i=j \\ -Np_ip_j, & i\neq j. \end{cases} \quad (13)$$

For $i=j$, the particle probability distribution function is the widest with respect to its mean when $p_i=0$ in the sense that $\langle (N_i-\langle N_i\rangle)^2\rangle/(\langle N_i\rangle)^2$ is maximum for $p_i\approx 0$. The probability distribution itself is the sharpest when $(\langle N_i\rangle/N)\approx 0$ or 1. Figure (1) plots $P_N(N_4)$ vs N_4 for $N=200$. The distributions are narrower for larger N. For $i\neq j$, Eq. (13) indicates there is a negative correlation between population fluctuations of different wave packets, as is expected from conservation of total number of particles.

Higher moments reveal departures of the population distributions from Gaussian. Here it is more useful to analyze the cumulants $\langle\langle N_i^p\rangle\rangle$ [11],

$$\langle\langle N_i^p \rangle\rangle \equiv \frac{\partial^p}{\partial c_i^p} \ln Q, \quad (14)$$

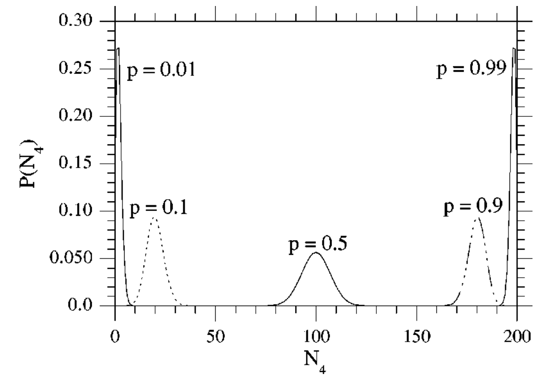

FIG. 2. Skewness and kurtosis of the wave packet population distribution $P_N(N_4)$ as a function of the mean number of particles in the 4WM wave packet.

rather than the moments of the probability distribution. The first few cumulants are given by

$$\langle\langle N_i \rangle\rangle = \langle N_i \rangle = Np_i, \quad (15)$$

$$\langle\langle N_i^2 \rangle\rangle = \langle (N_i-\langle N_i\rangle)^2\rangle = Np_i(1-p_i), \quad (16)$$

$$\langle\langle N_i^3 \rangle\rangle = \langle (N_i-\langle N_i\rangle)^3\rangle = Np_i(1-p_i)(1-2p_i), \quad (17)$$

$$\langle\langle N_i^4 \rangle\rangle = \langle (N_i-\langle N_i\rangle)^4\rangle - 3\langle (N_i-\langle N_i\rangle)^2\rangle^2 = Np_i(1-p_i)(1-6p_i+6p_i^2). \quad (18)$$

The first and second cumulants are the average and variance. The third and fourth cumulants are closely related to what is commonly called the skewness and kurtosis of the distribution [12]:

$$\Phi\ (\text{skewness}) = \frac{\langle\langle N_i^3\rangle\rangle}{\langle\langle N_i^2\rangle\rangle^{3/2}} = \frac{1-2p_i}{\sqrt{Np_i(1-p_i)}}, \quad (19)$$

$$\Phi\ (\text{kurtosis}) = \frac{\langle\langle N_i^4\rangle\rangle}{\langle\langle N_i^2\rangle\rangle^{2}} = \frac{1-6p_i+6p_i^2}{Np_i(1-p_i)}. \quad (20)$$

Skewness measures the departure of the distribution from symmetrical, while kurtosis measures the flatness of the top of the distribution. Both quantify how different the distribution is from Gaussian, and both are zero for a Gaussian distribution. Figure (2) plots $\sqrt{N}\Phi(\text{skewness})$ and $N\Phi(\text{kurtosis})$ versus p_i; these quantities are independent of N. The figure shows that the wave packet population distribution tends toward Gaussian with increasing N, especially away from the limits $p_i\rightarrow 0$ or 1. As shown in Fig. 2, the skewness indicates that the population tails toward high N_i when $p_i<1/2$ and toward low N_i when $p_i>1/2$, and the kurtosis indicates that the distribution is slightly flatter than Gaussian $[\Phi(\text{kurtosis})<0]$ when $p_i\approx 1/2$ and more spiked than Gaussian $[\Phi(\text{kurtosis})>0]$ in the limits of $p_i\rightarrow 0$ or 1.

When the total number of particles N is a random variable, the results we have obtained for the statistics of the atoms in the various wave packets must be folded with the statistics $P_i(N)$ of N initial bosons. The probabilities

$P(N_1,N_2,N_3,N_4)$ and $P(N_4)$ are then obtained as $P(N_1,N_2,N_3,N_4)=\Sigma_N P_i(N) P_N(N_1,N_2,N_3,N_4)$ and $P(N_4) = \Sigma_N P_i(N) P_N(N_4)$, respectively.

In the above discussion we described fluctuations of atomic boson systems. It is both interesting and instructive to mention the difference between atomic and photonic systems, especially in view of the close analogy between 4WM with BECs and 4WM in nonlinear optics. The difference is due to superselection rules [13] and conservation of barionic number in bosonic matter wave systems. In order to understand the relationship between the statistics of 4WM of matter waves and of photons in a nonlinear dispersive medium, it is instructive to consider what a beam splitter does to matter waves (i.e., an atomic beam splitter) and compare it to a photonic beam splitter. As we shall see, the difference arises not in the action of the beam splitter, which leads to the equivalent of the factor $P_N(N_4)$ given above, but in the initial probability distribution $P_i(N)$.

IV. BEAM-SPLITTER ANALOGY

To illustrate the difference between statistics of photonic and matter 4WM, we consider a simple model of a beam splitter for photons or for bosonic atoms. (The application of Bragg pulses to output couple part of a BEC may be viewed as a realization of a single-input atomic beam splitter.) Consider a beam splitter having two input and two output ports. The annihilation (and creation) operators for (photon or bosonic atom) particles in the input and output ports are related as follows:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \sqrt{p} & \sqrt{1-p} \\ -\sqrt{1-p} & \sqrt{p} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \tag{21}$$

where p is the probability of particles being directed from the first input port to the first output port. The input annihilation operators are denoted by b_1 and b_2, and the output annihilation operators by a_1 and a_2, respectively. Note that we have taken the probability *amplitudes* to be real; in general the matrix elements appearing on the right-hand side of Eq. (21) are complex. An input field can be injected into each port. The probability distribution for the number of particles in the jth output port is in general given by

$$P(N) = \langle \delta(N - a_j^\dagger a_j) \rangle. \tag{22}$$

Let us consider the case of electromagnetic input fields that are coherent states, and for simplicity, we (a) consider only one mode of the field being populated, and (b) take the initial input into the second port of the beam splitter to be the vacuum field. The product coherent input state is therefore given by $|\beta,0\rangle$ where β is the complex coherence coefficient for the first input port and 0 is the complex coherence coefficient for the second input port. The action of the input annihilation operators on the input state is given by

$$b_1|\beta,0\rangle = \beta|\beta,0\rangle,$$

$$b_2|\beta,0\rangle = 0. \tag{23}$$

To evaluate the average in Eq. (22) for the case of a coherent electromagnetic input field as defined here, we use the Fourier expansion of the delta function and the well-known expression of the exponential function in the normal-ordering form,

$$\exp[-i\xi a_1^\dagger a_1] = :\exp[(e^{-i\xi}-1)a_1^\dagger a_1]:, \tag{24}$$

where $:O:$ denotes normal ordering of an operator O. Since normal ordering of the a_i operators yields normal ordering of the b_i operators, when we take the expectation value in the equation after introducing normal ordering according to the above prescription, all the operators can be substituted by their expectation values. By expanding and integrating, term by term, we obtain

$$P(N) = \frac{(p\beta^2)^N}{N!} \exp(-p\beta^2). \tag{25}$$

Photons in the coherent state obey Poisson statistics and initial probability $P_i(n)$ for measuring n photons in the first port, before it passes through the beam splitter is therefore equal to

$$P_i(n) = \frac{\beta^{2n}}{n!} \exp(-\beta^2). \tag{26}$$

In a fashion similar to what was mentioned at the end of the previous section, the probability distribution after passing through the beam splitter is a convolution of the initial statistics and statistics of the beam splitter:

$$\begin{aligned} P(N) &= \sum_{n=N}^{\infty} P_i(n) \binom{n}{N} p^N (1-p)^{n-N} \\ &= \exp(-\beta^2) \frac{(\beta^2 p)^N}{N!} \sum_{n=N}^{\infty} \frac{[\beta^2(1-p)]^{n-N}}{(n-N)!}. \end{aligned} \tag{27}$$

This yields the result presented in Eq. (25). Hence, the statistics of photons passing though a beam splitter remains Poissonian, but the mean value $\langle N \rangle$ is lowered.

On the other hand, in the atomic bosonic case discussed in Sec. II, the initial zero temperature state of the BEC is a Fock state, with a well-defined number of particles. Hence, after application of the Bragg pulses to split the condensate, the particle distribution is binomial for bosonic atoms. In terms of our simple model of the beam splitter, we calculate the average in Eq. (22) using Fock states instead of coherent state:

$$\begin{aligned} P(N) &= \int_0^{2\pi} \frac{d\xi}{2\pi} \langle N,0| \exp[-i\xi a_1^\dagger a_1] |N,0\rangle \\ &= \binom{n}{N} p^n (1-p)^{N-n}. \end{aligned} \tag{28}$$

Our simple model of the beam splitter points out one of the differences in the statistics with photons and with coherent matter waves. The distribution $P_i(N)$ is in general different for the two cases. Another important difference arises from the fact that the nonlinearity in the mean-field approximation for BECs is not associated with any external medium. In contrast, for 4WM of electromagnetic fields in a dispersive nonlinear medium, the medium itself introduces a source of thermal fluctuations that may significantly affect the statistics of the measurements of particle number. A third potential source of difference is the fluctuations of the laser fields that produce the separate momentum wave packets of the BEC. The statistics of the Bragg pulses used to produce the high-momentum wave packets can introduce fluctuations in the number of atoms in the 4WM wave packet.

V. SUMMARY AND CONCLUSION

In summary, we have determined the fluctuations of the number of atoms in BEC wave packets produced in the 4WM of matter waves within the mean-field approximation. We showed that the number of atoms in the 4WM wave packet is given by a binomial distribution that, by the central-limit theorem, reduces to a Gaussian distribution in the strong 4WM conversion limit. We pointed out that skewness and kurtosis can be measured to establish the nature of the distribution of atoms in the various momentum wave packets. We discussed the possible differences of the statistics of 4WM for photons in nonlinear dispersive media and for coherent matter waves. The fluctuations described here will be difficult to observe in experiment, but their significance should not be underestimated, since their understanding is important for problems associated with entanglement, and because they provide a fundamental quantum limit. Finally, we emphasize that the statistical properties discussed here may be directly applied to any process in which the separation of the condensate into distinct subsystems occurs, e.g., the use of a sequence of optical light pulses to produce high-momentum component wave pacekts by Bragg scattering [3,4].

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