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Coherent Output, Stimulated Quantum Evaporation, and Pair Breaking in a Trapped Atomic Bose Gas

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We investigate the spectrum and coherence of an atomic beam slowly coupled out of an atomic trap which contains a partially condensed Bose gas at a finite temperature. The spectrum contains a coherent fraction emerging from the condensate and a thermal fraction emerging from the thermal excitations in the trap. We show the existence of a remarkable process involving the simultaneous creation of an output atom and an elementary excitation (quasiparticle) inside the trap. This process, which can serve as a probe to pair correlations in the condensate, can become dominant for a suitable choice of the coupling parameters. [S0031-9007(98)08353-7]

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The experimental realization of Bose-Einstein condensation of evaporatively cooled trapped alkali atoms [1–3] makes possible the generation of coherent atomic beams [4]. Previous papers have addressed the output coupling of a single-mode noninteracting condensate [5] or an interacting trapped condensate at $T = 0$ [6,7]. Under these conditions the output beam is a coherent matter wave packet which can be described by a single complex function of space and time. However, at finite temperatures one expects thermal excitations to play a major role. On the one hand, it is of interest to quantify and optimize the coherence properties of the output coupled atomic beam, as in the case of continuous wave and pulsed optical laser sources. On the other hand, an analysis of the properties of the output may serve as a probe of the nature of the ground state and excitations in the atomic gas, as quantum evaporation from the surface of ^{4}He is used to probe its internal dynamics [8]. Here we study a weak output coupling of a trapped atomic gas at finite temperatures below the critical temperature T_c. A qualitative analysis and a numerical illustration of the general analytical results reveal the following three mechanisms contributing to the output coupling: (i) coherent coupling of the condensate fraction, (ii) stimulated quantum evaporation of the thermal excitations, and (iii) the simultaneous appearance of an output atom and an internal excitation, indicating the existence of pair correlations in the ground state. The last process predicted here occurs even at $T = 0$ and may serve as a probe of the deviation of the ground state from a direct product of single atom states.

Here we consider output coupling by electromagnetically (EM) induced transition between the trapped atomic level $|t\rangle$ and a free output level $|f\rangle$ with different angular momentum, which is not confined by the trap. Typical mechanisms are a direct (one-photon) radio-frequency (rf) transition and an indirect (two-photon) stimulated Raman transition. In the dipole and rotating wave approximation the EM coupling mechanism is described by the following Hamiltonian:

$$H_{\mathrm{couple}} = \hbar \int d^3\mathbf{r}\, \lambda(\mathbf{r},t)\hat{\psi}_f^\dagger(\mathbf{r})\hat{\psi}_t(\mathbf{r}) + \mathrm{H.c.} \quad (1)$$

Here $\hat{\psi}_t(\mathbf{r})$ describes the annihilation of a trapped atom at $\mathbf{r}$, while $\hat{\psi}_f^\dagger(\mathbf{r})$ describes the creation of a free atom at the same point with amplitude $\lambda(\mathbf{r},t)$. In the EM induced processes the coupling amplitude can be written as $\lambda(\mathbf{r},t) = \tilde{\lambda}(\mathbf{r},t)e^{i(\mathbf{k}_{\mathrm{EM}}\cdot\mathbf{r}-\Delta_{\mathrm{EM}}t)}$ where $\tilde{\lambda}$ is slowly varying in space and time. Here $\hbar\mathbf{k}_{\mathrm{EM}}$ and $\hbar\Delta_{\mathrm{EM}}$ measure the momentum and energy transfer from the EM field to an output atom. In an rf coupling scheme, λ is the Rabi frequency $\Omega(\mathbf{r},t) = \langle\hat{p}\rangle\mathcal{E}(\mathbf{r},t)/\hbar$ [or $\langle\hat{\mu}\rangle\mathcal{B}(\mathbf{r},t)/\hbar$] corresponding to the flipping of the atomic electric (or magnetic) dipole $\langle\hat{p}\rangle$ (or $\langle\hat{\mu}\rangle$) in the electric (or magnetic) field $\mathcal{E}(\mathbf{r},t)$ [$\mathcal{B}(\mathbf{r},t)$], Δ_{EM} is the detuning of the EM field frequency from the transition frequency, and $\mathbf{k}_{\mathrm{EM}}$ is negligible compared to the initial momentum distribution of the atoms. In the stimulated Raman coupling, where two laser beams are used to induce a transition from $|t\rangle$ to $|f\rangle$ through an intermediate level $|i\rangle$, $\lambda = \Omega_1\Omega_2/\Delta_i$, where $\Omega_{1,2}$ are the Rabi frequencies corresponding to the intermediate transitions and Δ_i is their detuning from resonance with the two beams [9]. In this case Δ_{EM} and $\mathbf{k}_{\mathrm{EM}}$ are the differences between the frequencies and momenta associated with the two laser beams.

Here we consider a small rate of output from the trap, so that the output atoms are dilute enough to neglect the interactions between them outside the trap. It is still necessary to take into account their interaction with the trapped atomic gas left behind. The coupled equations for the field operators $\hat{\psi}_t$ and $\hat{\psi}_f$ take the form

$$i\hbar\dot{\hat{\psi}}_t(\mathbf{r}) = \mathcal{L}_t\hat{\psi}_t(\mathbf{r}) + U_0\hat{\psi}_t^{\dagger}(\mathbf{r})\hat{\psi}_t(\mathbf{r})\hat{\psi}_t(\mathbf{r}) + \hbar\lambda^*(\mathbf{r},t)\hat{\psi}_f(\mathbf{r}), \tag{2}$$

$$i\hbar\dot{\hat{\psi}}_f(\mathbf{r}) = \mathcal{L}_f\hat{\psi}_f(\mathbf{r}) + \hbar\lambda(\mathbf{r},t)\hat{\psi}_t(\mathbf{r}). \tag{3}$$

Here $\mathcal{L}_{t,f} \equiv -\hbar^2\nabla^2/2m + V_{t,f}(\mathbf{r})$, where $V_t(\mathbf{r})$ is the magnetic trapping potential, $V_f(\mathbf{r}) = \frac{1}{2}U_1\langle\hat{\psi}_t^{\dagger}(\mathbf{r})\hat{\psi}_t(\mathbf{r})\rangle$ is the effective potential induced by the collisions with the trapped gas, and $U_i = \frac{4\pi\hbar^2}{m}A_i$ are proportional to the s-wave scattering lengths A_i for collisions between the atoms. Equations (2) and (3) have been previously treated [6,7] in the Gross-Pitaevskii approximation, where the field operators are replaced by their mean values. This approximation neglects the existence of elementary excitations at finite temperatures and the fluctuations around the mean value even at $T = 0$, which are shown below to have an important effect.

The formal solution of Eq. (3) for $\hat{\psi}_f$ in terms of $\hat{\psi}_t$ is

$$\hat{\psi}_f(\mathbf{r},t) = \hat{\psi}_f^{(0)}(\mathbf{r},t) - i\int_0^t dt'\int d^3\mathbf{r}'\,G_f(\mathbf{r},\mathbf{r}',t-t')\lambda(\mathbf{r}',t')\hat{\psi}_t(\mathbf{r}',t'), \tag{4}$$

where $\hat{\psi}_f^{(0)}$ satisfies the time-dependent Schrödinger equation $i\hbar\dot{\hat{\psi}}_f^{(0)} = \mathcal{L}_f\hat{\psi}_f^{(0)}$ and the "free" Green's function $G_f(\mathbf{r},\mathbf{r}',t-t')$ can be written in terms of the solutions $\varphi_{\mathbf{k}}(\mathbf{r})$ of the corresponding time-independent equation $\hbar\omega_{\mathbf{k}}\varphi_{\mathbf{k}}(\mathbf{r}) = \mathcal{L}_f\varphi_{\mathbf{k}}(\mathbf{r})$: $G_f(\mathbf{r},\mathbf{r}',t-t') = \sum_{\mathbf{k}}\varphi_{\mathbf{k}}(\mathbf{r})\varphi_{\mathbf{k}}^*(\mathbf{r}')e^{-i\omega_{\mathbf{k}}(t-t')}\theta(t-t')$. The field operator $\hat{\psi}_t(\mathbf{r},t)$ in thermal equilibrium can be written in terms of the annihilation operators $\hat{\alpha}_0$ of the condensate state and $\hat{\alpha}_j$ of the elementary excitations, obtained from a Hartree-Fock-Bogoliubov approximation that puts the many-body Hamiltonian of the interacting system into a diagonal form [10]

$$\hat{\psi}_t(\mathbf{r},t) = e^{-i\mu t/\hbar}\Big\{\Psi_0^t(\mathbf{r})\hat{\alpha}_0 + \sum_j[u_j^t(\mathbf{r})\hat{\alpha}_j - v_j^{t*}(\mathbf{r})\hat{\alpha}_j^{\dagger}]\Big\}. \tag{5}$$

Here μ is the chemical potential, $\Psi_0^t(\mathbf{r})$ is the wave function of condensate atoms, and $u_j^t(\mathbf{r}), v_j^t(\mathbf{r})$ are the steady-state solution of the equations obtained from Eq. (2) in the absence of output coupling [10]. In thermal equilibrium $\hat{\alpha}_0$ can be replaced by $\sqrt{N_0}$, N_0 being the macroscopic mean number of condensate atoms [11], and the population of the excited modes is given by the Bose-Einstein distribution $N_j^{eq} \equiv \langle\hat{\alpha}_j^{\dagger}\hat{\alpha}_j\rangle_{eq} = [e^{\hbar\omega_j/T} - 1]^{-1}$. Equation (5) implies that the annihilation of a trapped particle at $\mathbf{r}$ is equivalent to the annihilation of a particle in the condensate [with amplitude $\Psi_0^t(\mathbf{r})$] or the annihilation of an elementary excitation (with amplitude u_j^t) or the *creation* of a new excitation (with amplitude $-v_j^{t*}$). The last amplitude is associated with the noncondensate component of the ground state of the system, which contains entangled pairs of correlated atoms in excited single-particle states, formed by binary collisions. The net effect of this annihilation, whose traces can be clearly seen in the output spectrum, as shown below, is the creation of a new quantum of excitation while the total number of trapped atoms is reduced by one. The wave functions $u_j^t(\mathbf{r})$ of a dilute weakly interacting Bose gas are nearly the energy eigenfunctions $\phi_n(\mathbf{r})$ of a single particle in the effective trapping potential $V_t(\mathbf{r}) + 2U_0N_0|\Psi_0^t(\mathbf{r})|^2$, whereas an expression for the functions $v_j^t(\mathbf{r})$ in terms of $u_j^t(\mathbf{r})$ is obtained by a formal solution of the equation for v_j^t in Ref. [10]

$$v_j(\mathbf{r}) = U_0N_0\sum_n\frac{\int d^3\mathbf{r}'\phi_n^*(\mathbf{r}')[\Psi_0^*(\mathbf{r}')]^2u_j^t(\mathbf{r}')}{E_n + \hbar\omega_j}\,\phi_n(\mathbf{r}), \tag{6}$$

implying that the function $-v_j^*(\mathbf{r})$ describes collisional scattering of two atoms in the condensate into two excited states. Such virtual processes are the essence of pairing effects in the ground state of a Bose gas.

Here we assume that the output rate is small compared to the energy separation $\delta\omega_j$ between the trap states. In this case the energies and wave functions of the condensate and elementary excitations do not vary significantly from their initial equilibrium values and the main contribution to the time dependence of $\hat{\psi}_t(\mathbf{r},t)$ comes from the operators $\hat{\alpha}_0, \hat{\alpha}_j$. Moreover, for short times we may assume that the system stays close to equilibrium and the population of the trap states decays exponentially with decay coefficients γ_0, γ_j. For $\delta\omega_j^{-1} \ll t \ll \gamma_0^{-1}, \gamma_j^{-1}$ we thus assume $\hat{\alpha}_0(t) = \hat{\alpha}_0(0)e^{-\gamma_0 t/2}$ and

$\hat{\alpha}_j(t) = \hat{\alpha}_j(0)e^{-i\omega_j t}e^{-\gamma_j t/2}$ [12], where γ_0, γ_j are estimated below. As shown below the exponential decay assumption is not always correct, and, in some cases, the number of excitations may even grow.

If at $t = 0$ all the atoms are in the trap, then, following from Eq. (4) with the free term $\hat{\psi}_t^{(0)}$ set to zero, we many express the field operator $\hat{\psi}_f(\mathbf{r}, t)$ of the free atoms at $t > 0$ in terms of the operators $\hat{\alpha}_0, \hat{\alpha}_j, \hat{\alpha}_j^\dagger$ appearing in Eq. (5). Then $\hat{\psi}_f$ has the form $\hat{\psi}_f = \Psi_0^f \hat{\alpha}_0 + \sum_j [u_j^f \hat{\alpha}_j - v_j^{f*} \hat{\alpha}_j^\dagger]$, where the components ξ^f of the vector $\vec{\xi}^f \equiv (\Psi_0^f, u_j^f, -v_j^{f*})$ are obtained from Eqs. (4) and (5) in terms of the corresponding components of $\vec{\xi}^t(\mathbf{r}) \equiv (\Psi_0^t, u_j^t, -v_j^{t*})$. An expansion in eigenfunctions $\varphi_\mathbf{k}(\mathbf{r})$ of a free atom gives

$$\xi^f(\mathbf{r}, t) = \sum_\mathbf{k} \tilde{\xi}_\mathbf{k}^f \varphi_\mathbf{k}(\mathbf{r}) e^{-i\omega_\mathbf{k} t} \frac{e^{i(\omega_\mathbf{k} - \omega^f - i\gamma^f/2)t} - 1}{\omega_\mathbf{k} - \omega^f - i\gamma^f/2}, \quad (7)$$

where

$$\tilde{\xi}_\mathbf{k}^f = \int d^3\mathbf{r}\, \varphi_\mathbf{k}^*(\mathbf{r}) \tilde{\lambda}(\mathbf{r}) e^{i\mathbf{k}_{\mathrm{EM}}\cdot\mathbf{r}} \xi^t(\mathbf{r}). \quad (8)$$

In Eq. (7) the frequencies ω^f are the components of the vector $\vec{\omega}^f = (\omega_0^f, \omega_{j+}^f, \omega_{j-}^f)$, where

$$\hbar\omega_0^f = \mu + \hbar\Delta_{\mathrm{EM}}, \qquad \hbar\omega_{j\pm}^f = \mu + \hbar\Delta_{\mathrm{EM}} \pm \hbar\omega_j. \quad (9)$$

After some time t the mean number of output atoms in a state with momentum $\mathbf{k}$ is given by $n_\mathbf{k} = \langle \hat{b}_\mathbf{k}^\dagger \hat{b}_\mathbf{k} \rangle$, where $\hat{b}_\mathbf{k} \equiv \int d^3\mathbf{r}\, \varphi_\mathbf{k}^*(\mathbf{r}) \hat{\psi}_f(\mathbf{r})$. From the form of the output operator $\hat{\psi}_f(\mathbf{r})$ and Eq. (7) we obtain

$$\begin{aligned} n_\mathbf{k}(t) = {} & |\tilde{\Psi}_{0\mathbf{k}}^f|^2 N_0^{eq} \mathcal{D}(\omega_\mathbf{k} - \omega_0^f, \gamma_0/2) \\ & + \sum_j [|\tilde{u}_{j\mathbf{k}}^f|^2 N_j^{eq} \mathcal{D}(\omega_\mathbf{k} - \omega_{j+}^f, \gamma_j/2) \\ & + |\tilde{v}_{j\mathbf{k}}^{f*}|^2 (N_j^{eq} + 1) \mathcal{D}(\omega_\mathbf{k} - \omega_{j-}^f, \gamma_j/2)], \end{aligned} \quad (10)$$

Here the time-dependent spectral line shapes $\mathcal{D}(\omega, \gamma/2) = \frac{|1 - e^{i\omega t} e^{-\gamma t/2}|^2}{\omega^2 + \gamma^2/4}$ tend to Lorentzians of width $\gamma/2$ in the limit $t \gg \gamma^{-1}$. However, the treatment here applies only to times $t \ll \gamma^{-1}$, where $\mathcal{D} \approx \sin^2(\omega t)/\omega^2$ has spectral width $\sim 1/t$, which may become narrow relative to the scale of variation of the functions $|\tilde{\xi}_\mathbf{k}^f|^2$ appearing in Eq. (10). Then

$$\mathcal{D}(\omega, \gamma) \approx 2\pi\delta(\omega)t \quad (11)$$

and the output rate $dn_\mathbf{k}/dt$ becomes constant in time.

The first term in Eq. (10) describes a coherent output component generated when atoms with energy μ in the trapped condensate are excited to a free momentum state with kinetic energy $\hbar\omega_\mathbf{k} = \mu + \hbar\Delta_{\mathrm{EM}}$ by absorbing energy $\hbar\Delta_{\mathrm{EM}}$ from the EM field. The second term describes a thermal output component generated by stimulated quantum evaporation, where an atom initially populating an elementary excitation with energy $\mu + \hbar\omega_j$ leaves the trap with kinetic energy $\hbar\omega_\mathbf{k} = \mu + \hbar\omega_j + \hbar\Delta_{\mathrm{EM}}$. The third term describes a process where the energy $\hbar\Delta_{\mathrm{EM}}$ from the EM field is sufficient to excite a ground state atom to a free momentum state with kinetic energy $\mu + \hbar\Delta_{\mathrm{EM}} - \hbar\omega_j$, thereby leaving a counterpart atom in an excited trap state with energy $\mu + \hbar\omega_j$. While a quantum of elementary excitation is annihilated from the trap in the second process of quantum evaporation, the third term describes a process where a new quantum of elementary excitation (a quasiparticle) is created. The factor $N_j^{eq} + 1$ in this term implies that this process occurs also in the absence of excitations (at $T = 0$), but it is amplified by the presence of excitations. According to the interpretation following Eq. (5) we may anticipate that this process points to the existence of pairs of correlated atoms in the ground state in the trap, which are broken when one atom is forced out. This process occurs also in the limit of weak (adiabatic) coupling and therefore it should be differentiated from nonadiabatic processes that may contribute to the generation of new elementary excitations [13].

In the limit where Eq. (11) holds, the above interpretation of the processes described by Eq. (10) leads to the following approximation for the exponential decay rates:

$$\gamma_0 = 2\pi \sum_\mathbf{k} |\tilde{\Psi}_{0\mathbf{k}}|^2 \delta(\omega_\mathbf{k} - \omega_0^f), \quad (12)$$

$$\gamma_j = 2\pi \sum_\mathbf{k} |\tilde{u}_{j\mathbf{k}}|^2 \delta(\omega_\mathbf{k} - \omega_{j+}^f) - \eta_j, \quad (13)$$

where $\eta_j = \sum_\mathbf{k} |\tilde{v}_\mathbf{k}^*|^2 \delta(\omega_\mathbf{k} - \omega_{j-}^f)$ is the rate of creation of the jth excitation. When this last process is dominant γ_j may become negative and the number of excitations grows. A more detailed analysis of the dynamics will be given elsewhere [14].

Energy conservation requires $\hbar\omega_\mathbf{k} = \mu \pm \hbar\omega_j + \hbar\Delta_{\mathrm{EM}}$ and momentum conservation follows from Eq. (8). In the first two processes $\mathbf{k}_f = \mathbf{k}_t + \mathbf{k}_{\mathrm{EM}}$, i.e., the momentum $\mathbf{k}_t$ of an initially trapped atom plus the momentum $\mathbf{k}_{\mathrm{EM}}$ supplied by the EM field equals the momentum $\mathbf{k}_f$ of a free output atom at the trap, before it is accelerated by the repulsive interaction with the atoms left in the trap. Far from the trap its momentum is given by $|\mathbf{k}| \approx \sqrt{k_f^2 + mU_1N/V}$, $U_1N/2V$ being the effective repulsive potential. In the third process $\mathbf{k}_f + \mathbf{k}_t = \mathbf{k}_{\mathrm{EM}}$, implying that the total momentum of the resulting output atom-quasiparticle pair is equal to the momentum supplied by the field. The contribution of a specific term in Eq. (10) to the output rate of atoms with a corresponding asymptotic kinetic energy $\hbar^2k^2/2m = \mu \pm \hbar\omega_j + \hbar\Delta_{\mathrm{EM}}$ is proportional to the number of atoms with momentum $\hbar\mathbf{k}_t$ satisfying $\hbar^2(\mathbf{k}_t \pm \mathbf{k}_{\mathrm{EM}})^2/2m + U_1N/2V = \mu \pm \hbar\omega_j + \hbar\Delta_{\mathrm{EM}}$ in the initial momentum distribution of the corresponding trap state and to the density of free states with the corresponding output momentum. The main contribution of the condensate wave function Ψ_0^* is near $\mathbf{k}_t = 0$, corresponding to an EM field detuning $\hbar\Delta_{\rm EM} = \hbar\omega_{\mathbf{k}} - \mu \sim U_1 N/2V + \hbar^2 k_{\rm EM}^2/2m - \mu$. The width of this distribution as $\Delta_{\rm EM}$ is varied is governed by the width Δk_t of the initial atomic distribution and the additional momentum $\mathbf{k}_{\rm EM}$. The contribution of processes revealed in the second and third terms in Eq. (10) is dominant at field detuning $\hbar\Delta_{\rm EM} \sim U_1 N/2V + \hbar^2 k_{\rm EM}^2/2m - \mu \pm \hbar\omega_j$. By tuning the value of $\Delta_{\rm EM}$ we can select one of the three processes discussed above to be dominant. Low (or negative) values of $\Delta_{\rm EM}$ give rise to a thermal output leading to an evaporative cooling of the trapped gas. High values of $\Delta_{\rm EM}$ give rise to an increase in the number of excitations. Output atomic beams with the best coherence properties are expected for medium values of $\Delta_{\rm EM}$, where the condensate atoms are coupled most efficiently out of the trap.

The output rates corresponding to the different terms are demonstrated in Fig. 1 for a one-dimensional system of $N = 2000$ atoms in a harmonic trapping potential of frequency $\omega_{\rm trap}$ and a repulsive interaction strength $NU_0 = NU_1 = 10\sqrt{\hbar^3\omega_{\rm trap}/2m}$. The corresponding critical temperature is $T_c \approx 300\hbar\omega_{\rm trap}$.

The coherence properties of the output atomic beam are characterized by the correlation functions of the output field operator like the first order coherence function $g^{(1)}(\mathbf{r},\mathbf{r}',t) = \langle\hat{\psi}_f^\dagger(\mathbf{r},t)\hat{\psi}_f(\mathbf{r}',t)\rangle$. The output beam at a finite temperature is a mixture of quasimonochromatic partial beams whose energies are given in Eq. (9). We thus have $g^{(1)}(\mathbf{r},\mathbf{r}',t) = N_0\Psi_0^{f*}(\mathbf{r})\Psi_0^f(\mathbf{r}') + \sum_j[N_j \times u_j^{f*}(\mathbf{r})u_j^f(\mathbf{r}') + (N_j + 1)v_j^f(\mathbf{r})v_j^{f*}(\mathbf{r}')]$. Far from the trap, each component $\xi^f(\mathbf{r})$ [see Eq. (7)] is roughly given by a wave of finite spatial extent $\xi^f(\mathbf{r}) \sim \theta(r - v_f t)\sum_{\mathbf{k}}\delta(\hbar\mathbf{k}^2/2m - \omega_{\mathbf{k}})\xi_{\mathbf{k}}^f e^{i\mathbf{k}\cdot\mathbf{r}}$, where $v_f = \sqrt{2\hbar\omega^f/m}$. The two-point coherence at $\mathbf{r},\mathbf{r}'$ is determined by the number of terms with $v_f > r/t$ and $v_f > r'/t$ appearing in $g^{(1)}$. The coherence is maximal if only one of the terms is dominant. Higher order coherence involves higher order correlation functions. If the term originating from the condensate is dominant at a point $\mathbf{r}$, then the beam is expected to be coherent to a high order, while it will not be coherent if terms originating in thermal excitations are significant. The significance of the different terms is visualized in Fig. 1. It shows that the best separation between the different terms is achieved for $\mathbf{k}_{\rm EM} = 0$, where the width of the output rates as a function of $\Delta_{\rm EM}$ reflects the initial momentum distribution inside the trap.

Optimal coherence properties may be achieved by tuning the spatial shape of $\tilde{\lambda}(\mathbf{r})$ in Eq. (8) by focusing the laser beams used in the stimulated Raman transition to overlap with the condensate wave function $\Psi_0(\mathbf{r})$.

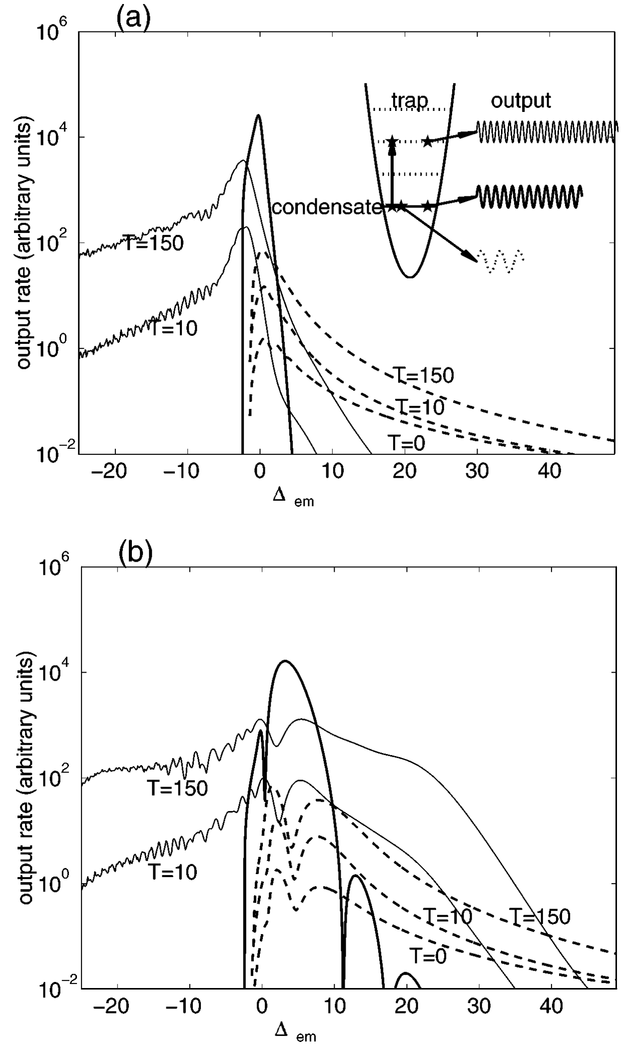

FIG. 1. The relative output rate from the condensate (thick solid), thermal excitations (thin solid), and pair breaking (dashed) for 2000 atoms in a harmonic 1D trapping potential as a function of the detuning $\Delta_{\rm EM}$ of the coupling EM field from the atomic transition. The detuning and temperatures are given in units of the trap frequency $\omega_{\rm trap}$ for (a) $k_{\rm EM} = 0$ and (b) $k_{\rm EM} = 2\sqrt{2m\omega_{\rm trap}/\hbar}$. A constant density of free states was used. Inset: illustration of the main effects contributing to the output.

To conclude, this Letter shows that the measurement of the spectrum of a weakly coupled output as a function of the coupling parameters may be an excellent tool for the analysis of the quantum state of a trapped atomic Bose gas at a finite temperature. The remarkable effect of output from pair breaking in the ground state demonstrates that the output spectrum may as well reveal properties of the condensate that are not included in mean field theories.

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